GOVERNMENT OF TAMIL NADU

STANDARD NINE

TERM - II

VOLUME 2

MATHEMATICS

Untouchability is Inhuman and a Crime

A publication under Free Textbook Programme of Government of Tamil Nadu

Department Of School Education

Government of Tamil Nadu

First Edition - 2018

(Published under Uniform System of School Education Scheme in Trimester Pattern)

NOT FOR SALE

Content Creation

State Council of Educational Research and Training

Printing & Publishing

Tamil NaduTextbook and Educational Services Corporation

www.textbooksonline.tn.nic in

SYMBOLS

$=$	equal to
$\neq$	not equal to
$<$	less than
$\leq$	less than or equal to
$>$	greater than
$\geq$	greater than or equal to
$\approx$	equivalent to
$\cup$	union
$\cap$	intersection
$\mathbb{U}$	universal Set
$\in$	belongs to
$\notin$	does not belong to
$\subset$	proper subset of
$\subseteq$	subset of or is contained in
$\not\subset$	not a proper subset of
$\not\subseteq$	not a subset of or is not contained in
A' (or) A^c	complement of A
$\varnothing$ (or) $\{\ \}$	empty set or null set or void set
$n(A)$	number of elements in the set A
$P(A)$	power set of A
$\sum$	summation

$\|\|\|^{ly}$	similarly
Δ	symmetric difference
$\mathbb{N}$	natural numbers
$\mathbb{W}$	whole numbers
$\mathbb{Z}$	integers
$\mathbb{R}$	real numbers
$\triangle$	triangle
$\angle$	angle
$\perp$	perpendicular to
$\parallel$	parallel to
$\Longrightarrow$	implies
$\therefore$	therefore
$\because$	since (or) because
$\lvert\ \rvert$	absolute value
$\simeq$	approximately equal to
$\mid$ (or) :	such that
$\equiv$ (or) $\cong$	congruent
$\equiv$	identically equal to
π	pi
$\pm$	plus or minus

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CONTENTS

Captions used in this Textbook

எண்ணென்ப ஏனை எழுத்தென்ப இவ்விரண்டும்
கண்ணென்ப வாழும் உயிர்க்கு – குறள் 392

Numbers and letters, they are known as
eyes to humans, they are. Kural 392

Learning Outcomes

To transform the classroom processes into learning centric with a set of bench marks

Note

To provide additional inputs for students in the content

Activity / Project

To encourage students to involve in activities to learn mathematics

ICT Corner

To encourage learner's understanding of content through application of technology

Thinking Corner

To kindle the inquisitiveness of students in learning mathematics. To make the students to have a diverse thinking

Points to Remember

To recall the points learnt in the topic

Multiple Choice Questions

To provide additional assessment items on the content

Progress Check

Self evaluation of the learner's progress

Exercise

To evaluate the learners' in understanding the content

"The essence of mathematics is not to make simple things complicated but to make complicated things simple" -S. Gudder

QR GUIDE

Let's use the QR code in the text book ! How?

- Download the QR code scanner from the Google PlayStore/ Apple App Store into your smartphone
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- Once the camera detects the QR code, a url appears in the screen.Click the url and go to the content page.

1 SET LANGUAGE

"In mathematics, the art of asking questions is more valuable than solving problems". *- Georg Cantor*

John Venn
(1834-1923)

John Venn was an English mathematician. He invented Venn diagrams which pictorially represent the relations between sets. Venn diagrams are used in the field of Set Theory, Probability, Statistics, Logic and Computer Science.

Learning Outcomes

- To explain the commutative property among set operations.
- To interpret the associative property among set operations.
- To infer the distributive property among set operations.
- To verify De Morgan's laws.
- To use set language in solving life oriented word problems.

1.1 Introduction

A set, we know, is a "well-defined collection of objects". We are aware of different sets such as empty set, finite set, infinite set, subset, power set, equal sets, equivalent sets and universal set. We have performed set operations like union, intersection and difference between two sets.

It is an interesting investigation to find out if operations among sets (like union, intersection, etc) follow mathematical properties such as Commutativity, Associativity, etc., We have seen numbers having many of these properties; whether sets also possess these, is to be explored.

1.2 Properties of Set Operations

We first take up the properties of set operations on union and intersection.

1.2.1 Commutative Property

In set language, commutative situations can be seen when we perform operations. For example, we can look into the Union (and Intersection) of sets to find out if the operation is commutative.

