Exercise 4.1 Page: 73

1. Check whether the following are quadratic equations:
(i) $(x + 1)^2 = 2(x – 3)$
(ii) $x^2 – 2x = (–2) (3 – x)$
(iii) $(x – 2)(x + 1) = (x – 1)(x + 3)$
(iv) $(x – 3)(2x +1) = x(x + 5)$
(v) $(2x – 1)(x – 3) = (x + 5)(x – 1)$
(vi) $x^2 + 3x + 1 = (x – 2)^2$
(vii) $(x + 2)^3 = 2x (x^2 – 1)$
(viii) $x^3 – 4x^2 – x + 1 = (x – 2)^3$

Solutions:

(i) Given,
$(x + 1)^2 = 2(x – 3)$
By using the formula for $(a+b)^2 = a^2+2ab+b^2$
$\Rightarrow x^2 + 2x + 1 = 2x – 6$
$\Rightarrow x^2 + 7 = 0$
Since the above equation is in the form of $ax^2 + bx + c = 0$.
Therefore, the given equation is quadratic equation.

(ii) Given, $x^2 – 2x = (–2) (3 – x)$
By using the formula for $(a+b)^2 = a^2+2ab+b^2$
$\Rightarrow x^2 – 2x = -6 + 2x$
$\Rightarrow x^2 – 4x + 6 = 0$
Since the above equation is in the form of $ax^2 + bx + c = 0$.
Therefore, the given equation is quadratic equation.

(iii) Given, $(x – 2)(x + 1) = (x – 1)(x + 3)$
By using the formula for $(a+b)^2 = a^2+2ab+b^2$
$\Rightarrow x^2 – x – 2 = x^2 + 2x – 3$
$\Rightarrow 3x – 1 = 0$
Since the above equation is not in the form of $ax^2 + bx + c = 0$.
Therefore, the given equation is not a quadratic equation.

(iv) Given, $(x – 3)(2x +1) = x(x + 5)$
By using the formula for $(a+b)^2=a^2+2ab+b^2$
$\Rightarrow 2x^2 – 5x – 3 = x^2 + 5x$
$\Rightarrow x^2 – 10x – 3 = 0$
Since the above equation is in the form of $ax^2 + bx + c = 0$.
Therefore, the given equation is quadratic equation.

(v) Given, $(2x – 1)(x – 3) = (x + 5)(x – 1)$
By using the formula for $(a+b)^2=a^2+2ab+b^2$
$\Rightarrow 2x^2 – 7x + 3 = x^2 + 4x – 5$
$\Rightarrow x^2 – 11x + 8 = 0$
Since the above equation is in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is quadratic equation.

(vi) Given, $x^2 + 3x + 1 = (x - 2)^2$

By using the formula for $(a+b)^2=a^2+2ab+b^2$

$\Rightarrow x^2 + 3x + 1 = x^2 + 4 - 4x$

$\Rightarrow 7x - 3 = 0$

Since the above equation is not in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is not a quadratic equation.

(vii) Given, $(x + 2)^3 = 2x(x^2 - 1)$

By using the formula for $(a+b)^2 = a^2+2ab+b^2$

$\Rightarrow x^3 + 8 + x^2 + 12x = 2x^3 - 2x$

$\Rightarrow x^3 + 14x - 6x^2 - 8 = 0$

Since the above equation is not in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is not a quadratic equation.

(viii) Given, $x^3 - 4x^2 - x + 1 = (x - 2)^3$

By using the formula for $(a+b)^2 = a^2+2ab+b^2$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$

$\Rightarrow 2x^2 - 13x + 9 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$.

Therefore, the given equation is quadratic equation.

2. Represent the following situations in the form of quadratic equations:

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken

Solutions:

(i) Let us consider,

Breadth of the rectangular plot $= x$ m

Thus, the length of the plot $= (2x + 1)$ m.

As we know,

Area of rectangle = length × breadth = 528 m^2
Putting the value of length and breadth of the plot in the formula, we get,

$(2x + 1) \times x = 528$

$\Rightarrow 2x^2 + x = 528$

$\Rightarrow 2x^2 + x - 528 = 0$

Therefore, the length and breadth of plot, satisfies the quadratic equation, $2x^2 + x - 528 = 0$, which is the required representation of the problem mathematically.

