

Exercise 14.2

1. The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Solution:

To find out the modal class, let us consider the class interval with high frequency

Here, the greatest frequency = 23, so the modal class = 35 – 45,

$$l = 35,$$

$$\text{class width (h)} = 10,$$

$$f_m = 23,$$

$$f_1 = 21 \text{ and } f_2 = 14$$

The formula to find the mode is

$$\text{Mode} = l + [(f_m - f_1) / (2f_m - f_1 - f_2)] \times h$$

Substitute the values in the formula, we get

$$\text{Mode} = 35 + [(23 - 21) / (46 - 21 - 14)] \times 10$$

$$\text{Mode} = 35 + (20 / 11) = 35 + 1.8$$

$$\text{Mode} = 36.8 \text{ year}$$

So the mode of the given data = 36.8 year

Calculation of Mean:

First find the midpoint using the formula, $x_i = (\text{upper limit} + \text{lower limit}) / 2$

Class Interval	Frequency (f_i)	Mid-point (x_i)	$f_i x_i$
5-15	6	10	60
15-25	11	20	220
25-35	21	30	630
35-45	23	40	920
45-55	14	50	700
55-65	5	60	300
	Sum $f_i = 80$		Sum $f_i x_i = 2830$

The mean formula is

$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= 2830 / 80$$

$$= 35.37 \text{ years}$$

Therefore, the mean of the given data = 35.37 years

2. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Lifetime (in hours)	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Solution:

From the given data the modal class is 60–80.

$$l = 60,$$

The frequencies are:

$$f_m = 61, f_1 = 52, f_2 = 38 \text{ and } h = 20$$

The formula to find the mode is

$$\text{Mode} = l + \left[\frac{(f_m - f_1)}{(2f_m - f_1 - f_2)} \right] \times h$$

Substitute the values in the formula, we get

$$\text{Mode} = 60 + \left[\frac{(61 - 52)}{(2 \times 61 - 52 - 38)} \right] \times 20$$

$$\text{Mode} = 60 + \left(\frac{9 \times 20}{32} \right)$$

$$\text{Mode} = 60 + \left(\frac{45}{8} \right) = 60 + 5.625$$

Therefore, modal lifetime of the components = 65.625 hours.

3. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure:

Expenditure	Number of families
1000-1500	24
1500-2000	40
2000-2500	33
2500-3000	28
3000-3500	30
3500-4000	22
4000-4500	16
4500-5000	7

Solution:

Given data:

Modal class = 1500-2000,

$$l = 1500,$$

Frequencies:

$$f_m = 40, f_1 = 24, f_2 = 33 \text{ and}$$

$$h = 500$$

Mode formula:

$$\text{Mode} = l + \left[\frac{(f_m - f_1)}{(2f_m - f_1 - f_2)} \right] \times h$$

Substitute the values in the formula, we get

$$\text{Mode} = 1500 + \left[\frac{(40 - 24)}{(80 - 24 - 33)} \right] \times 500$$

$$\text{Mode} = 1500 + \left(\frac{16 \times 500}{23} \right)$$

$$\text{Mode} = 1500 + \left(\frac{8000}{23} \right) = 1500 + 347.83$$

Therefore, modal monthly expenditure of the families = Rupees 1847.83

Calculation for mean:

First find the midpoint using the formula, $x_i = (\text{upper limit} + \text{lower limit})/2$

Let us assume a mean, A be 2750

Class Interval	f_i	x_i	$d_i = x_i - a$	$u_i = d_i/h$	$f_i u_i$
1000-1500	24	1250	-1500	-3	-72
1500-2000	40	1750	-1000	-2	-80
2000-2500	33	2250	-500	-1	-33
2500-3000	28	2750	0	0	0
3000-3500	30	3250	500	1	30
3500-4000	22	3750	1000	2	44
4000-4500	16	4250	1500	3	48
4500-5000	7	4750	2000	4	28
	$f_i = 200$				$f_i u_i = -35$

