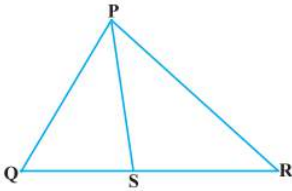


Exercise 6.6

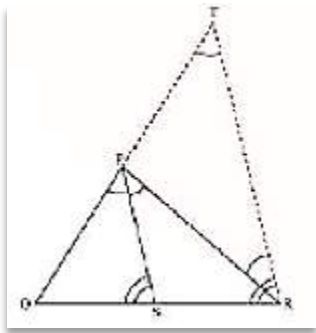
1. In Figure, PS is the bisector of $\angle QPR$ of ΔPQR . Prove that $QS/PQ = SR/PR$



Solution:

Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T.

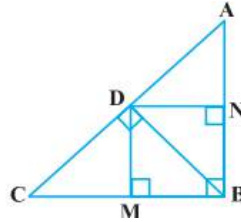
Given, PS is the angle bisector of $\angle QPR$. Therefore,
 $\angle QPS = \angle SPR$(i)



As per the constructed figure,
 $\angle SPR = \angle PRT$ (Since, $PS \parallel TR$).....(ii)
 $\angle QPS = \angle QRT$ (Since, $PS \parallel TR$)(iii)
 From the above equations, we get,
 $\angle PRT = \angle QTR$
 Therefore,
 $PT = PR$

In ΔQTR , by basic proportionality theorem,
 $QS/SR = QP/PT$
 Since, $PT = PR$
 Therefore,
 $QS/SR = PQ/PR$
 Hence, proved.

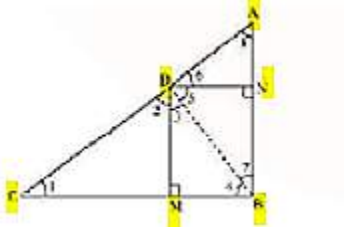
2. In Fig. 6.57, D is a point on hypotenuse AC of $\triangle ABC$, such that $BD \perp AC$, $DM \perp BC$ and $DN \perp AB$. Prove



that: (i) $DM^2 = DN \cdot MC$ (ii) $DN^2 = DM \cdot AN$.

Solution:

(i) Let us join Point D and B.



Given,

$BD \perp AC$, $DM \perp BC$ and $DN \perp AB$

Now from the figure we have,

$DN \parallel CB$, $DM \parallel AB$ and $\angle B = 90^\circ$

Therefore, $DMBN$ is a rectangle.

So, $DN = MB$ and $DM = NB$

The given condition which we have to prove, is when D is the foot of the perpendicular drawn from B to AC.

$$\therefore \angle CDB = 90^\circ \Rightarrow \angle 2 + \angle 3 = 90^\circ \dots\dots\dots (i)$$

$$\text{In } \triangle CDM, \angle 1 + \angle 2 + \angle DMC = 180^\circ$$

$$\Rightarrow \angle 1 + \angle 2 = 90^\circ \dots\dots\dots (ii)$$

$$\text{In } \triangle DMB, \angle 3 + \angle DMB + \angle 4 = 180^\circ$$

$$\Rightarrow \angle 3 + \angle 4 = 90^\circ \dots\dots\dots (iii)$$

From equation (i) and (ii), we get

$$\angle 1 = \angle 3$$

From equation (i) and (iii), we get

$$\angle 2 = \angle 4$$

In $\triangle DCM$ and $\triangle BDM$,

$$\angle 1 = \angle 3 \text{ (Already Proved)}$$

$$\angle 2 = \angle 4 \text{ (Already Proved)}$$

$\therefore \triangle DCM \sim \triangle BDM$ (AA similarity criterion)

$$BM/DM = DM/MC$$

$$\Rightarrow DN/DM = DM/MC \quad (BM = DN)$$

$$\Rightarrow DM^2 = DN \times MC$$

Hence, proved.