Let $A=\{2,3,8,10\}$ and $B=\{1,3,10,13\}$ be two sets.

Then, $A\cup B=\{1,2,3,8,10,13\}$ and

$B\cup A=\{1,2,3,8,10,13\}$

From the above, we see that $A\cup B = B\cup A$.

This is called Commutative property of union of sets.

Now, $A\cap B=\{3,10\}$ and $B\cap A=\{3,10\}$. Then, we see that $A\cap B=B\cap A$.

This is called Commutative property of intersection of sets.

Note

For any set A,

- $A\cup A=A$ and $A\cap A=A$ [Idempotent Laws].
- $A\cup\phi=A$ and $A\cap U=A$ [Identity Laws].

Commutative property: For any two sets A and B

(i) $A\cup B=B\cup A$ (ii) $A\cap B=B\cap A$

Example 1.1 If $A=\{b,e,f,g\}$ and $B=\{c,e,g,h\}$, then verify the commutative property of (i) union of sets (ii) intersection of sets.

Solution

Given, $A=\{b,e,f,g\}$ and $B=\{c,e,g,h\}$

(i) $A\cup B=\{b,c,e,f,g,h\}$ ... (1)

$B\cup A=\{b,c,e,f,g,h\}$ ... (2)

From (1) and (2) we have $A\cup B=B\cup A$

It is verified that union of sets is commutative.

(ii) $A\cap B=\{e,g\}$ ... (3)

$B\cap A=\{e,g\}$ ... (4)

From (3) and (4) we get, $A\cap B=B\cap A$

It is verified that intersection of sets is commutative.

Thinking Corner

Given, $P=\{l,n,p\}$ and $P\cup Q=\{j,l,m,n,o,p\}$. If P and Q are disjoint sets, then what will be Q and $P\cap Q$?

Note

Recall that subtraction on numbers is not commutative. Is set difference commutative? We expect that the set difference is not commutative as well. For instance, consider $A=\{a,b,c\}$, $B=\{b,c,d\}$. $A-B=\{a\}$, $B-A=\{d\}$; we see that $A-B \neq B-A$.

Progress Check

(1) If $A=\{1,2,3,4\}$ and $B=\{2,4,6,8\}$, then find:

(i) $A\cup B$ (ii) $B\cup A$ (iii) $A\cap B$ (iv) $B\cap A$

(2) Use these to verify commutative property of union and intersection of sets.

1.2.2 Associative Property

Now, we perform operations on union and intersection for three sets.

Let $A=\{-1,0,1,2\}$, $B=\{-3,0,2,3\}$ and $C=\{0,1,3,4\}$ be three sets.

Now, $B\cup C = \{-3,0,1,2,3,4\}$

$A\cup(B\cup C) = \{-1,0,1,2\}\cup\{-3,0,1,2,3,4\}$

$= \{-3,-1,0,1,2,3,4\}$ ... (1)

Then, $A\cup B = \{-3,-1,0,1,2,3\}$

$(A\cup B)\cup C = \{-3,-1,0,1,2,3\}\cup\{0,1,3,4\}$

$= \{-3,-1,0,1,2,3,4\}$ ... (2)

From (1) and (2), $A\cup(B\cup C)=(A\cup B)\cup C$

This is associative property of union among sets A, B, and C.

Now, $B\cap C = \{0,3\}$

$A\cap(B\cap C) = \{-1,0,1,2\}\cap\{0,3\}$

$= \{0\}$ ... (3)

Then, $A\cap B = \{0,2\}$

$(A\cap B)\cap C = \{0,2\}\cap\{0,1,3,4\}$

$= \{0\}$ ... (4)

From (3) and (4), $A\cap(B\cap C)=(A\cap B)\cap C$

This is associative property of intersection among sets A, B and C.

Associative property: For any three sets A, B and C

(i) $A\cup(B\cup C)=(A\cup B)\cup C$ (ii) $A\cap(B\cap C)=(A\cap B)\cap C$

Example 1.2 If $A=\{2,3,4,5\}, B=\{2,3,5,7\}$ and $C=\{1,3,5\}$, then verify $A\cup(B\cup C)=(A\cup B)\cup C$.

