Exercise 4.1 Page: 73

1. Check whether the following are quadratic equations:

(i) $(x + 1)^2 = 2(x – 3)$
(ii) $x^2 – 2x = (–2) (3 – x)$
(iii) $(x – 2)(x + 1) = (x – 1)(x + 3)$
(iv) $(x – 3)(2x +1) = x(x + 5)$
(v) $(2x – 1)(x – 3) = (x + 5)(x – 1)$
(vi) $x^2 + 3x + 1 = (x – 2)^2$
(vii) $(x + 2)^3 = 2x (x^2 – 1)$
(viii) $x^3 – 4x^2 – x + 1 = (x – 2)^3$

Solutions:

(i) Given,

$(x + 1)^2 = 2(x – 3)$

By using the formula for $(a+b)^2 = a^2+2ab+b^2$

$\Rightarrow x^2 + 2x + 1 = 2x – 6$

$\Rightarrow x^2 + 7 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is quadratic equation.

(ii) Given, $x^2 – 2x = (–2) (3 – x)$

By using the formula for $(a+b)^2 = a^2+2ab+b^2$

$\Rightarrow x^2 – 2x = -6 + 2x$

$\Rightarrow x^2 – 4x + 6 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is quadratic equation.

(iii) Given, $(x – 2)(x + 1) = (x – 1)(x + 3)$

By using the formula for $(a+b)^2 = a^2+2ab+b^2$

$\Rightarrow x^2 – x – 2 = x^2 + 2x – 3$

$\Rightarrow 3x – 1 = 0$

Since the above equation is not in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is not a quadratic equation.

(iv) Given, $(x – 3)(2x +1) = x(x + 5)$

By using the formula for $(a+b)^2=a^2+2ab+b^2$

$\Rightarrow 2x^2 – 5x – 3 = x^2 + 5x$

$\Rightarrow x^2 – 10x – 3 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is quadratic equation.

(v) Given, $(2x – 1)(x – 3) = (x + 5)(x – 1)$

By using the formula for $(a+b)^2=a^2+2ab+b^2$

$\Rightarrow 2x^2 – 7x + 3 = x^2 + 4x – 5$

$\Rightarrow x^2 – 11x + 8 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is quadratic equation.

(vi) Given, $x^2 + 3x + 1 = (x - 2)^2$

By using the formula for $(a+b)^2=a^2+2ab+b^2$

$\Rightarrow x^2 + 3x + 1 = x^2 + 4 - 4x$

$\Rightarrow 7x - 3 = 0$

Since the above equation is not in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is not a quadratic equation.

(vii) Given, $(x + 2)^3 = 2x(x^2 - 1)$

By using the formula for $(a+b)^2 = a^2+2ab+b^2$

$\Rightarrow x^3 + 8 + x^2 + 12x = 2x^3 - 2x$

$\Rightarrow x^3 + 14x - 6x^2 - 8 = 0$

Since the above equation is not in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is not a quadratic equation.

(viii) Given, $x^3 - 4x^2 - x + 1 = (x - 2)^3$

By using the formula for $(a+b)^2 = a^2+2ab+b^2$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$

$\Rightarrow 2x^2 - 13x + 9 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is quadratic equation.

2. Represent the following situations in the form of quadratic equations:

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken

Solutions:

(i) Let us consider,

Breadth of the rectangular plot = x m

Thus, the length of the plot = $(2x + 1)$ m.

As we know,

Area of rectangle = length × breadth = 528 m^2
Putting the value of length and breadth of the plot in the formula, we get,

$(2x + 1) \times x = 528$

$\Rightarrow 2x^2 + x = 528$

$\Rightarrow 2x^2 + x - 528 = 0$

Therefore, the length and breadth of plot, satisfies the quadratic equation, $2x^2 + x - 528 = 0$, which is the required representation of the problem mathematically.

(ii) Let us consider,

The first integer number = x

Thus, the next consecutive positive integer will be = $x + 1$

Product of two consecutive integers = $x \times (x+1) = 306$
$\Rightarrow x^2 + x = 306$

$\Rightarrow x^2 + x - 306 = 0$

Therefore, the two integers x and x+1, satisfies the quadratic equation, $x^2 + x - 306 = 0$, which is the required representation of the problem mathematically.

(iii) Let us consider,

Age of Rohan's = x years

Therefore, as per the given question,

Rohan's mother's age = $x + 26$

After 3 years,

Age of Rohan's = $x + 3$

Age of Rohan's mother will be = $x + 26 + 3 = x + 29$

The product of their ages after 3 years will be equal to 360, such that

$(x + 3)(x + 29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 = 360$

$\Rightarrow x^2 + 32x + 87 - 360 = 0$

$\Rightarrow x^2 + 32x - 273 = 0$

Therefore, the age of Rohan and his mother, satisfies the quadratic equation, $x^2 + 32x - 273 = 0$, which is the required representation of the problem mathematically.

(iv) Let us consider,

The speed of train $= x$ km/h

And

Time taken to travel 480 km $= 480/x$ km/hr

As per second condition, the speed of train $= (x - 8)$ km/h

Also given, the train will take 3 hours to cover the same distance.

Therefore, time taken to travel 480 km $= 480/(x+3)$ km/h

As we know,

Speed $\times$ Time $=$ Distance

Therefore,

$(x - 8)(480/(x + 3) = 480$

$\Rightarrow 480 + 3x - 3840/x - 24 = 480$

$\Rightarrow 3x - 3840/x = 24$

$\Rightarrow 3x^2 - 8x - 1280 = 0$

Therefore, the speed of the train, satisfies the quadratic equation, $3x^2 - 8x - 1280 = 0$, which is the required representation of the problem mathematically.