Exercise 8.1 Page: 181

1. In Δ ABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine:

(i) sin A, cos A

(ii) sin C, cos C

Solution:

In a given triangle ABC, right angled at B = ∠B = 90°

Given: AB = 24 cm and BC = 7 cm

According to the Pythagoras Theorem,

In a right- angled triangle, the squares of the hypotenuse side is equal to the sum of the squares of the other two sides.

By applying Pythagoras theorem, we get

$AC^2=AB^2+BC^2$

$AC^2 = (24)^2+7^2$

$AC^2 = (576+49)$

$AC^2 = 625cm^2$

$AC = \sqrt{625} = 25$

Therefore, AC = 25 cm

(i) To find Sin (A), Cos (A)

We know that sine (or) Sin function is the equal to the ratio of length of the opposite side to the hypotenuse side. So it becomes

Sin (A) = Opposite side /Hypotenuse = BC/AC = 7/25

Cosine or Cos function is equal to the ratio of the length of the adjacent side to the hypotenuse side and it becomes,

Cos (A) = Adjacent side/Hypotenuse = AB/AC = 24/25

(ii) To find Sin (C), Cos (C)

Sin (C) = AB/AC = 24/25

Cos (C) = BC/AC = 7/25

2. In Fig. 8.13, find tan P – cot R

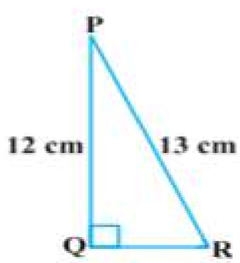

Fig. 8.13

Solution:

In the given triangle PQR, the given triangle is right angled at Q and the given measures are:
PR = 13cm,
PQ = 12cm
Since the given triangle is right angled triangle, to find the side QR, apply the Pythagorean theorem
According to Pythagorean theorem,
In a right- angled triangle, the squares of the hypotenuse side is equal to the sum of the squares of the other two sides.
$PR^2 = QR^2 + PQ^2$
Substitute the values of PR and PQ
$13^2 = QR^2+12^2$
$169 = QR^2+144$
Therefore, $QR^2 = 169-144$
$QR^2 = 25$
$QR = \sqrt{25} = 5$
Therefore, the side QR = 5 cm
To find tan P – cot R:
According to the trigonometric ratio, the tangent function is equal to the ratio of the length of the opposite side to the adjacent sides, the value of tan (P) becomes
tan (P) = Opposite side /Hypotenuse = QR/PQ = 5/12
Since cot function is the reciprocal of the tan function, the ratio of cot function becomes,
Cot (R) = Adjacent side/Hypotenuse = QR/PQ = 5/12

Therefore,
tan (P) – cot (R) = 5/12 – 5/12 = 0
Therefore, tan(P) – cot(R) = 0

3. If sin A = 3/4, Calculate cos A and tan A.

Solution:
Let us assume a right angled triangle ABC, right angled at B
Given: Sin A = 3/4
We know that, Sin function is the equal to the ratio of length of the opposite side to the hypotenuse side.

Therefore, Sin A = Opposite side /Hypotenuse= 3/4
Let BC be 3k and AC will be 4k
where k is a positive real number.
According to the Pythagoras theorem, the squares of the hypotenuse side is equal to the sum of the squares of the other two sides of a right angle triangle and we get,
$AC^2=AB^2 + BC^2$
Substitute the value of AC and BC
$(4k)^2=AB^2 + (3k)^2$
$16k^2-9k^2 =AB^2$
$AB^2=7k^2$
Therefore, $AB = \sqrt{7}k$
Now, we have to find the value of cos A and tan A
We know that,

Cos (A) = Adjacent side/Hypotenuse
Substitute the value of AB and AC and cancel the constant k in both numerator and denominator, we get
AB/AC = √7k/4k = √7/4
Therefore, cos (A) = √7/4

tan(A) = Opposite side/Adjacent side
Substitute the Value of BC and AB and cancel the constant k in both numerator and denominator, we get,
BC/AB = 3k/√7k = 3/√7
Therefore, tan A = 3/√7

4. Given 15 cot A = 8, find sin A and sec A.

Solution:
Let us assume a right angled triangle ABC, right angled at B
Given: 15 cot A = 8
So, Cot A = 8/15
We know that, cot function is the equal to the ratio of length of the adjacent side to the opposite side.

