Exercise 6.4 Page: 143

1. Let ΔABC ~ ΔDEF and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.

Solution: Given, ΔABC ~ ΔDEF,
Area of ΔABC = 64 cm^2
Area of ΔDEF = 121 cm^2
EF = 15.4 cm

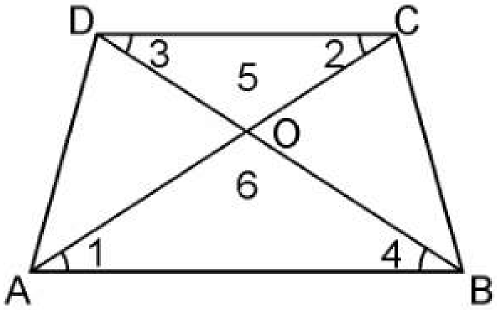
$$\therefore \frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \frac{AB^2}{DE^2}$$

As we know, if two triangles are similar, ratio of their areas are equal to the square of the ratio of their corresponding sides,
= $AC^2/DF^2 = BC^2/EF^2$

∴ $64/121 = BC^2/EF^2$
⇒ $(8/11)^2 = (BC/15.4)^2$
⇒ 8/11 = BC/15.4
⇒ BC = 8×15.4/11
⇒ BC = 8 × 1.4
⇒ BC = 11.2 cm

2. Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.

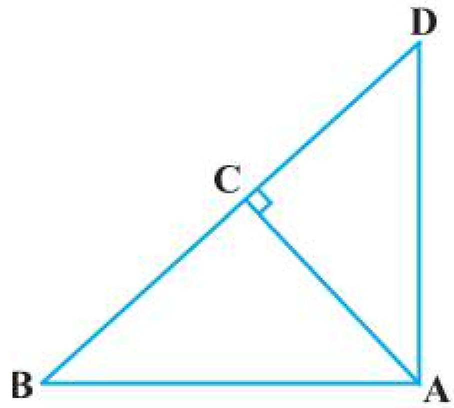
Solution:
Given, ABCD is a trapezium with AB || DC. Diagonals AC and BD intersect each other at point O.



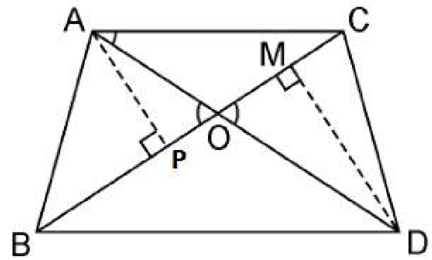
In ΔAOB and ΔCOD, we have
∠1 = ∠2 (Alternate angles)
∠3 = ∠4 (Alternate angles)
∠5 = ∠6 (Vertically opposite angle)
∴ ΔAOB ~ ΔCOD [AAA similarity criterion]

As we know, If two triangles are similar then the ratio of their areas are equal to the square of the ratio of their corresponding sides. Therefore,
Area of (ΔAOB)/Area of (ΔCOD) = AB^2/CD^2
= $(2CD)^2/CD^2$ [∴ AB = 2CD]
∴ Area of (ΔAOB)/Area of (ΔCOD)
= $4CD^2/CD^2 = 4/1$
Hence, the required ratio of the area of ΔAOB and ΔCOD = 4:1

3. In the figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that area (ΔABC)/area (ΔDBC) = AO/DO.



Solution:
Given, ABC and DBC are two triangles on the same base BC. AD intersects BC at O.
We have to prove: Area (ΔABC)/Area (ΔDBC) = AO/DO
Let us draw two perpendiculars AP and DM on line BC.



We know that area of a triangle = 1/2 × Base × Height

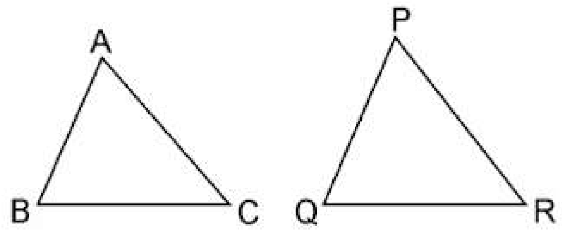
$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{\frac{1}{2}BC \times AP}{\frac{1}{2}BC \times DM} = \frac{AP}{DM}$$

In ΔAPO and ΔDMO,
∠APO = ∠DMO (Each 90°)
∠AOP = ∠DOM (Vertically opposite angles)
∴ ΔAPO ~ ΔDMO (AA similarity criterion)

∴ AP/DM = AO/DO
⇒ Area (ΔABC)/Area (ΔDBC) = AO/DO.