(ii) In right triangle DBN,

$$\angle 5 + \angle 7 = 90^\circ \dots\dots\dots (iv)$$

In right triangle DAN,

$$\angle 6 + \angle 8 = 90^\circ \dots\dots\dots (v)$$

D is the point in triangle, which is foot of the perpendicular drawn from B to AC.

$$\therefore \angle ADB = 90^\circ \Rightarrow \angle 5 + \angle 6 = 90^\circ \dots\dots\dots (vi)$$

From equation (iv) and (vi), we get,

$$\angle 6 = \angle 7$$

From equation (v) and (vi), we get,

$$\angle 8 = \angle 5$$

In $\triangle DNA$ and $\triangle BND$,

$$\angle 6 = \angle 7 \text{ (Already proved)}$$

$$\angle 8 = \angle 5 \text{ (Already proved)}$$

$\therefore \triangle DNA \sim \triangle BND$ (AA similarity criterion)

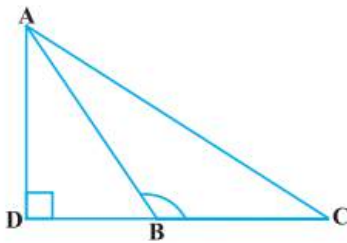
$$AN/DN = DN/NB$$

$$\Rightarrow DN^2 = AN \times NB$$

$$\Rightarrow DN^2 = AN \times DM \text{ (Since, } NB = DM)$$

Hence, proved.

3. In Figure, ABC is a triangle in which $\angle ABC > 90^\circ$ and $AD \perp CB$ produced. Prove that $AC^2 = AB^2 + BC^2 + 2 BC \cdot BD$.



Solution:

By applying Pythagoras Theorem in $\triangle ADB$, we get,

$$AB^2 = AD^2 + DB^2 \dots\dots\dots (i)$$

Again, by applying Pythagoras Theorem in $\triangle ADC$, we get,

$$AC^2 = AD^2 + DC^2$$

$$AC^2 = AD^2 + (DB + BC)^2$$

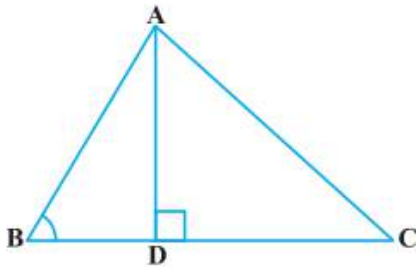
$$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$$

From equation (i), we can write,

$$AC^2 = AB^2 + BC^2 + 2DB \times BC$$

Hence, proved.

4. In Figure, ABC is a triangle in which $\angle ABC < 90^\circ$ and $AD \perp BC$. Prove that $AC^2 = AB^2 + BC^2 - 2 BC \cdot BD$.



Solution:

By applying Pythagoras Theorem in $\triangle ADB$, we get,

$$AB^2 = AD^2 + DB^2$$

We can write it as;

$$\Rightarrow AD^2 = AB^2 - DB^2 \dots\dots\dots (i)$$

By applying Pythagoras Theorem in $\triangle ADC$, we get,

$$AD^2 + DC^2 = AC^2$$

From equation (i),

$$AB^2 - BD^2 + DC^2 = AC^2$$

$$AB^2 - BD^2 + (BC - BD)^2 = AC^2$$

$$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$$

$$AC^2 = AB^2 + BC^2 - 2BC \times BD$$

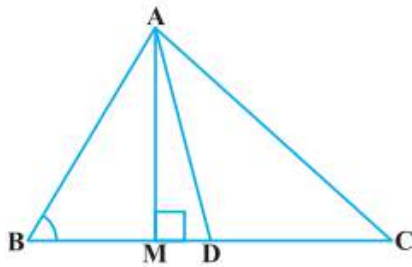
Hence, proved.