Therefore, cot A = Adjacent side/Opposite side = AB/BC = 8/15

Let AB be 8k and BC will be 15k
Where, k is a positive real number.
According to the Pythagoras theorem, the squares of the hypotenuse side is equal to the sum of the squares of the other two sides of a right angle triangle and we get,
$AC^2=AB^2 + BC^2$
Substitute the value of AB and BC
$AC^2= (8k)^2 + (15k)^2$
$AC^2= 64k^2 + 225k^2$
$AC^2= 289k^2$
Therefore, AC = 17k
Now, we have to find the value of sin A and sec A
We know that,
Sin (A) = Opposite side /Hypotenuse

Substitute the value of BC and AC and cancel the constant k in both numerator and denominator, we get
Sin A = BC/AC = 15k/17k = 15/17

Therefore, sin A = 15/17
Since secant or sec function is the reciprocal of the cos function which is equal to the ratio of the length of the hypotenuse side to the adjacent side.

Sec (A) = Hypotenuse/Adjacent side
Substitute the Value of BC and AB and cancel the constant k in both numerator and denominator, we get,

AC/AB = 17k/8k = 17/8
Therefore sec (A) = 17/8

5. Given sec θ = 13/12 . Calculate all other trigonometric ratios

Solution:
We know that sec function is the reciprocal of the cos function which is equal to the ratio of the length of the hypotenuse side to the adjacent side
Let us assume a right angled triangle ABC, right angled at B
sec θ =13/12 = Hypotenuse/Adjacent side = AC/AB
Let AC be 13k and AB will be 12k
Where, k is a positive real number.
According to the Pythagoras theorem, the squares of the hypotenuse side is equal to the sum of the squares of the other two sides of a right angle triangle and we get,
$AC^2=AB^2 + BC^2$
Substitute the value of AB and AC
$(13k)^2= (12k)^2 + BC^2$
$169k^2= 144k^2 + BC^2$
$169k^2= 144k^2 + BC^2$
$BC^2 = 169k^2 - 144k^2$
$BC^2= 25k^2$
Therefore, BC = 5k

Now, substitute the corresponding values in all other trigonometric ratios
So,
Sin θ = Opposite Side/Hypotenuse = BC/AC = 5/13
Cos θ = Adjacent Side/Hypotenuse = AB/AC = 12/13
tan θ = Opposite Side/Adjacent Side = BC/AB = 5/12

Cosec θ = Hypotenuse/Opposite Side = AC/BC = 13/5

cot θ = Adjacent Side/Opposite Side = AB/BC = 12/5

6. If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠ A = ∠ B.
Solution:

Let us assume the triangle ABC in which CD⊥AB
Give that the angles A and B are acute angles, such that
Cos (A) = cos (B)
As per the angles taken, the cos ratio is written as
AD/AC = BD/BC
Now, interchange the terms, we get
AD/BD = AC/BC
Let take a constant value

AD/BD = AC/BC = k

Now consider the equation as
AD = k BD …(1)
AC = k BC …(2)
By applying Pythagoras theorem in △CAD and △CBD we get,
$CD^2 = BC^2 – BD^2$ … (3)
$CD^2 = AC^2 − AD^2$ ….(4)
From the equations (3) and (4) we get,
$AC^2−AD^2 = BC^2−BD^2$
Now substitute the equations (1) and (2) in (3) and (4)
$K^2(BC^2−BD^2)=(BC^2−BD^2)$ $k^2=1$
Putting this value in equation, we obtain
AC = BC
∠A=∠B (Angles opposite to equal side are equal-isosceles triangle)

7. If cot θ = 7/8, evaluate :
(i) (1 + sin θ)(1 - sin θ)/(1+cos θ)(1-cos θ)
(ii) $\cot^2 \theta$

Solution:
Let us assume a △ABC in which ∠B = 90° and ∠C = θ
Given:
cot θ = BC/AB = 7/8
Let BC = 7k and AB = 8k, where k is a positive real number
According to Pythagoras theorem in △ABC we get.

$AC^2 = AB^2+BC^2$
$AC^2 = (8k)^2+(7k)^2$
$AC^2 = 64k^2+49k^2$
$AC^2 = 113k^2$
$AC = \sqrt{113}\ k$

According to the sine and cos function ratios, it is written as

sin θ = AB/AC = Opposite Side/Hypotenuse = 8k/√113 k = 8/√113 and

cos θ = Adjacent Side/Hypotenuse = BC/AC = 7k/√113 k = 7/√113

Now apply the values of sin function and cos function:

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{49}{64}$$

(ii) $\cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

8. If 3 cot A = 4, check whether $(1-\tan^2 A)/(1+\tan^2 A) = \cos^2 A - \sin^2 A$ or not.

Solution:

Let △ABC in which ∠B=90°

We know that, cot function is the reciprocal of tan function and it is written as

cot(A) = AB/BC = 4/3

Let AB = 4k an BC =3k, where k is a positive real number.