(ii) Let us consider,

The first integer number = x

Thus, the next consecutive positive integer will be = $x + 1$

Product of two consecutive integers = $x \times (x+1) = 306$
$\Rightarrow x^2 + x = 306$

$\Rightarrow x^2 + x - 306 = 0$

Therefore, the two integers x and x+1, satisfies the quadratic equation, $x^2 + x - 306 = 0$, which is the required representation of the problem mathematically.

(iii) Let us consider,

Age of Rohan's = x years

Therefore, as per the given question,

Rohan's mother's age = $x + 26$

After 3 years,

Age of Rohan's = $x + 3$

Age of Rohan's mother will be = $x + 26 + 3 = x + 29$

The product of their ages after 3 years will be equal to 360, such that

$(x + 3)(x + 29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 = 360$

$\Rightarrow x^2 + 32x + 87 - 360 = 0$

$\Rightarrow x^2 + 32x - 273 = 0$

Therefore, the age of Rohan and his mother, satisfies the quadratic equation, $x^2 + 32x - 273 = 0$, which is the required representation of the problem mathematically.

(iv) Let us consider,

The speed of train = x km/h

And

Time taken to travel 480 km = 480/x km/hr

As per second condition, the speed of train = $(x - 8)$ km/h

Also given, the train will take 3 hours to cover the same distance.

Therefore, time taken to travel 480 km = 480/(x+3) km/h

As we know,

Speed × Time = Distance

Therefore,

$(x - 8)(480/(x + 3) = 480$

$\Rightarrow 480 + 3x - 3840/x - 24 = 480$

$\Rightarrow 3x - 3840/x = 24$

$\Rightarrow 3x^2 - 8x - 1280 = 0$

Therefore, the speed of the train, satisfies the quadratic equation, $3x^2 - 8x - 1280 = 0$, which is the required representation of the problem mathematically.

Exercise 4.2

1. Find the roots of the following quadratic equations by factorisation:
(i) $x^2 – 3x – 10 = 0$
(ii) $2x^2 + x – 6 = 0$
(iii) $\sqrt{2}\, x^2 + 7x + 5\sqrt{2} = 0$
(iv) $2x^2 – x + 1/8 = 0$
(v) $100x^2 – 20x + 1 = 0$

Solutions:

(i) Given, $x^2 – 3x – 10 = 0$
Taking LHS,
$\Rightarrow x^2 - 5x + 2x – 10$
$\Rightarrow x(x - 5) + 2(x - 5)$
$\Rightarrow (x - 5)(x + 2)$
The roots of this equation, $x^2 – 3x – 10 = 0$ are the values of x for which $(x - 5)(x + 2) = 0$
Therefore, $x - 5 = 0$ or $x + 2 = 0$
$\Rightarrow x = 5$ or $x = -2$

(ii) Given, $2x^2 + x – 6 = 0$
Taking LHS,
$\Rightarrow 2x^2 + 4x - 3x - 6$
$\Rightarrow 2x(x + 2) - 3(x + 2)$
$\Rightarrow (x + 2)(2x - 3)$
The roots of this equation, $2x^2 + x – 6=0$ are the values of x for which $(x - 5)(x + 2) = 0$
Therefore, $x + 2 = 0$ or $2x - 3 = 0$
$\Rightarrow x = -2$ or $x = 3/2$

(iii) $\sqrt{2}\, x^2 + 7x + 5\sqrt{2}=0$
Taking LHS,
$\Rightarrow \sqrt{2}\, x^2 + 5x + 2x + 5\sqrt{2}$
$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5)= (\sqrt{2}x + 5)(x + \sqrt{2})$
The roots of this equation, $\sqrt{2}\, x^2 + 7x + 5\sqrt{2}=0$ are the values of x for which $(x - 5)(x + 2) = 0$
Therefore, $\sqrt{2}x + 5 = 0$ or $x + \sqrt{2} = 0$
$\Rightarrow x = -5/\sqrt{2}$ or $x = -\sqrt{2}$