4. If the areas of two similar triangles are equal, prove that they are congruent.

Solution:
Say ΔABC and ΔPQR are two similar triangles and equal in area

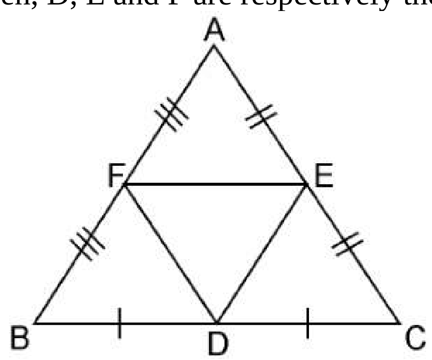


Now let us prove ΔABC ≅ ΔPQR.

Since, ΔABC ~ ΔPQR
∴ Area of (ΔABC)/Area of (ΔPQR) = BC^2/QR^2
⇒ BC^2/QR^2 =1 [Since, Area(ΔABC) = (ΔPQR)
⇒ BC^2/QR^2
⇒ BC = QR
Similarly, we can prove that
AB = PQ and AC = PR
Thus, ΔABC ≅ ΔPQR [SSS criterion of congruence]

5. D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the area of ΔDEF and ΔABC.

Solution:
Given, D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC.



In ΔABC,
F is the mid-point of AB (Already given)
E is the mid-point of AC (Already given)

So, by the mid-point theorem, we have,
FE || BC and FE = 1/2BC
⇒ FE || BC and FE || BD [BD = 1/2BC]
Since, opposite sides of parallelogram are equal and parallel
∴ BDEF is parallelogram.

Similarly, in ΔFBD and ΔDEF, we have
FB = DE (Opposite sides of parallelogram BDEF)
FD = FD (Common sides)
BD = FE (Opposite sides of parallelogram BDEF)
∴ ΔFBD ≅ ΔDEF

Similarly, we can prove that
ΔAFE ≅ ΔDEF
ΔEDC ≅ ΔDEF

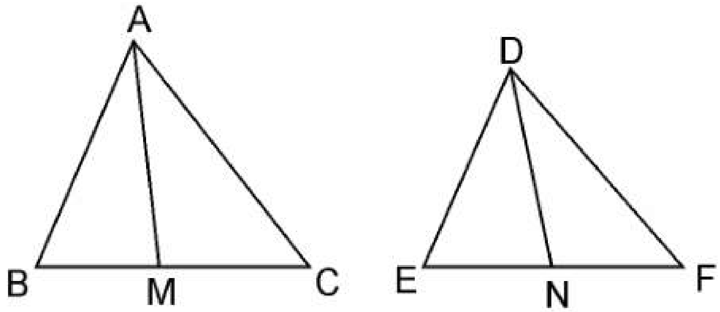
As we know, if triangles are congruent, then they are equal in area.
So,
Area(ΔFBD) = Area(ΔDEF) …………………………...**(i)**
Area(ΔAFE) = Area(ΔDEF) ……………………………….**(ii)**
and,
Area(ΔEDC) = Area(ΔDEF) ………………………….**(iii)**
Now,
Area(ΔABC) = Area(ΔFBD) + Area(ΔDEF) + Area(ΔAFE) + Area(ΔEDC) ……...**(iv)**
Area(ΔABC) = Area(ΔDEF) + Area(ΔDEF) + Area(ΔDEF) + Area(ΔDEF)
From equation **(i)**, **(ii)** and **(iii)**,
⇒ Area(ΔDEF) = (1/4)Area(ΔABC)
⇒ Area(ΔDEF)/Area(ΔABC) = 1/4

Hence, Area(ΔDEF): Area(ΔABC) = 1:4

6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Solution:

Given: AM and DN are the medians of triangles ABC and DEF respectively and ΔABC ~ ΔDEF.