According to the Pythagorean theorem,

$AC^2=AB^2+BC^2$

$AC^2=(4k)^2+(3k)^2$

$AC^2=16k^2+9k^2$

$AC^2=25k^2$

AC=5k

Now, apply the values corresponding to the ratios

tan(A) = BC/AB = 3/4

sin (A) = BC/AC = 3/5

cos (A) = AB/AC = 4/5

Now compare the left hand side(LHS) with right hand side(RHS)

$$\text{L.H.S.} = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{7}{25}$$

$$\text{R.H.S.} = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

Since, both the LHS and RHS = 7/25

R.H.S. =L.H.S.

Hence, **$(1-\tan^2 A)/(1+\tan^2 A) = \cos^2 A - \sin^2 A$** is proved

9. In triangle ABC, right-angled at B, if tan A = 1/√3 find the value of:

(i) sin A cos C + cos A sin C

(ii) cos A cos C – sin A sin C

Solution:

Let ΔABC in which $\angle B=90°$

tan A = BC/AB = $1/\sqrt{3}$
Let BC = 1k and AB = $\sqrt{3}$ k,
Where k is the positive real number of the problem
By Pythagoras theorem in ΔABC we get:
$AC^2=AB^2+BC^2$
$AC^2=(\sqrt{3}\ k)^2+(k)^2$
$AC^2=3k^2+k^2$
$AC^2=4k^2$
AC = 2k
Now find the values of cos A, Sin A
Sin A = BC/AC = 1/2
Cos A = AB/AC = $\sqrt{3}/2$
Then find the values of cos C and sin C
Sin C = AB/AC = $\sqrt{3}/2$
Cos C = BC/AC = 1/2
Now, substitute the values in the given problem

(i) sin A cos C + cos A sin C = $(1/2) \times (1/2) + \sqrt{3}/2 \times \sqrt{3}/2 = 1/4 + 3/4 = 1$
(ii) cos A cos C – sin A sin C = $(\sqrt{3}/2)(1/2) - (1/2)(\sqrt{3}/2) = 0$

10. In Δ PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P

Solution:

In a given triangle PQR, right angled at Q, the following measures are
PQ = 5 cm
PR + QR = 25 cm
Now let us assume, QR = x
PR = 25-QR
PR = 25- x
According to the Pythagorean Theorem,
$PR^2 = PQ^2 + QR^2$
Substitute the value of PR as x
$(25- x)^2 = 5^2 + x^2$
$25^2 + x^2 - 50x = 25 + x^2$
$625 + x^2 - 50x - 25 - x^2 = 0$
-50x = -600
x= -600/-50
x = 12 = QR
Now, find the value of PR
PR = 25- QR
Substitute the value of QR
PR = 25-12
PR = 13
Now, substitute the value to the given problem

(1) sin p = Opposite Side/Hypotenuse = QR/PR = 12/13
(2) Cos p = Adjacent Side/Hypotenuse = PQ/PR = 5/13
(3) tan p =Opposite Side/Adjacent side = QR/PQ = 12/5

11. State whether the following are true or false. Justify your answer.
(i) The value of tan A is always less than 1.
(ii) sec A = 12/5 for some value of angle A.
(iii)cos A is the abbreviation used for the cosecant of angle A.
(iv) cot A is the product of cot and A.
(v) sin θ = 4/3 for some angle θ.
Solution:

(i) The value of tan A is always less than 1.
Answer: **False**
Proof: In ΔMNC in which ∠N = 90∘,
MN = 3, NC = 4 and MC = 5
Value of tan M = 4/3 which is greater than.
The triangle can be formed with sides equal to 3, 4 and hypotenuse = 5 as it will follow the Pythagoras theorem.
$MC^2=MN^2+NC^2$
$5^2=3^2+4^2$
25=9+16
25 = 25

(ii) sec A = 12/5 for some value of angle A
Answer: **True**
Justification: Let a ΔMNC in which ∠N = 90º,
MC=12k and MB=5k, where k is a positive real number.
By Pythagoras theorem we get,
$MC^2=MN^2+NC^2$
$(12k)^2=(5k)^2+NC^2$
$NC^2+25k^2=144k^2$
$NC^2=119k^2$
Such a triangle is possible as it will follow the Pythagoras theorem.

(iii) cos A is the abbreviation used for the cosecant of angle A.
Answer: **False**
Justification: Abbreviation used for cosecant of angle M is cosec M. cos M is the abbreviation used for cosine of angle M.

(iv) cot A is the product of cot and A.
Answer: **False**
Justification: cot M is not the product of cot and M. It is the cotangent of ∠M.

(v) sin θ = 4/3 for some angle θ.

Answer**: False**

Justification: sin θ = Height/Hypotenuse

We know that in a right angled triangle, Hypotenuse is the longest side.

∴ sin θ will always less than 1 and it can never be 4/3 for any value of θ.