(iv) $2x^2 – x + 1/8 = 0$
Taking LHS,
$=1/8\ (16x^2 - 8x + 1)$
$= 1/8\ (16x^2 - 4x - 4x + 1)$
$= 1/8\ (4x(4x - 1) - 1(4x - 1))$

$= 1/8\ (4x - 1)^2$
The roots of this equation, $2x^2 – x + 1/8 = 0$, are the values of x for which $(4x - 1)^2= 0$

Therefore, $(4x - 1) = 0$ or $(4x - 1) = 0$
$\Rightarrow x = 1/4$ or $x = 1/4$

(v) Given, $100x^2 - 20x + 1=0$
Taking LHS,
$= 100x^2 - 10x - 10x + 1$
$= 10x(10x - 1) -1(10x - 1)$
$= (10x - 1)^2$
The roots of this equation, $100x^2 - 20x + 1=0$, are the values of x for which $(10x - 1)^2= 0$
$\therefore (10x - 1) = 0$ or $(10x - 1) = 0$
$\Rightarrow x = 1/10$ or $x = 1/10$

2. Solve the problems given in Example 1.
Represent the following situations mathematically:
(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.
(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ` 750. We would like to find out the number of toys produced on that day.

Solutions:

(i) Let us say, the number of marbles John have = x.
Therefore, number of marble Jivanti have = $45 - x$
After losing 5 marbles each,
Number of marbles John have = $x - 5$
Number of marble Jivanti have = $45 - x - 5 = 40 - x$
Given that the product of their marbles is 124.
$\therefore (x - 5)(40 - x) = 124$
$\Rightarrow x^2 - 45x + 324 = 0$
$\Rightarrow x^2 - 36x - 9x + 324 = 0$
$\Rightarrow x(x - 36) -9(x - 36) = 0$
$\Rightarrow (x - 36)(x - 9) = 0$
Thus, we can say,
$x - 36 = 0$ or $x - 9 = 0$
$\Rightarrow x = 36$ or $x = 9$
Therefore,
If, John's marbles = 36,
Then, Jivanti's marbles = 45 - 36 = 9

And if John's marbles = 9,
Then, Jivanti's marbles = 45 - 9 = 36

(ii) Let us say, number of toys produced in a day be x.
Therefore, cost of production of each toy = Rs$(55 - x)$
Given, total cost of production of the toys = Rs 750
$\therefore x(55 - x) = 750$

$\Rightarrow x^2 – 55x + 750 = 0$
$\Rightarrow x^2 – 25x - 30x + 750 = 0$
$\Rightarrow x(x - 25) -30(x - 25) = 0$
$\Rightarrow (x - 25)(x - 30) = 0$
Thus, either $x -25 = 0$ or $x - 30 = 0$
$\Rightarrow x = 25$ or $x = 30$
Hence, the number of toys produced in a day, will be either 25 or 30.

3. Find two numbers whose sum is 27 and product is 182.

Solution:
Let us say, first number be x and the second number is $27 - x$.
Therefore, the product of two numbers
$x(27 - x) = 182$
$\Rightarrow x^2 – 27x - 182 = 0$
$\Rightarrow x^2 – 13x - 14x + 182 = 0$
$\Rightarrow x(x - 13) -14(x - 13) = 0$
$\Rightarrow (x - 13)(x -14) = 0$
Thus, either, $x = -13 = 0$ or $x - 14 = 0$
$\Rightarrow x = 13$ or $x = 14$
Therefore, if first number = 13, then second number = 27 - 13 = 14
And if first number = 14, then second number = 27 - 14 = 13
Hence, the numbers are 13 and 14.

4. Find two consecutive positive integers, sum of whose squares is 365.

Solution:
Let us say, the two consecutive positive integers be x and $x + 1$.
Therefore, as per the given questions,
$x^2 + (x + 1)^2 = 365$
$\Rightarrow x^2 + x^2 + 1 + 2x = 365$
$\Rightarrow 2x^2 + 2x - 364 = 0$
$\Rightarrow x^2 + x - 182 = 0$
$\Rightarrow x^2 + 14x - 13x - 182 = 0$
$\Rightarrow x(x + 14) -13(x + 14) = 0$
$\Rightarrow (x + 14)(x - 13) = 0$
Thus, either, $x + 14 = 0$ or $x - 13 = 0$,
$\Rightarrow x = - 14$ or $x = 13$
since, the integers are positive, so x can be 13, only.
$\therefore x + 1 = 13 + 1 = 14$
Therefore, two consecutive positive integers will be 13 and 14.