5. In Figure, AD is a median of a triangle ABC and $AM \perp BC$. Prove that :

(i) $AC^2 = AD^2 + BC \cdot DM + 2 \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 - BC \cdot DM + 2 \left(\frac{BC}{2}\right)^2$

(iii) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$



Solution:

(i) By applying Pythagoras Theorem in $\triangle AMD$, we get,

$$AM^2 + MD^2 = AD^2 \dots\dots\dots (i)$$

Again, by applying Pythagoras Theorem in $\triangle AMC$, we get,

$$AM^2 + MC^2 = AC^2$$

$$AM^2 + (MD + DC)^2 = AC^2$$

$$(AM^2 + MD^2) + DC^2 + 2MD \cdot DC = AC^2$$

From equation(i), we get,

$$AD^2 + DC^2 + 2MD.DC = AC^2$$

Since, $DC=BC/2$, thus, we get,

$$AD^2 + (BC/2)^2 + 2MD.(BC/2) = AC^2$$

$$AD^2 + (BC/2)^2 + 2MD \times BC = AC^2$$

Hence, proved.

(ii) By applying Pythagoras Theorem in ΔABM , we get;

$$AB^2 = AM^2 + MB^2$$

$$= (AD^2 - DM^2) + MB^2$$

$$= (AD^2 - DM^2) + (BD - MD)^2$$

$$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$$

$$= AD^2 + BD^2 - 2BD \times MD$$

$$= AD^2 + (BC/2)^2 - 2(BC/2) \times MD$$

$$= AD^2 + (BC/2)^2 - BC \times MD$$

Hence, proved.

(iii) By applying Pythagoras Theorem in ΔABM , we get,

$$AM^2 + MB^2 = AB^2 \dots\dots\dots (i)$$

By applying Pythagoras Theorem in ΔAMC , we get,

$$AM^2 + MC^2 = AC^2 \dots\dots\dots (ii)$$

Adding both the equations (i) and (ii), we get,

$$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$$

$$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$$

$$2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$$

$$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$$

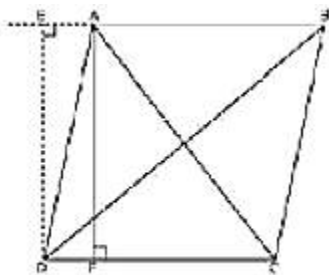
$$2(AM^2 + MD^2) + (BC/2)^2 + (BC/2)^2 + 2MD(-BC/2 + BC/2) = AB^2 + AC^2$$

$$2AD^2 + BC^2/2 = AB^2 + AC^2$$

6. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Solution:

Let us consider, ABCD be a parallelogram. Now, draw perpendicular DE on extended side of AB, and draw a perpendicular AF meeting DC at point F.



By applying Pythagoras Theorem in $\triangle DEA$, we get,

$$DE^2 + EA^2 = DA^2 \dots\dots\dots (i)$$

By applying Pythagoras Theorem in $\triangle DEB$, we get,

$$DE^2 + EB^2 = DB^2$$

$$DE^2 + (EA + AB)^2 = DB^2$$

$$(DE^2 + EA^2) + AB^2 + 2EA \times AB = DB^2$$

$$DA^2 + AB^2 + 2EA \times AB = DB^2 \dots\dots\dots (ii)$$

By applying Pythagoras Theorem in $\triangle ADF$, we get,

$$AD^2 = AF^2 + FD^2$$

Again, applying Pythagoras theorem in $\triangle AFC$, we get,

$$AC^2 = AF^2 + FC^2 = AF^2 + (DC - FD)^2$$

$$= AF^2 + DC^2 + FD^2 - 2DC \times FD$$

$$= (AF^2 + FD^2) + DC^2 - 2DC \times FD$$

$$AC^2 = AD^2 + DC^2 - 2DC \times FD \dots\dots\dots (iii)$$

Since ABCD is a parallelogram,

$$AB = CD \dots\dots\dots (iv)$$

$$\text{And } BC = AD \dots\dots\dots (v)$$

In $\triangle DEA$ and $\triangle ADF$,

$$\angle DEA = \angle AFD \text{ (Each } 90^\circ)$$

$$\angle EAD = \angle ADF \text{ (EA } \parallel \text{ DF)}$$

$$AD = AD \text{ (Common Angles)}$$

$$\therefore \triangle EAD \cong \triangle FDA \text{ (AAS congruence criterion)}$$

