MATHEMATICS PAPER

Code: SS-15-Mathematics

Time: $3\frac{1}{4}$ **Hours** **M.M. 80**

GENERAL INSTRUCTIONS TO THE EXAMINEES:

1. Candidate must write first his / her Roll No. on the question paper compulsorily.

2. All the questions are compulsory.

3. Write the answer to each question in the given answer-book only.

4. For questions having more than one part the answers to those parts are to be written together in continuity.

5. If there is any error / difference / contradiction in Hindi & English versions of the question paper, the question of Hindi version should be treated valid.

6.

Section	**Q. Nos.**	**Marks per questions**
A	1-10	1
B	11 - 25	3
C	26 - 30	5

7. There are internal choices in Q. Nos. 11, 12, 15, 17, 29 and 30. You have to attempt only one of the alternatives in these questions.

8. Draw the graph of Q. No. 23 on the graph paper

SECTION - A

1. Find the value of $\sin\left(\frac{\pi}{3}+\sin^{-1}\left(-\frac{1}{2}\right)\right)$

2. If $A=\begin{bmatrix}2 & 4\\ -3 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}$, then find $2A-B$.

3. If $A=\begin{bmatrix}2 & -4 & 3\end{bmatrix}$ and $B=\begin{bmatrix}2\\-4\\8\end{bmatrix}$ then find $(AB)'$.

4. Find : $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx$

5. Find the general solution of the differential equation :

$$\frac{dy}{dx}=\frac{2x}{y^2}$$

6. If vector $\vec{a}=2\hat{i}-2\hat{j}+2\hat{k}$ and vector $\vec{b}=\hat{i}+\hat{j}-\hat{k}$ then find unit vector along the vector $\left(\vec{a}+\vec{b}\right)$.

7. Find the Cartesian form of equation of the line passing through the points (1, 0, 2) and (4, 5, 6)

8. If a line makes 120°, 45° and 90° angles with the x, y and z-axis respectively then find its direction-cosines.

9. Show the region of feasible solution under the following constraints:

$x+3y\geq 6; x\geq 0, y\geq 0$ in answer book.

10. If $P\left(\frac{B}{A}\right)=0.2$ and P(A) = 0.8, then find $P\left(A\cap B\right)$.

SECTION - B

11. Prove that the relation R defined on set Z as $aRb \Leftrightarrow a-b$ is divisible by 3 is an equivalence relation.

OR

If function $f,g:R\rightarrow R$ are defined as $f\left(x\right)=x^2, g\left(x\right)=2x$ then find fog(x), gof(x) and fof(3)

12. Express the function $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right);\frac{\pi}{4}<x<\frac{3\pi}{4}$ in the simplest form.

OR

Prove that : $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$

13. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$; then prove that $A^2-5A+7I_2=0$, where I_2 is the identity matrix of order 2.

14. Examine the continuity of function $f\left(x\right)=\begin{cases}x+5 & x\le 1\\ x-5 & x>1\end{cases}$ at point x = 1.

15. Find the equation of the tangent to the curve $y=x^3-x+1$ at the point whose x coordinate is 1.

OR

The length x of a rectangle is decreasing at the rate 3 cm/minute and the width y is increasing at the rate 5cm/minute. When x = 10 cm and y = 6 cm, find the area of the rectangle.

16. Find the maximum profit that a company can make, if the profit function is given by $P\left(x\right)=51-72x-18x^2$.

17. Find : $\int\frac{dx}{x\left(x^5+1\right)}$

OR

Find: $\int\frac{x\sin^{-1}}{\sqrt{1-x^2}}dx$

18. Find: $\int \frac{\sec^2 x dx}{\sqrt{\tan^2 x + 4}}$

19. Find the area of the region bounded by parabola $y^2 = 16x$ and the lines $x = 1, x = 4$ and x-axis in the first quadrant.

20. Using integration find the area of region bounded by the triangle ABC whose vertices are $A(1,0), B(2,2)$ and $C(3,1)$

21. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$, then find the angle between the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$

22. Find the area of a parallelogram whose adjacent sides are vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

23. By graphical method solve the following linear programming problem for minimize.

Objective function $Z = 5x + 7y$

Constraints $2x + y \geq 8$

$x + 2y \geq 10$

24. Given three identical boxes I, II and III each containing two coins. In box I both coins are gold coins in box II both are silver coins and in the box III there is one gold and one silver coin. A person chooses a box at random and take out a coin. If the coin is of silver what is the probability that the other coin in the box is also of silver.

25. Find the variance of the number obtained on a throw of an unbiased die.

SECTION – C

26. Show that $\begin{vmatrix} a & a^2 & 1+pa^3 \\ b & b^2 & 1+pb^3 \\ c & c^2 & 1+pc^3 \end{vmatrix} = (1+pabc)(a-b)(b-c)(c-a)$

27. If $y = x^x + x^p + p^x + p^p$, $p > 0$ and $x > 0$, then find $\frac{dy}{dx}$

28. Show that $\int_0^{\pi} \frac{xdx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{\pi^2}{2ab}$

29. Find the solution of the differential equation $(x-y)dy-(x+y)dx=0$.

OR

Find the solution of the differential equation

$$\cos^2 x.\frac{dy}{dx}+y=\tan x\left(0\le x\le\frac{\pi}{2}\right)$$

30. Find the shortest distance between the lines.

$\vec{r}=\left(\hat{i}-2\hat{j}+3\hat{k}\right)+\lambda\left(-\hat{i}+\hat{j}-2\hat{k}\right)$ and $\vec{r}=\left(\hat{i}-\hat{j}-\hat{k}\right)+\mu\left(\hat{i}+2\hat{j}-2\hat{k}\right)$

OR

Find the equation of the plane that contains the point (2, -1, 3) and is perpendicular to each of the planes $2x+3y-2z=5$ and $x+2y-3z=8$.