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Solution:
Let us say, the base of the right triangle be x cm.

Given, the altitude of right triangle = (x - 7) cm
From Pythagoras theorem, we know,
$\text{Base}^2 + \text{Altitude}^2 = \text{Hypotenuse}^2$
$\therefore x^2 + (x - 7)^2 = 132$
$\Rightarrow x^2 + x^2 + 49 - 14x = 169$
$\Rightarrow 2x^2 - 14x - 120 = 0$
$\Rightarrow x^2 - 7x - 60 = 0$
$\Rightarrow x^2 - 12x + 5x - 60 = 0$
$\Rightarrow x(x - 12) + 5(x - 12) = 0$
$\Rightarrow (x - 12)(x + 5) = 0$

Thus, either $x - 12 = 0$ or $x + 5 = 0$,
$\Rightarrow x = 12$ or $x = - 5$
Since sides cannot be negative, x can only be 12.
Therefore, the base of the given triangle is 12 cm and the altitude of this triangle will be (12 - 7) cm = 5 cm.

6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs.90, find the number of articles produced and the cost of each article.

Solution:
Let us say, the number of articles produced be x.
Therefore, cost of production of each article = Rs $(2x + 3)$
Given , total cost of production is Rs.90
$\therefore x(2x + 3) = 90$
$\Rightarrow 2x^2 + 3x - 90 = 0$
$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$
$\Rightarrow x(2x + 15) - 6(2x + 15) = 0$
$\Rightarrow (2x + 15)(x - 6) = 0$
Thus, either $2x + 15 = 0$ or $x - 6 = 0$
$\Rightarrow x = -15/2$ or $x = 6$
As the number of articles produced can only be a positive integer, therefore, x can only be 6.
Hence, number of articles produced = 6
Cost of each article = $2 \times 6 + 3$ = Rs 15.

Exercise 4.3 Page: 87

1. Find the roots of the following quadratic equations, if they exist, by the method of completing the square:
(i) $2x^2 – 7x +3 = 0$
(ii) $2x^2 + x – 4 = 0$
(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$
(iv) $2x^2 + x + 4 = 0$

Solutions:

(i) $2x^2 – 7x + 3 = 0$
$\Rightarrow 2x^2 – 7x = - 3$
Dividing by 2 on both sides, we get
$\Rightarrow x^2 -7x/2 = -3/2$
$\Rightarrow x^2 -2 \times x \times 7/4 = -3/2$

On adding $(7/4)^2$ to both sides of equation, we get
$\Rightarrow (x)^2-2\times x\times 7/4 +(7/4)^2 = (7/4)^2-3/2$
$\Rightarrow (x-7/4)^2 = (49/16) – (3/2)$
$\Rightarrow (x-7/4)^2 = 25/16$
$\Rightarrow (x-7/4)^2 = \pm 5/4$
$\Rightarrow x = 7/4 \pm 5/4$
$\Rightarrow x = 7/4 + 5/4$ or $x = 7/4 – 5/4$
$\Rightarrow x = 12/4$ or $x = 2/4$
$\Rightarrow x = 3$ or $x = 1/2$

(ii) $2x^2 + x – 4 = 0$
$\Rightarrow 2x^2 + x = 4$
Dividing both sides of the equation by 2, we get
$\Rightarrow x^2 +x/2 = 2$
Now on adding $(1/4)^2$ to both sides of the equation, we get,

$\Rightarrow (x)^2 + 2 \times x \times 1/4 + (1/4)^2 = 2 + (1/4)^2$
$\Rightarrow (x + 1/4)^2 = 33/16$
$\Rightarrow x + 1/4 = \pm \sqrt{33}/4$
$\Rightarrow x = \pm \sqrt{33}/4 - 1/4$
$\Rightarrow x = \pm \sqrt{33}-1/4$

Therefore, either $x = \sqrt{33}-1/4$ or $x = -\sqrt{33}-1/4$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Converting the equation into $a^2+2ab+b^2$ form, we get,

⇒ $(2x)^2 + 2 \times 2x \times \sqrt{3} + (\sqrt{3})^2 = 0$
⇒ $(2x + \sqrt{3})^2 = 0$
⇒ $(2x + \sqrt{3}) = 0$ and $(2x + \sqrt{3}) = 0$
Therefore, either $x = -\sqrt{3}/2$ or $x = -\sqrt{3}/2$.