$$\Rightarrow EA = DF \dots\dots\dots (vi)$$

Adding equations (i) and (iii), we get,

$$DA^2 + AB^2 + 2EA \times AB + AD^2 + DC^2 - 2DC \times FD = DB^2 + AC^2$$

$$DA^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2DC \times FD = DB^2 + AC^2$$

From equation (iv) and (vi),

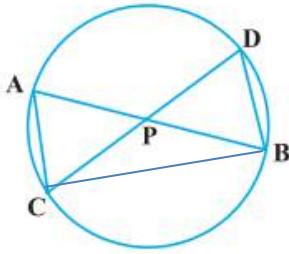
$$BC^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2AB \times EA = DB^2 + AC^2$$

$$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$$

7. In Figure, two chords AB and CD intersect each other at the point P. Prove that :

(i) $\triangle APC \sim \triangle DPB$

(ii) $AP \cdot PB = CP \cdot DP$



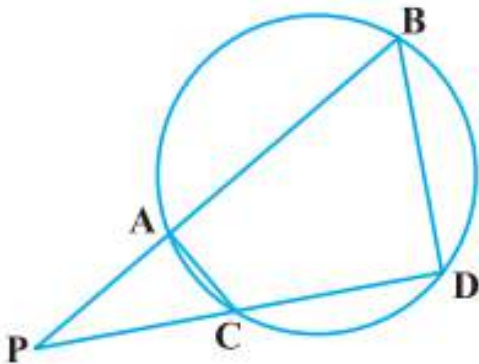
Solution:

Firstly, let us join CB, in the given figure.

- (i) In $\triangle APC$ and $\triangle DPB$,
 $\angle APC = \angle DPB$ (Vertically opposite angles)
 $\angle CAP = \angle BDP$ (Angles in the same segment for chord CB)
 Therefore,
 $\triangle APC \sim \triangle DPB$ (AA similarity criterion)

- (ii) In the above, we have proved that $\triangle APC \sim \triangle DPB$
 We know that the corresponding sides of similar triangles are proportional.
 $\therefore AP/DP = PC/PB = CA/BD$
 $\Rightarrow AP/DP = PC/PB$
 $\therefore AP \cdot PB = PC \cdot DP$
 Hence, proved.

- 8. In Fig. 6.62, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that:**
 (i) $\triangle PAC \sim \triangle PDB$
 (ii) $PA \cdot PB = PC \cdot PD$.



Solution:

- (i) In $\triangle PAC$ and $\triangle PDB$,

$\angle P = \angle P$ (Common Angles)

As we know, exterior angle of a cyclic quadrilateral is $\angle PCA$ and $\angle PBD$ is opposite interior angle, which are both equal.

$\angle PAC = \angle PDB$

Thus, $\Delta PAC \sim \Delta PDB$ (AA similarity criterion)

(ii) We have already proved above,

$\Delta APC \sim \Delta DPB$

We know that the corresponding sides of similar triangles are proportional.

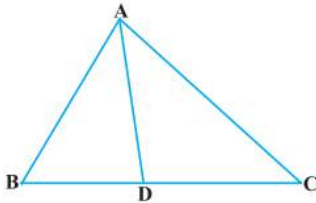
Therefore,

$AP/DP = PC/PB = CA/BD$

$AP/DP = PC/PB$

$\therefore AP \cdot PB = PC \cdot DP$

9. In Figure, D is a point on side BC of ΔABC such that $BD/CD = AB/AC$. Prove that AD is the bisector of $\angle BAC$.

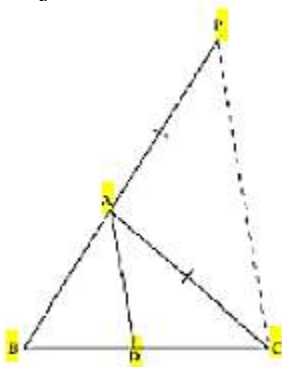


Solution:

In the given figure, let us extend BA to P such that;

$AP = AC$.