Solution

Given, $A=\{2,3,4,5\}, B=\{2,3,5,7\}$ and $C=\{1,3,5\}$

Now, $B\cup C = \{1,2,3,5,7\}$

$A\cup(B\cup C) = \{1,2,3,4,5,7\}$ ... (1)

Then, $A\cup B = \{2,3,4,5,7\}$

$(A\cup B)\cup C = \{1,2,3,4,5,7\}$ ... (2)

From (1) and (2), it is verified that $A\cup(B\cup C) = (A\cup B)\cup C$

Example 1.3 If $A=\left\{-\frac{1}{2},0,\frac{1}{4},\frac{3}{4},2\right\}$, $B=\left\{0,\frac{1}{4},\frac{3}{4},2,\frac{5}{2}\right\}$ and $C=\left\{-\frac{1}{2},\frac{1}{4},1,2,\frac{5}{2}\right\}$, then verify $A\cap(B\cap C)=(A\cap B)\cap C$.

Solution

Now, $(B\cap C) = \left\{\frac{1}{4}, 2, \frac{5}{2}\right\}$

$A\cap(B\cap C) = \left\{\frac{1}{4}, 2\right\}$ ... (1)

Then, $A\cap B = \left\{0, \frac{1}{4}, \frac{3}{4}, 2\right\}$

$(A\cap B)\cap C = \left\{\frac{1}{4}, 2\right\}$ ... (2)

From (1) and (2), it is verified that

$(A\cap B)\cap C = A\cap(B\cap C)$

Note

The set difference in general is not associative

that is, $(A-B)-C \neq A-(B-C)$.

But, if the sets A, B and C are mutually disjoint then the set difference is associative

that is, $(A-B)-C = A-(B-C)$.

Exercise 1.1

1. If $P=\{1,2,5,7,9\}$, $Q=\{2,3,5,9,11\}$, $R=\{3,4,5,7,9\}$ and $S=\{2,3,4,5,8\}$, then find

   (i) $(P\cup Q)\cup R$ (ii) $(P\cap Q)\cap S$ (iii) $(Q\cap S)\cap R$

2. Test for the commutative property of union and intersection of the sets
   $P=\{\,x : x$ is a real number between 2 and 7$\}$ and
   $Q=\{\,x : x$ is an irrational number between 2 and 7$\}$.

3. If $A = \{p,q,r,s\}$, $B = \{m,n,q,s,t\}$ and $C = \{m,n,p,q,s\}$, then verify the associative property of union of sets.

4. Verify the associative property of intersection of sets for $A=\left\{-11, \sqrt{2}, \sqrt{5}, 7\right\}$, $B=\left\{\sqrt{3}, \sqrt{5}, 6, 13\right\}$ and $C=\left\{\sqrt{2}, \sqrt{3}, \sqrt{5}, 9\right\}$.

5. If $A=\{x : x=2^n, n \in W$ and $n<4\}$, $B=\{x : x=2n, n \in \mathbb{N}$ and $n \leq 4\}$ and $C=\{0,1,2,5,6\}$, then verify the associative property of intersection of sets.

1.2.3 Distributive Property

In lower classes, we have studied distributive property of multiplication over addition on numbers. That is, $a \times (b+c) = (a \times b) + (a \times c)$.

We now attempt to verify distributive of intersection of **sets over union**:

Consider three sets, $A=\left\{x, y, z\right\}$, $B=\left\{t, u, x, z\right\}$ and $C=\left\{s, t, x, y\right\}$

Now, $B \cup C = \left\{s, t, u, x, y, z\right\}$

$A \cap (B \cup C) = \left\{x, y, z\right\}$ ... (1)

Then, $A \cap B = \left\{x, z\right\}$ and $A \cap C = \left\{x, y\right\}$

$(A \cap B) \cup (A \cap C) = \left\{x, y, z\right\}$ ... (2)

From (1) and (2), we get $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

This is called distributive property of intersection over union.

Union of sets also distributes over intersection :

Now, $B \cap C = \{t, x\}$

$A \cup (B \cap C) = \{t, x, y, z\}$ ... (3)

Then, $A \cup B = \{t, u, x, y, z\}$ and $A \cup C = \{s, t, x, y, z\}$

$(A \cup B) \cap (A \cup C) = \{t, x, y, z\}$ ... (4)

From (3) and (4) we get $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

This is called the distributive property of union over intersection.

Distributive property: For any three sets A, B and C

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [Intersection over union]

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [Union over intersection]

Example 1.4

If $A=\{0,2,4,6,8\}, B=\{x: x$ is a prime number and $x<11\}$ and $C=\{x: x\in N$ and $5\le x<9\}$ then verify $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$.