(iv) $2x^2 + x + 4 = 0$
⇒ $2x^2 + x = -4$
Dividing both sides of the equation by 2, we get
⇒ $x^2 + 1/2x = 2$
⇒ $x^2 + 2 \times x \times 1/4 = -2$
By adding $(1/4)^2$ to both sides of the equation, we get
⇒ $(x)^2 + 2 \times x \times 1/4 + (1/4)^2 = (1/4)^2 - 2$
⇒ $(x + 1/4)^2 = 1/16 - 2$
⇒ $(x + 1/4)^2 = -31/16$

As we know, the square of numbers cannot be negative.
Therefore, there is no real root for the given equation, $2x^2 + x + 4 = 0$.

2. Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

(i) $2x^2 – 7x + 3 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get,

$a = 2$, $b = -7$ and $c = 3$
By using quadratic formula, we get,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

⇒ $x = (7\pm\sqrt{(49 – 24)})/4$
⇒ $x = (7\pm\sqrt{25})/4$
⇒ $x = (7\pm5)/4$
⇒ $x = (7+5)/4$ or $x = (7-5)/4$
⇒ $x = 12/4$ or $2/4$
∴ $x = 3$ or $1/2$

(ii) $2x^2 + x - 4 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get,

a = 2, b = 1 and $c = -4$

By using quadratic formula, we get,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

⇒x = -1±√1+32/4
⇒x = -1±√33/4
∴ x = -1+√33/4 or x = -1-√33/4

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get
$a = 4$, $b = 4\sqrt{3}$ and c = 3

By using quadratic formula, we get,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

⇒ x = -4√3±√48-48/8
⇒ x = -4√3±0/8
∴ x = -√3/2 or x = -√3/2

(iv) $2x^2 + x + 4 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get,

$a = 2$, $b = 1$ and $c = 4$

By using quadratic formula, we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

⇒ x = -1±√1-32/4
⇒ x = -1±√-31/4
As we know, the square of a number can never be negative. Therefore, there is no real solution for the given equation.

3. Find the roots of the following equations:

(i) x-1/x = 3, $x \neq 0$

(ii) 1/x+4 - 1/x-7 = 11/30, x = -4, 7

Solution:

(i) $x-1/x = 3$
⇒ $x^2 - 3x - 1 = 0$
On comparing the given equation with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = -3$ and $c = -1$
By using quadratic formula, we get,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

⇒ $x = 3\pm\sqrt{9+4}/2$

⇒ $x = 3\pm\sqrt{13}/2$

∴ $x = 3+\sqrt{13}/2$ or $x = 3-\sqrt{13}/2$

(ii) $1/x+4 - 1/x-7 = 11/30$

⇒ $x-7-x-4/(x+4)(x-7) = 11/30$

⇒ $-11/(x+4)(x-7) = 11/30$
⇒ $(x+4)(x-7) = -30$
⇒ $x^2 - 3x - 28 = 30$
⇒ $x^2 - 3x + 2 = 0$
We can solve this equation by factorization method now,
⇒ $x^2 - 2x - x + 2 = 0$
⇒ $x(x - 2) - 1(x - 2) = 0$
⇒ $(x - 2)(x - 1) = 0$
⇒ $x = 1$ or 2

4. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $1/3$. Find his present age.
Solution:

Let us say, present age of Rahman is x years.
Three years ago, Rehman's age was $(x - 3)$ years.
Five years after, his age will be $(x + 5)$ years.

Given, the sum of the reciprocals of Rehman's ages 3 years ago and after 5 years is equal to $1/3$.