Now join PC.



Given, $BD/CD = AB/AC$

$\Rightarrow BD/CD = AP/AC$

By using the converse of basic proportionality theorem, we get,

$AD \parallel PC$

$\angle BAD = \angle APC$ (Corresponding angles) (i)

And, $\angle DAC = \angle ACP$ (Alternate interior angles) (ii)

By the new figure, we have;

$$AP = AC$$

$$\Rightarrow \angle APC = \angle ACP \dots\dots\dots (iii)$$

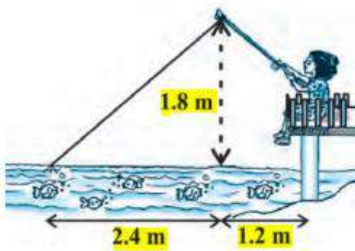
On comparing equations (i), (ii), and (iii), we get,

$$\angle BAD = \angle APC$$

Therefore, AD is the bisector of the angle BAC.

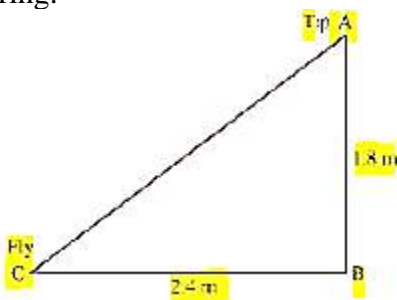
Hence, proved.

10. Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Figure)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



Solution:

Let us consider, AB is the height of the tip of the fishing rod from the water surface and BC is the horizontal distance of the fly from the tip of the fishing rod. Therefore, AC is now the length of the string.



To find AC, we have to use Pythagoras theorem in $\triangle ABC$, is such way;

$$AC^2 = AB^2 + BC^2$$

$$AB^2 = (1.8 \text{ m})^2 + (2.4 \text{ m})^2$$

$$AB^2 = (3.24 + 5.76) \text{ m}^2$$

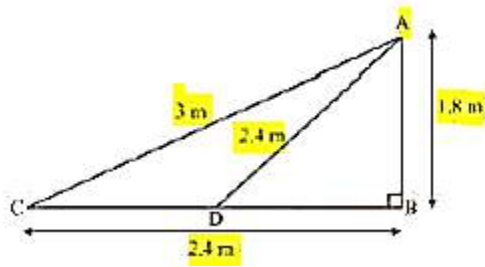
$$AB^2 = 9.00 \text{ m}^2$$

$$\Rightarrow AB = \sqrt{9} \text{ m} = 3 \text{ m}$$

Thus, the length of the string out is 3 m.

As its given, she pulls the string at the rate of 5 cm per second.

Therefore, string pulled in 12 seconds = $12 \times 5 = 60 \text{ cm} = 0.6 \text{ m}$



Let us say now, the fly is at point D after 12 seconds.

Length of string out after 12 seconds is AD.

$AD = AC - \text{String pulled by Nazima in 12 seconds}$

$$= (3.00 - 0.6) \text{ m}$$

$$= 2.4 \text{ m}$$

In $\triangle ADB$, by Pythagoras Theorem,

$$AB^2 + BD^2 = AD^2$$

$$(1.8 \text{ m})^2 + BD^2 = (2.4 \text{ m})^2$$

$$BD^2 = (5.76 - 3.24) \text{ m}^2 = 2.52 \text{ m}^2$$

$$BD = 1.587 \text{ m}$$

Horizontal distance of fly = $BD + 1.2 \text{ m}$

$$= (1.587 + 1.2) \text{ m} = 2.787 \text{ m}$$

$$= 2.79 \text{ m}$$