Solution

Given $A=\{0,2,4,6,8\}$, $B=\{2,3,5,7\}$ and $C=\{5,6,7,8\}$

First, we find $B\cap C=\{5,7\}$

$A\cup(B\cap C)=\{0,2,4,5,6,7,8\}$ ... (1)

Next, $A\cup B=\{0,2,3,4,5,6,7,8\}$, $A\cup C=\{0,2,4,5,6,7,8\}$

Then, $(A\cup B)\cap(A\cup C)=\{0,2,4,5,6,7,8\}$ ... (2)

From (1) and (2), it is verified that $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$.

Example 1.5

Verify $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$ using Venn diagrams.

Solution

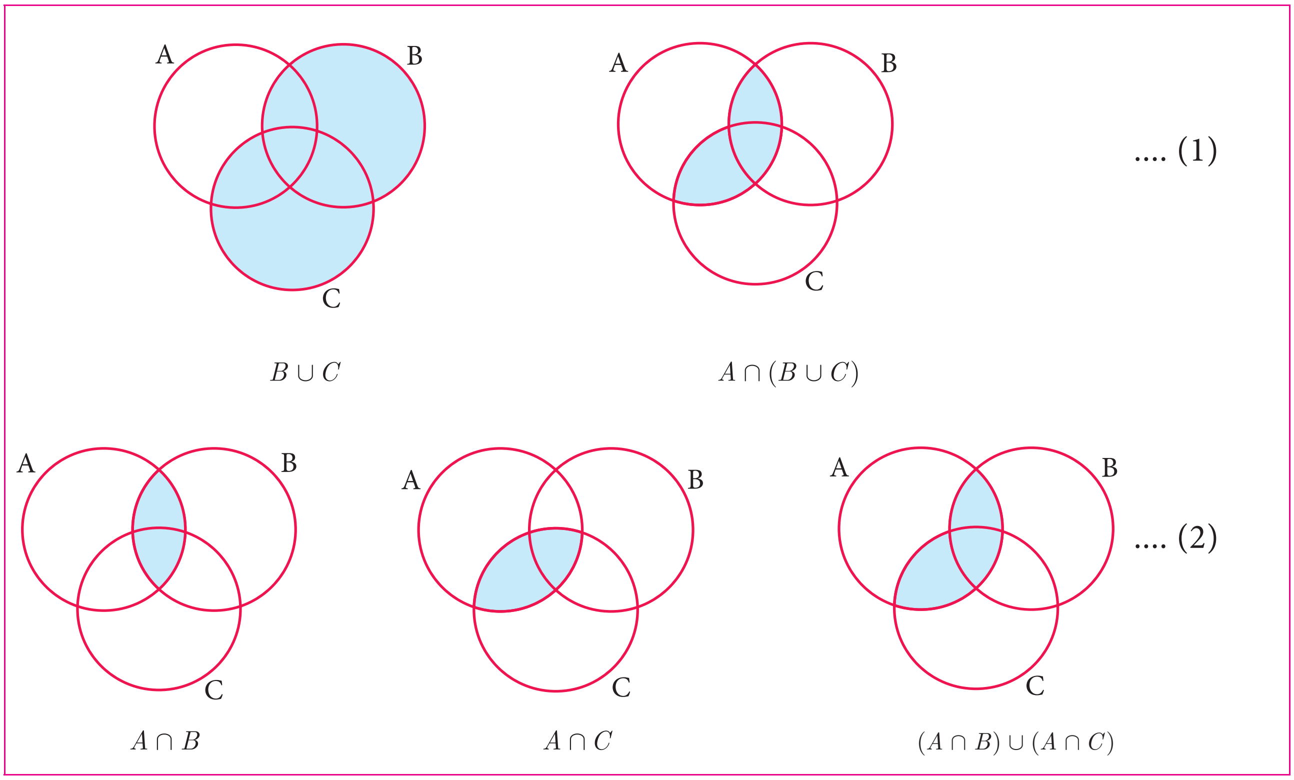


Fig.1.1

From (1) and (2), $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$ is verified.