∴ $1/x-3 + 1/x-5 = 1/3$

$(x+5+x-3)/(x-3)(x+5) = 1/3$

$(2x+2)/(x-3)(x+5) = 1/3$

⇒ $3(2x + 2) = (x-3)(x+5)$
⇒ $6x + 6 = x^2 + 2x - 15$

⇒ $x^2 - 4x - 21 = 0$
⇒ $x^2 - 7x + 3x - 21 = 0$
⇒ $x(x - 7) + 3(x - 7) = 0$
⇒ $(x - 7)(x + 3) = 0$
⇒ $x = 7, -3$
As we know, age cannot be negative.
Therefore, Rahman's present age is 7 years.

5. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Solution:
Let us say, the marks of Shefali in Maths be x.
Then, the marks in English will be $30 - x$.
As per the given question,
$(x + 2)(30 - x - 3) = 210$
$(x + 2)(27 - x) = 210$
⇒ $-x^2 + 25x + 54 = 210$
⇒ $x^2 - 25x + 156 = 0$
⇒ $x^2 - 12x - 13x + 156 = 0$
⇒ $x(x - 12) -13(x - 12) = 0$
⇒ $(x - 12)(x - 13) = 0$
⇒ $x = 12, 13$
Therefore, if the marks in Maths are 12, then marks in English will be 30 - 12 = 18 and the marks in Maths are 13, then marks in English will be 30 - 13 = 17**.**

6. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Solution:
Let us say, the shorter side of the rectangle be x m.
Then, larger side of the rectangle = $(x + 30)$ m
Diagonal of the rectangle = $\sqrt{x^2 + (x + 30)^2}$
As given, the length of the diagonal is = x + 30 m
Therefore,

$$\sqrt{x^2 + (x + 30)^2} = x + 60$$

⇒ $x^2 + (x + 30)^2 = (x + 60)^2$
⇒ $x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$
⇒ $x^2 - 60x - 2700 = 0$
⇒ $x^2 - 90x + 30x - 2700 = 0$
⇒ $x(x - 90) + 30(x -90) = 0$
⇒ $(x - 90)(x + 30) = 0$
⇒ $x = 90, -30$

However, side of the field cannot be negative. Therefore, the length of the shorter side will be 90 m. and the length of the larger side will be (90 + 30) m = 120 m.

7. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Solution:
Let us say, the larger and smaller number be x and y respectively.
As per the question given,
$x^2 - y^2 = 180$ and $y^2 = 8x$
$\Rightarrow x^2 - 8x = 180$
$\Rightarrow x^2 - 8x - 180 = 0$
$\Rightarrow x^2 - 18x + 10x - 180 = 0$
$\Rightarrow x(x - 18) +10(x - 18) = 0$
$\Rightarrow (x - 18)(x + 10) = 0$
$\Rightarrow x = 18, -10$

However, the larger number cannot considered as negative number, as 8 times of the larger number will be negative and hence, the square of the smaller number will be negative which is not possible.
Therefore, the larger number will be 18 only.
$x = 18$
$\therefore y^2 = 8x = 8 \times 18 = 144$
$\Rightarrow y = \pm\sqrt{144} = \pm 12$
$\therefore$ Smaller number = ± 12
Therefore, the numbers are 18 and 12 or 18 and -12.

8. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution:
Let us say, the speed of the train be x km/hr.
Time taken to cover 360 km = $360/x$ hr.
As per the question given,
$\Rightarrow (x + 5)(360-1/x) = 360$
$\Rightarrow 360 - x + 1800-5/x = 360$
$\Rightarrow x^2 + 5x + 10x - 1800 = 0$
$\Rightarrow x(x + 45) -40(x + 45) = 0$
$\Rightarrow (x + 45)(x - 40) = 0$
$\Rightarrow x = 40, -45$
As we know, the value of speed cannot be negative.
Therefore, the speed of train is 40 km/h.

9. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Solution:

Let the time taken by the smaller pipe to fill the tank = x hr.
Time taken by the larger pipe = (x - 10) hr
Part of tank filled by smaller pipe in 1 hour = $1/x$
Part of tank filled by larger pipe in 1 hour = $1/(x – 10)$

As given, the tank can be filled in $9\frac{3}{8}$ = 75/8 hours by both the pipes together.
Therefore,
$1/x + 1/x-10 = 8/75$
$x-10+x/x(x-10) = 8/75$
$\Rightarrow 2x-10/x(x-10) = 8/75$
$\Rightarrow 75(2x - 10) = 8x^2 - 80x$
$\Rightarrow 150x - 750 = 8x^2 - 80x$
$\Rightarrow 8x^2 - 230x +750 = 0$
$\Rightarrow 8x^2 - 200x - 30x + 750 = 0$
$\Rightarrow 8x(x - 25) -30(x - 25) = 0$
$\Rightarrow (x - 25)(8x -30) = 0$
$\Rightarrow x = 25, 30/8$

Time taken by the smaller pipe cannot be 30/8 = 3.75 hours, as the time taken by the larger pipe will become negative, which is logically not possible.

Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and 25 - 10 =15 hours respectively.

10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speeds of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Solution:

Let us say, the average speed of passenger train = x km/h.
Average speed of express train = (x + 11) km/h
Given, time taken by the express train to cover 132 km is 1 hour less than the passenger train to cover the same distance. Therefore,
(132/x) – (132/(x+11)) = 1

132(x+11-x)/(x(x+11)) = 1
132 × 11 /(x(x+11)) = 1

$\Rightarrow 132 \times 11 = x(x + 11)$
$\Rightarrow x^2 + 11x - 1452 = 0$
$\Rightarrow x^2 + 44x - 33x - 1452 = 0$
$\Rightarrow x(x + 44) - 33(x + 44) = 0$
$\Rightarrow (x + 44)(x - 33) = 0$
$\Rightarrow x = -44, 33$

As we know, Speed cannot be negative.

Therefore, the speed of the passenger train will be 33 km/h and thus, the speed of the express train will be 33 + 11 = 44 km/h.

11. Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Solution:
Let the sides of the two squares be x m and y m.
Therefore, their perimeter will be $4x$ and $4y$ respectively
And area of the squares will be x^2 and y^2 respectively.
Given,
$4x - 4y = 24$
$x - y = 6$
$x = y + 6$
Also, $x^2 + y^2 = 468$
$\Rightarrow (6 + y^2) + y^2 = 468$
$\Rightarrow 36 + y^2 + 12y + y^2 = 468$
$\Rightarrow 2y^2 + 12y + 432 = 0$
$\Rightarrow y^2 + 6y - 216 = 0$
$\Rightarrow y^2 + 18y - 12y - 216 = 0$
$\Rightarrow y(y + 18) - 12(y + 18) = 0$
$\Rightarrow (y + 18)(y - 12) = 0$
$\Rightarrow y = -18, 12$
As we know, the side of a square cannot be negative.
Hence, the sides of the squares are 12 m and (12 + 6) m = 18 m.

Exercise 4.4

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1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them;
(i) $2x^2 - 3x + 5 = 0$
(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$
(iii) $2x^2 - 6x + 3 = 0$
Solutions:

(i) Given,
$2x^2 - 3x + 5 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get
$a = 2$, $b = -3$ and $c = 5$
We know, Discriminant $= b^2 - 4ac$
$= (-3)^2 - 4(2)(5) = 9 - 40$
$= -31$
As you can see, $b^2 - 4ac < 0$
Therefore, no real root is possible for the given equation, $2x^2 - 3x + 5 = 0$.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get
$a = 3$, $b = -4\sqrt{3}$ and $c = 4$
We know, Discriminant $= b^2 - 4ac$
$= (-4\sqrt{3})^2 - 4(3)(4)$
$= 48 - 48 = 0$

As $b^2 - 4ac = 0$,
Real roots exist for the given equation and they are equal to each other.
Hence the roots will be $-b/2a$ and $-b/2a$.
$-b/2a = -(-4\sqrt{3})/2\times3 = 4\sqrt{3}/6 = 2\sqrt{3}/3 = 2/\sqrt{3}$
Therefore, the roots are $2/\sqrt{3}$ and $2/\sqrt{3}$.

