

Exercise 3.5

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1. Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross multiplication method.

(i) $x - 3y - 3 = 0$ and $3x - 9y - 2 = 0$

(iii) $3x - 5y = 20$ and $6x - 10y = 40$

(ii) $2x + y = 5$ and $3x + 2y = 8$

(iv) $x - 3y - 7 = 0$ and $3x - 3y - 15 = 0$

Solutions:

(i) Given, $x - 3y - 3 = 0$ and $3x - 9y - 2 = 0$

$$a_1/a_2 = 1/3, \quad b_1/b_2 = -3/-9 = 1/3, \quad c_1/c_2 = -3/-2 = 3/2$$

$$(a_1/a_2) = (b_1/b_2) \neq (c_1/c_2)$$

Since, the given set of lines are parallel to each other they will not intersect each other and therefore there is no solution for these equations.

(ii) Given, $2x + y = 5$ and $3x + 2y = 8$

$$a_1/a_2 = 2/3, \quad b_1/b_2 = 1/2, \quad c_1/c_2 = -5/-8$$

$$(a_1/a_2) \neq (b_1/b_2)$$

Since they intersect at a unique point these equations will have a unique solution by cross multiplication method:

$$x/(b_1c_2 - c_1b_2) = y/(c_1a_2 - c_2a_1) = 1/(a_1b_2 - a_2b_1)$$

$$x/(-8 - (-10)) = y/(15 + 16) = 1/(4 - 3)$$

$$x/2 = y/1 = 1$$

$$\therefore x = 2 \text{ and } y = 1$$

(iii) Given, $3x - 5y = 20$ and $6x - 10y = 40$

$$(a_1/a_2) = 3/6 = 1/2$$

$$(b_1/b_2) = -5/-10 = 1/2$$

$$(c_1/c_2) = 20/40 = 1/2$$

$$a_1/a_2 = b_1/b_2 = c_1/c_2$$

Since the given sets of lines are overlapping each other there will be infinite number of solutions for this pair of equation.

(iv) Given, $x - 3y - 7 = 0$ and $3x - 3y - 15 = 0$

$$(a_1/a_2) = 1/3$$

$$(b_1/b_2) = -3/-3 = 1$$

$$(c_1/c_2) = -7/-15$$

$$a_1/a_2 \neq b_1/b_2$$

Since this pair of lines are intersecting each other at a unique point, there will be a unique solution.

By cross multiplication,

$$x/(45-21) = y/(-21+15) = 1/(-3+9)$$

$$x/24 = y/-6 = 1/6$$

$$x/24 = 1/6 \text{ and } y/-6 = 1/6$$

$$\therefore x = 4 \text{ and } y = 1.$$

2. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$$2x + 3y = 7$$

$$(a - b)x + (a + b)y = 3a + b - 2$$

(ii) For which value of k will the following pair of linear equations have no solution?

$$3x + y = 1$$

$$(2k - 1)x + (k - 1)y = 2k + 1$$

Solution:

(i) $3y + 2x - 7 = 0$

$$(a + b)y + (a - b)x - (3a + b - 2) = 0$$

$$a_1/a_2 = 2/(a - b), \quad b_1/b_2 = 3/(a + b), \quad c_1/c_2 = -7/-(3a + b - 2)$$

For infinitely many solutions,

$$a_1/a_2 = b_1/b_2 = c_1/c_2$$

$$\text{Thus } 2/(a - b) = 7/(3a + b - 2)$$

$$6a + 2b - 4 = 7a - 7b$$

$$a - 9b = -4 \dots\dots\dots(i)$$

$$2/(a - b) = 3/(a + b)$$

$$2a + 2b = 3a - 3b$$

$$a - 5b = 0 \dots\dots\dots(ii)$$

Subtracting (i) from (ii), we get

$$4b = 4$$

$$b = 1$$

Substituting this eq. in (ii), we get

$$a - 5 \times 1 = 0$$

$$a = 5$$

Thus at $a = 5$ and $b = 1$ the given equations will have infinite solutions.

$$(ii) 3x + y - 1 = 0$$

$$(2k - 1)x + (k - 1)y - 2k - 1 = 0$$

$$a_1/a_2 = 3/(2k - 1), \quad b_1/b_2 = 1/(k - 1), \quad c_1/c_2 = -1/(-2k - 1) = 1/(2k + 1)$$

For no solutions

$$a_1/a_2 = b_1/b_2 \neq c_1/c_2$$

$$3/(2k - 1) = 1/(k - 1) \neq 1/(2k + 1)$$

$$3/(2k - 1) = 1/(k - 1)$$

$$3k - 3 = 2k - 1$$

$$k = 2$$

Therefore, for $k = 2$ the given pair of linear equations will have no solution.

3. Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$$8x + 5y = 9$$

$$3x + 2y = 4$$

Solution:

$$8x + 5y = 9 \dots\dots\dots(1)$$

$$3x + 2y = 4 \dots\dots\dots(2)$$

From equation (2) we get

$$x = (4 - 2y) / 3 \dots\dots\dots (3)$$

Using this value in equation 1, we get

$$8(4 - 2y) / 3 + 5y = 9$$

$$32 - 16y + 15y = 27$$

$$-y = -5$$

$$y = 5 \dots\dots\dots(4)$$

Using this value in equation (2), we get

$$3x + 10 = 4$$

$$x = -2$$

Thus, $x = -2$ and $y = 5$.

Now, Using Cross Multiplication method:

$$8x + 5y - 9 = 0$$

$$3x + 2y - 4 = 0$$

$$x/(-20+18) = y/(-27 + 32) = 1/(16-15)$$

$$-x/2 = y/5 = 1/1$$

$$\therefore x = -2 \text{ and } y = 5.$$

4. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs.1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs.1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Solutions:

(i) Let x be the fixed charge and y be the charge of food per day.

According to the question,

$$x + 20y = 1000 \dots\dots\dots (i)$$

$$x + 26y = 1180 \dots\dots\dots(ii)$$

Subtracting (i) from (ii) we get

$$6y = 180$$

$$y = \text{Rs.}30$$

Using this value in equation (ii) we get

$$x = 1180 - 26 \times 30$$

$$x = \text{Rs.}400.$$

Therefore, fixed charges is Rs.400 and charge per day is Rs.30.

(ii) Let the fraction be x/y .

So, as per the question given,

$$(x - 1)/y = 1/3 \Rightarrow 3x - y = 3 \dots\dots\dots(1)$$

$$x/(y + 8) = 1/4 \Rightarrow 4x - y = 8 \dots\dots\dots(2)$$

Subtracting equation (1) from (2) , we get

$$x = 5 \dots\dots\dots(3)$$

Using this value in equation (2), we get,

$$(4 \times 5) - y = 8$$

$$y = 12$$

Therefore, the fraction is $5/12$.

(iii) Let the number of right answers is x and number of wrong answers be y

According to the given question;

$$3x - y = 40 \dots\dots\dots(1)$$

$$4x - 2y = 50$$

$$\Rightarrow 2x - y = 25 \dots\dots\dots(2)$$

Subtracting equation (2) from equation (1), we get;

$$x = 15 \dots\dots\dots(3)$$

Putting this in equation (2), we obtain;

$$30 - y = 25$$

$$\text{Or } y = 5$$

Therefore, number of right answers = 15 and number of wrong answers = 5

Hence, total number of questions = 20

(iv) Let x be the number of correct numbers and y be the number of incorrect answers.

According to the question given,

$$3x - y = 40 \dots\dots\dots(i)$$

$$4x - 2y = 50$$

$$\text{And } 2x - y = 25 \dots\dots\dots(ii)$$

Subtracting equation (ii) from (i), we get

$$x = 15 \dots\dots\dots(iii)$$

Using this in equation (i), we get,

$$3(15) - 40 = y$$

$$y = 5$$

Therefore, the number of correct answers = 15

And the number of incorrect answers = 5

The total number of questions = 20

(v) Let,

The length of rectangle = x unit

And breadth of the rectangle = y unit

Now, as per the question given,

$$(x - 5)(y + 3) = xy - 9$$

$$3x - 5y - 6 = 0 \dots\dots\dots(1)$$

$$(x + 3)(y + 2) = xy + 67$$

$$2x + 3y - 61 = 0 \dots\dots\dots(2)$$

Using cross multiplication method, we get,

$$x/(-305 + 18) = y/(-12 + 183) = 1/(-9 + 10)$$

$$x/323 = y/171 = 1/19$$

Therefore, $x = 17$ and $y = 9$.

Hence, the length of rectangle = 17 units

And breadth of the rectangle = 9 units