Exercise 1.2

1. If $K = \{a, b, d, e, f\}$, $L = \{b, c, d, g\}$ and $M = \{a, b, c, d, h\}$, then find the following:

   (i) $K \cup (L \cap M)$ (ii) $K \cap (L \cup M)$

   (iii) $(K \cup L) \cap (K \cup M)$ (iv) $(K \cap L) \cup (K \cap M)$

2. Draw Venn diagram for each of the following:

   (i) $A \cup (B \cap C)$ (ii) $A \cap (B \cup C)$

   (iii) $(A \cup B) \cap C$ (iv) $(A \cap B) \cup C$

3. If $A = \{11, 13, 14, 15, 16, 18\}$, $B = \{11, 12, 15, 16, 17, 19\}$ and $C = \{13, 15, 16, 17, 18, 20\}$, then verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

4. If $A = \{x : x \in \mathbb{Z}, -2 < x \leq 4\}$, $B = \{x : x \in \mathbb{W}, x \leq 5\}$, $C = \{-4, -1, 0, 2, 3, 4\}$, then verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

5. Verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ using Venn diagrams.

1.3 De Morgan's Laws

Augustus De Morgan (1806 – 1871) was a British mathematician. He was born on 27th June 1806 in Madurai, Tamilnadu, India. His father was posted in India by the East India Company. When he was seven months old, his family moved back to England. De Morgan was educated at Trinity College, Cambridge, London. He formulated laws for set difference and complementation. These are called De Morgan's laws.

1.3.1 De Morgan's Laws for Set Difference

These laws relate the set operations union, intersection and set difference.

Let us consider three sets A, B and C as $A = \{-5, -2, 1, 3\}$, $B = \{-3, -2, 0, 3, 5\}$ and $C = \{-2, -1, 0, 4, 5\}$.

Now, $B \cup C = \{-3, -2, -1, 0, 3, 4, 5\}$

$$A - (B \cup C) = \{-5, 1\} \quad \text{... (1)}$$

Then, $A - B = \{-5, 1\}$ and $A - C = \{-5, 1, 3\}$

$$(A - B) \cup (A - C) = \{-5, 1, 3\} \quad \text{... (2)}$$

$$(A - B) \cap (A - C) = \{-5, 1\} \quad \text{... (3)}$$

From (1) and (2), we see that

$$A-(B\cup C) \neq (A-B)\cup(A-C)$$

But note that from (1) and (3), we see that

$$A-(B\cup C) = (A-B)\cap(A-C)$$

Now, $B\cap C = \{-2,0,5\}$

$$A-(B\cap C) = \{-5,1,3\} \quad \ldots (4)$$

From (3) and (4) we see that

$$A-(B\cap C) \neq (A-B)\cap(A-C)$$

But note that from (2) and (4), we get $A-(B\cap C)=(A-B)\cup(A-C)$

Thinking Corner

$(A-B)\cap(B-A)=$____

Thinking Corner

$(A-B)\cup(A-C)\cup(A\cap B)=$____

De Morgan's laws for set difference : For any three sets A, B and C

(i) $A-(B\cup C)=(A-B)\cap(A-C)$ (ii) $A-(B\cap C)=(A-B)\cup(A-C)$

Example 1.6 Verify $A-(B\cup C)=(A-B)\cap(A-C)$ using Venn diagrams.