(iii) $2x^2 - 6x + 3 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get
$a = 2$, $b = -6$, $c = 3$
As we know, Discriminant $= b^2 - 4ac$
$= (-6)^2 - 4(2)(3)$
$= 36 - 24 = 12$
As $b^2 - 4ac > 0$,
Therefore, there are distinct real roots exist for this equation, $2x^2 - 6x + 3 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

= (-(-6) ± √(-6^2-4(2)(3)))/ 2(2)
= (6±2√3)/4
= (3±√3)/2
Therefore the roots for the given equation are (3+√3)/2 and (3-√3)/2

2. Find the values of *k* for each of the following quadratic equations, so that they have two equal roots.
(i) $2x^2 + kx + 3 = 0$
(ii) $kx(x - 2) + 6 = 0$

Solutions:
(i) $2x^2 + kx + 3 = 0$
Comparing the given equation with $ax^2 + bx + c = 0$, we get,
$a = 2$, b = k and $c = 3$
As we know, Discriminant = $b^2 - 4ac$
= $(k)^2 - 4(2)(3)$
= $k^2 - 24$
For equal roots, we know,
Discriminant = 0
$k^2 - 24 = 0$
$k^2 = 24$
k = ±√24 = ±2√6

(ii) $kx(x - 2) + 6 = 0$
or $kx^2 - 2kx + 6 = 0$
Comparing the given equation with $ax^2 + bx + c = 0$, we get
$a = k$, $b = -2k$ and $c = 6$

We know, Discriminant = $b^2 - 4ac$
= $(-2k)^2 - 4(k)(6)$
= $4k^2 - 24k$

For equal roots, we know,
$b^2 - 4ac = 0$
$4k^2 - 24k = 0$
$4k(k - 6) = 0$
Either $4k = 0$ or $k = 6 = 0$
$k = 0$ or $k = 6$
However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'.
Therefore, if this equation has two equal roots, k should be 6 only.

3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth.

Solution:

Let the breadth of mango grove be l.
Length of mango grove will be $2l$.
Area of mango grove = $(2l)(l) = 2l^2$
$2l^2 = 800$
$l^2 = 800/2 = 400$
$l^2 - 400 = 0$
Comparing the given equation with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = 0$, $c = 400$
As we know, Discriminant = $b^2 - 4ac$
$\Rightarrow (0)^2 - 4 \times (1) \times (-400) = 1600$
Here, $b^2 - 4ac > 0$
Thus, the equation will have real roots. And hence, the desired rectangular mango grove can be designed.
$l = \pm 20$
As we know, the value of length cannot be negative.
Therefore, breadth of mango grove = 20 m
Length of mango grove = 2 × 20 = 40 m

4. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Solution:
Let's say, the age of one friend be x years.
Then, the age of the other friend will be (20 - x) years.
Four years ago,
Age of First friend = $(x - 4)$ years
Age of Second friend = $(20 - x - 4) = (16 - x)$ years

As per the given question, we can write,
$(x - 4)(16 - x) = 48$
$16x - x^2 - 64 + 4x = 48$
$-x^2 + 20x - 112 = 0$
$x^2 - 20x + 112 = 0$

Comparing the equation with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = -20$ and $c = 112$

Discriminant = $b^2 - 4ac$
$\Rightarrow (-20)^2 - 4 \times 112$
$\Rightarrow 400 - 448 = -48$
$b^2 - 4ac < 0$

Therefore, there will be no real solution possible for the equations. Hence, condition doesn't exist.

5. Is it possible to design a rectangular park of perimeter 80 and area 400 m2? If so find its length and breadth.

Solution:
Let the length and breadth of the park be l and b.
Perimeter of the rectangular park = 2 (l + b) = 80
So, $l + b = 40$
Or, $b = 40 - l$
Area of the rectangular park = $l \times b = l(40 - l) = 40l - l^2 = 400$
$l^2 - 40l + 400 = 0$, which is a quadratic equation.

Comparing the equation with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = -40$, $c = 400$

Since, Discriminant = $b^2 - 4ac$
$=> (-40)^2 - 4 \times 400$
=> 1600 - 1600 = 0
Thus, $b^2 - 4ac = 0$
Therefore, this equation has equal real roots. Hence, the situation is possible.

Root of the equation,
$l = -b/2a$
$l = (40)/2(1) = 40/2 = 20$
Therefore, length of rectangular park, $l = 20$ m
And breadth of the park, $b = 40 - l = 40 - 20 = 20$ m.