Exercise 8.3

1. Evaluate :
(i) sin 18°/cos 72°
(ii) tan 26°/cot 64°
(iii) cos 48° – sin 42°
(iv) cosec 31° – sec 59°

Solution:

(i) sin 18°/cos 72°
To simplify this, convert the sin function into cos function
We know that, 18° is written as 90° - 18°, which is equal to the cos 72°.
= sin (90° - 18°) /cos 72°
Substitute the value, to simplify this equation
= cos 72° /cos 72° = 1

(ii) tan 26°/cot 64°
To simplify this, convert the tan function into cot function
We know that, 26° is written as 90° - 36°, which is equal to the cot 64°.
= tan (90° - 36°)/cot 64°
Substitute the value, to simplify this equation
= cot 64°/cot 64° = 1

(iii) cos 48° - sin 42°
To simplify this, convert the cos function into sin function
We know that, 48° is written as 90° - 42°, which is equal to the sin 42°.
= cos (90° - 42°) - sin 42°
Substitute the value, to simplify this equation
= sin 42° - sin 42° = 0

(iv) cosec 31° - sec 59°
To simplify this, convert the cosec function into sec function
We know that, 31° is written as 90° - 59°, which is equal to the sec 59°
= cosec (90° - 59°) - sec 59°
Substitute the value, to simplify this equation
= sec 59° - sec 59° = 0

2. Show that:
(i) tan 48° tan 23° tan 42° tan 67° = 1
(ii) cos 38° cos 52° – sin 38° sin 52° = 0
Solution:

(i) tan 48° tan 23° tan 42° tan 67°
Simplify the given problem by converting some of the tan functions to the cot functions

We know that, tan 48° = tan (90° - 42°) = cot 42°
tan 23° = tan (90° - 67°) = cot 67°
= tan (90° - 42°) tan (90° - 67°) tan 42° tan 67°
Substitute the values
= cot 42° cot 67° tan 42° tan 67°
= (cot 42° tan 42°) (cot 67° tan 67°) = 1×1 = 1

(ii) cos 38° cos 52° - sin 38° sin 52°
Simplify the given problem by converting some of the cos functions to the sin functions
We know that,
cos 38° = cos (90° - 52°) = sin 52°
cos 52°= cos (90°-38°) = sin 38°
= cos (90° - 52°) cos (90°-38°) - sin 38° sin 52°
Substitute the values
= sin 52° sin 38° - sin 38° sin 52° = 0

3. If tan 2A = cot (A – 18°), where 2A is an acute angle, find the value of A.

Solution:
tan 2A = cot (A- 18°)
We know that tan 2A = cot (90° - 2A)
Substitute the above equation in the given problem
⇒ cot (90° - 2A) = cot (A -18°)
Now, equate the angles,
⇒ 90° - 2A = A- 18° ⇒ 108° = 3A
A = 108° / 3
Therefore, the value of A = 36°

4. If tan A = cot B, prove that A + B = 90°.

Solution:
tan A = cot B
We know that cot B = tan (90° - B)
To prove A + B = 90°, substitute the above equation in the given problem
tan A = tan (90° - B)
A = 90° - B
A + B = 90°
Hence Proved.

5. If sec 4A = cosec (A – 20°), where 4A is an acute angle, find the value of A.
Solution:

sec 4A = cosec (A - 20°)
We know that sec 4A = cosec (90° - 4A)
To find the value of A, substitute the above equation in the given problem
cosec (90° - 4A) = cosec (A - 20°)

Now, equate the angles
90° - 4A= A- 20°
110° = 5A
A = 110°/ 5 = 22°
Therefore, the value of A = 22°

6. If A, B and C are interior angles of a triangle ABC, then show that
sin (B+C/2) = cos A/2

Solution:

We know that, for a given triangle, sum of all the interior angles of a triangle is equal to 180°
A + B + C = 180° ….(1)
To find the value of (B+ C)/2, simplify the equation (1)
⇒ B + C = 180° - A
⇒ (B+C)/2 = (180°-A)/2
⇒ (B+C)/2 = (90°-A/2)
Now, multiply both sides by sin functions, we get
⇒ sin (B+C)/2 = sin (90°-A/2)
Since sin (90°-A/2) = = cos A/2, the above equation is equal to
sin (B+C)/2 = cos A/2
Hence proved.

7. Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

Solution:

Given:
sin 67° + cos 75°
In term of sin as cos function and cos as sin function, it can be written as follows
sin 67° = sin (90° - 23°)
cos 75° = cos (90° - 15°)
= sin (90° - 23°) + cos (90° - 15°)
Now, simplify the above equation
= cos 23° + sin 15°
Therefore, sin 67° + cos 75° is also expressed as cos 23° + sin 15°