Solution

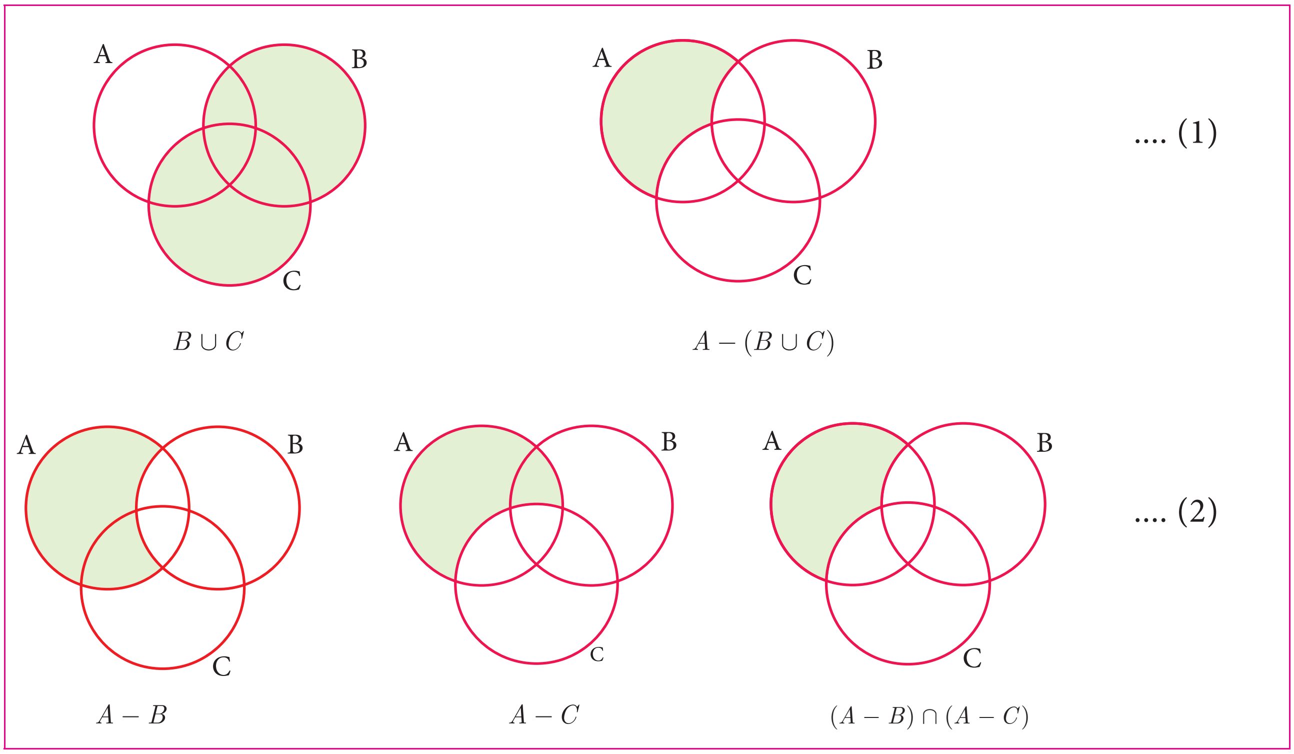


Fig.1.2

From (1) and (2), we get $A-(B\cup C)=(A-B)\cap(A-C)$. Hence it is verified.

Example 1.7 If $P = \{x : x \in \mathbb{W}$ and $0 < x < 10\}$, $Q = \{x : x = 2n+1, n \in \mathbb{W}$ and $n<5\}$ and $R = \{2,3,5,7,11,13\}$, then verify $P - (Q \cap R) = (P - Q) \cup (P - R)$

Solution

The roster form of sets P, Q and R are

$P = \{1,2,3,4,5,6,7,8,9\}$, $Q = \{1,3,5,7,9\}$

and $R = \{2,3,5,7,11,13\}$

First, we find $(Q \cap R) = \{3,5,7\}$

Then, $P - (Q \cap R) = \{1,2,4,6,8,9\}$ ... (1)

Next, $P - Q = \{2,4,6,8\}$ and

$P - R = \{1,4,6,8,9\}$

and so, $(P - Q) \cup (P - R) = \{1,2,4,6,8,9\}$ ... (2)

Hence from (1) and (2), it is verified that $P - (Q \cap R) = (P - Q) \cup (P - R)$.

> Finding the elements of set Q
>
> Given, $x = 2n + 1$
>
> $n = 0 \rightarrow x = 2(0) + 1 = 0 + 1 = 1$
>
> $n = 1 \rightarrow x = 2(1) + 1 = 2 + 1 = 3$
>
> $n = 2 \rightarrow x = 2(2) + 1 = 4 + 1 = 5$
>
> $n = 3 \rightarrow x = 2(3) + 1 = 6 + 1 = 7$
>
> $n = 4 \rightarrow x = 2(4) + 1 = 8 + 1 = 9$
>
> Therefore, x takes values such as 1, 3, 5, 7 and 9.

Progress Check

1. If $A = \{a,b,c,d\}$, $B = \{b,d,e,f\}$ and $C = \{a,b,d,f\}$, then find
   (i) $B \cup C$ (ii) $A - (B \cup C)$ (iii) $A - B$ (iv) $A - C$
2. If P, Q and R are three sets, then draw Venn diagram for the following:
   (i) $Q \cup R$ (ii) $P - (Q \cap R)$ (iii) $P - Q$ (iv) $P - R$

1.3.2 De Morgan's Laws for Complementation

These laws relate the set operations on union, intersection and complementation.