We have to prove: Area(ΔABC)/Area(ΔDEF) = AM^2/DN^2

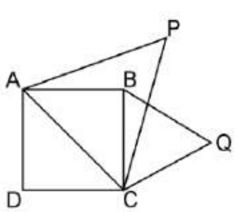
Since, ΔABC ~ ΔDEF (Given)
∴ Area(ΔABC)/Area(ΔDEF) = (AB^2/DE^2) …………………………...**(i)**
and, AB/DE = BC/EF = CA/FD ……………………………………...**(ii)**

$$\Rightarrow \frac{AB}{DE} = \frac{\frac{1}{2}BC}{\frac{1}{2}EF} = \frac{CD}{FD}$$

In ΔABM and ΔDEN,
Since ΔABC ~ ΔDEF
∴ ∠B = ∠E
AB/DE = BM/EN [Already Proved in equation **(i)**]
∴ ΔABC ~ ΔDEF [SAS similarity criterion]
⇒ AB/DE = AM/DN …………………………………………………..**(iii)**
∴ ΔABM ~ ΔDEN
As the areas of two similar triangles are proportional to the squares of the corresponding sides.
∴ area(ΔABC)/area(ΔDEF) = AB^2/DE^2 = AM^2/DN^2
Hence, proved.

7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Solution:
Given, ABCD is a square whose one diagonal is AC. ΔAPC and ΔBQC are two equilateral triangles described on the diagonals AC and side BC of the square ABCD.



Area(ΔBQC) = ½ Area(ΔAPC)

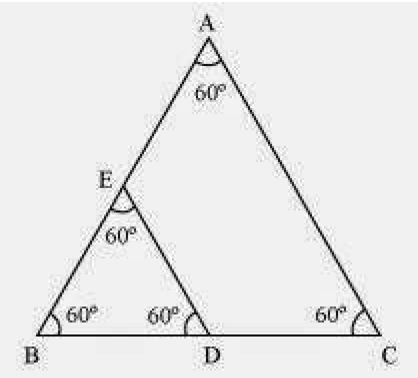
Since, ΔAPC and ΔBQC are both equilateral triangles, as per given,
∴ ΔAPC ~ ΔBQC [AAA similarity criterion]
∴ area(ΔAPC)/area(ΔBQC) = (AC^2/BC^2) = AC^2/BC^2
Since, Diagonal = √2 side = √2 BC = AC

$(\frac{\sqrt{2}BC}{BC})^2 = 2$

⇒ area(ΔAPC) = 2 × area(ΔBQC)
⇒ area(ΔBQC) = 1/2area(ΔAPC)
Hence, proved.

Tick the correct answer and justify:
8. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is
(A) 2 : 1
(B) 1 : 2
(C) 4 : 1
(D) 1 : 4

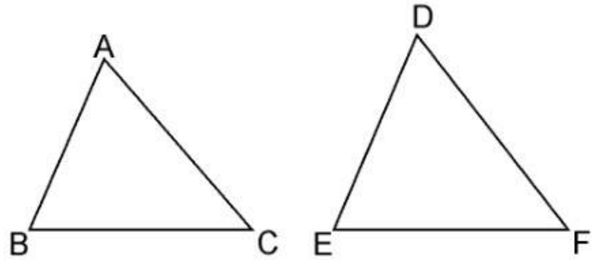
Solution:
Given**,** ΔABC and ΔBDE are two equilateral triangle. D is the midpoint of BC.



∴ BD = DC = 1/2BC
Let each side of triangle is 2*a*.
As, ΔABC ~ ΔBDE
∴ Area(ΔABC)/Area(ΔBDE) = $AB^2/BD^2 = (2a)^2/(a)^2 = 4a^2/a^2$ = 4/1 = 4:1
Hence, the correct answer is (C).

9. Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio
(A) 2 : 3
(B) 4 : 9
(C) 81 : 16
(D) 16 : 81

Solution:
Given, Sides of two similar triangles are in the ratio 4 : 9.



Let ABC and DEF are two similar triangles, such that,

$\Delta ABC \sim \Delta DEF$

And AB/DE = AC/DF = BC/EF = 4/9

As, the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides,

$\therefore$ Area(ΔABC)/Area(ΔDEF) = AB^2/DE^2

$\therefore$ Area(ΔABC)/Area(ΔDEF) = $(4/9)^2$ = 16/81 = 16:81

Hence, the correct answer is (D).