The formula to calculate the mean,

$$\text{Mean} = \bar{x} = a + (\sum f_i u_i / \sum f_i) \times h$$

Substitute the values in the given formula

$$= 2750 + (-35/200) \times 500$$

$$= 2750 - 87.50$$

$$= 2662.50$$

So, the mean monthly expenditure of the families = Rupees 2662.50

4. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures

No of Students per teacher	Number of states / U.T
15-20	3
20-25	8
25-30	9
30-35	10
35-40	3
40-45	0
45-50	0
50-55	2

Solution:

Given data:

Modal class = 30 – 35,

$$l = 30,$$

Class width (h) = 5,

$$f_m = 10, f_1 = 9 \text{ and } f_2 = 3$$

Mode Formula:

$$\text{Mode} = l + \left[\frac{(f_m - f_1)}{(2f_m - f_1 - f_2)} \right] \times h$$

Substitute the values in the given formula

$$\text{Mode} = 30 + \left(\frac{10 - 9}{(20 - 9 - 3)} \right) \times 5$$

$$\text{Mode} = 30 + (5/8) = 30 + 0.625$$

$$\text{Mode} = 30.625$$

Therefore, the mode of the given data = 30.625

Calculation of mean:

Find the midpoint using the formula, $x_i = (\text{upper limit} + \text{lower limit})/2$

Class Interval	Frequency (f_i)	Mid-point (x_i)	$f_i x_i$
15-20	3	17.5	52.5
20-25	8	22.5	180.0
25-30	9	27.5	247.5
30-35	10	32.5	325.0
35-40	3	37.5	112.5
40-45	0	42.5	0
45-50	0	47.5	0
50-55	2	52.5	105.5
	Sum $f_i = 35$		Sum $f_i x_i = 1022.5$

$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= 1022.5/35$$

$$= 29.2$$

Therefore, mean = 29.2

5. The given distribution shows the number of runs scored by some top batsmen of the world in one- day international cricket matches.

Run Scored	Number of Batsman
3000-4000	4
4000-5000	18
5000-6000	9
6000-7000	7
7000-8000	6
8000-9000	3
9000-10000	1
10000-11000	1

Find the mode of the data.

Solution:

Given data:

Modal class = 4000 – 5000,

$$l = 4000,$$

$$\text{class width (h)} = 1000,$$

$$f_m = 18, f_1 = 4 \text{ and } f_2 = 9$$

Mode Formula:

$$\text{Mode} = l + \left[\frac{(f_m - f_1)}{(2f_m - f_1 - f_2)} \right] \times h$$

Substitute the values

$$\text{Mode} = 4000 + \left(\frac{(18 - 4)}{(36 - 4 - 9)} \right) \times 1000$$

$$\text{Mode} = 4000 + \left(\frac{14000}{23} \right) = 4000 + 608.695$$

$$\text{Mode} = 4608.695$$

$$\text{Mode} = 4608.7 \text{ (approximately)}$$

Thus, the mode of the given data is 4608.7 runs

6. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarized it in the table given below. Find the mode of the data:

Number of cars	Frequency
0-10	7
10-20	14
20-30	13
30-40	12
40-50	20
50-60	11
60-70	15
70-80	8

Solution:

Given Data:

$$\text{Modal class} = 40 - 50, l = 40,$$

$$\text{Class width (h)} = 10, f_m = 20, f_1 = 12 \text{ and } f_2 = 11$$

$$\text{Mode} = l + \left[\frac{(f_m - f_1)}{(2f_m - f_1 - f_2)} \right] \times h$$

Substitute the values

$$\text{Mode} = 40 + \left(\frac{(20 - 12)}{(40 - 12 - 11)} \right) \times 10$$

$$\text{Mode} = 40 + \left(\frac{80}{17} \right) = 40 + 4.7 = 44.7$$

Thus, the mode of the given data is 44.7 cars