Let us consider universal set $U=\{0,1,2,3,4,5,6\}$, $A=\{1,3,5\}$ and $B=\{0,3,4,5\}$.

Now, $A \cup B = \{0,1,3,4,5\}$

Then, $(A \cup B)' = \{2,6\}$ .....(1)

Next, $A' = \{0,2,4,6\}$ and $B' = \{1,2,6\}$

Then, $A' \cap B' = \{2,6\}$ .....(2)

From (1) and (2), we get $(A \cup B)' = A' \cap B'$

> **Thinking Corner**
>
> Check whether $A - B = A \cap B'$

Also, $A \cap B = \{3, 5\}$,

$$(A \cap B)' = \{0, 1, 2, 4, 6\} \quad .....(3)$$

$$A' = \{0, 2, 4, 6\} \text{ and } B' = \{1, 2, 6\}$$

$$A' \cup B' = \{0, 1, 2, 4, 6\} \quad .....(4)$$

From (3) and (4), we get $(A \cap B)' = A' \cup B'$

Thinking Corner

$(A - B) \cup (B - A') =$ _____

De Morgan's laws for complementation : Let 'U' be the universal set containing finite sets A and B. Then (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Example 1.8 Verify $(A \cup B)' = A' \cap B'$ using Venn diagrams.

Solution

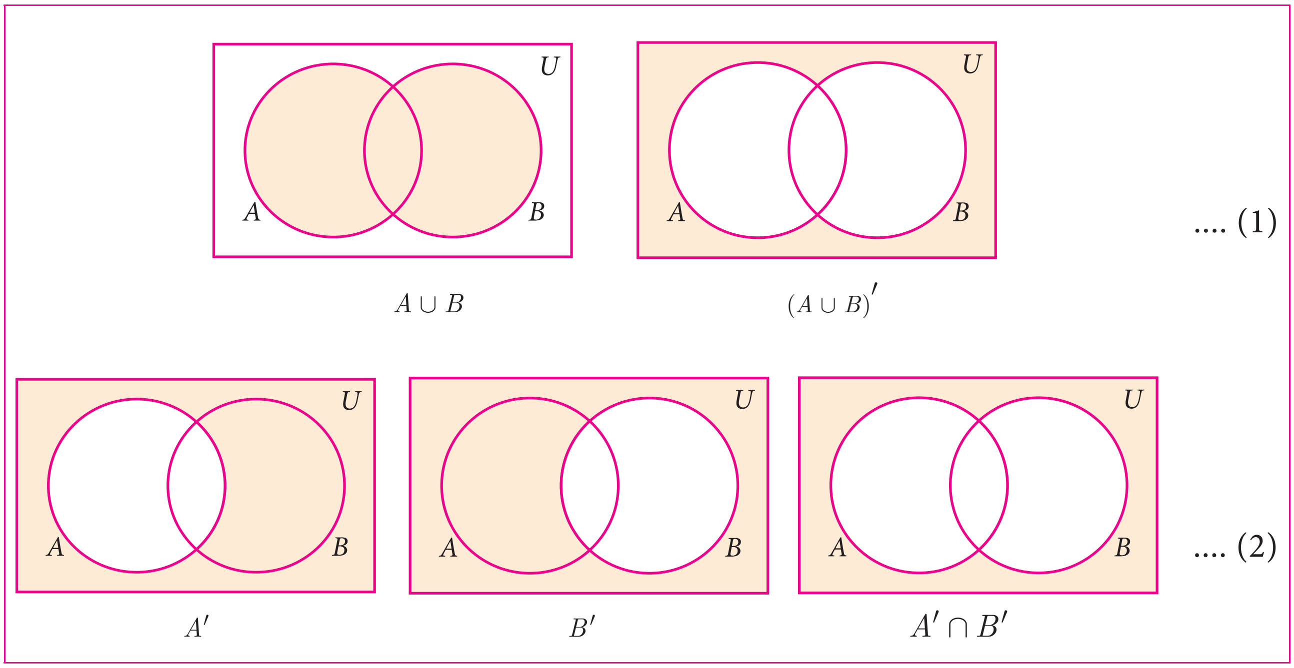


Fig.1.3

From (1) and (2), it is verified that $(A \cup B)' = A' \cap B'$

Example 1.9 If $U = \{x : x \in \mathbb{Z}, -2 \le x \le 10\}$,

$A = \{x : x = 2p + 1, \ p \in \mathbb{Z}, -1 \le p \le 4\}$, $B = \{x : x = 3q + 1, \ q \in \mathbb{Z}, -1 \le q < 4\}$,

verify De Morgan's laws for complementation.

Solution

Given $U = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,

$A = \{-1, 1, 3, 5, 7, 9\}$ and $B = \{-2, 1, 4, 7, 10\}$

Thinking Corner

$A \cap (A \cup B)' =$ _____

Law (i) $(A \cup B)' = A' \cap B'$

Now, $A \cup B = \{-2,-1,1,3,4,5,7,9,10\}$

$(A \cup B)' = \{0,2,6,8\}$ ..... (1)

Then, $A' = \{-2,0,2,4,6,8,10\}$ and $B' = \{-1,0,2,3,5,6,8,9\}$

$A' \cap B' = \{0,2,6,8\}$ ..... (2)

From (1) and (2), it is verified that $(A \cup B)' = A' \cap B'$

Thinking Corner

$(A \cup B)' \cup (A' \cap B) =$ ____

Law (ii) $(A \cap B)' = A' \cup B'$

Now, $A \cap B = \{1,7\}$

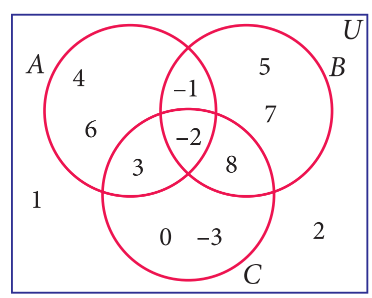
$(A \cap B)' = \{-2,-1,0,2,3,4,5,6,8,9,10\}$ ..... (3)

Then, $A' \cup B' = \{-2,-1,0,2,3,4,5,6,8,9,10\}$ ..... (4)

From (3) and (4), it is verified that $(A \cap B)' = A' \cup B'$

Exercise 1.3

1. Using the adjacent Venn diagram, find the following sets:



(i) $A - B$ (ii) $B - C$ (iii) $A' \cup B'$

(iv) $A' \cap B'$ (v) $(B \cup C)'$

(vi) $A - (B \cup C)$ (vii) $A - (B \cap C)$

2. If A, B and C are overlapping sets, then draw Venn diagram for the following sets:

(i) $(A - B) \cap C$ (ii) $(A \cup C) - B$ (iii) $A - (A \cap C)$

(iv) $(B \cup C) - A$ (v) $A \cap B \cap C$

3. If $A = \{b,c,e,g,h\}$, $B = \{a,c,d,g,i\}$ and $C = \{a,d,e,g,h\}$, then show that $A - (B \cap C) = (A - B) \cup (A - C)$.

4. If $A = \{x : x = 6n,\ n \in \mathbb{W}$ and $n < 6\}$, $B = \{x : x = 2n,\ n \in \mathbb{N}$ and $2 < n \leq 9\}$ and $C = \{x : x = 3n,\ n \in \mathbb{N}$ and $4 \leq n < 10\}$, then show that $A - (B \cap C) = (A - B) \cup (A - C)$

5. If $A = \{-2,0,1,3,5\}$, $B = \{-1,0,2,5,6\}$ and $C = \{-1,2,5,6,7\}$, then show that $A - (B \cup C) = (A - B) \cap (A - C)$.

6. If $A = \{y : y = \frac{a+1}{2},\ a \in \mathbb{W}$ and $a \leq 5\}$, $B = \{y : y = \frac{2n-1}{2},\ n \in \mathbb{W}$ and $n < 5\}$ and $C = \left\{-1, -\frac{1}{2}, 1, \frac{3}{2}, 2\right\}$, then show that $A - (B \cup C) = (A - B) \cap (A - C)$.

7. Verify $A-(B\cap C)=(A-B)\cup(A-C)$ using Venn diagrams.

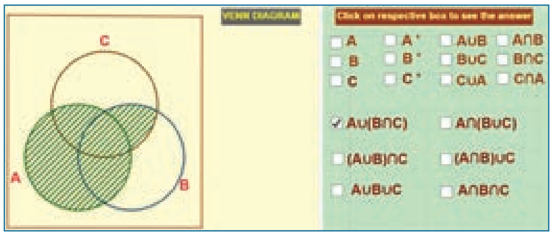
8. If U = {4,7,8,10,11,12,15,16}, A={7,8,11,12,} and B = {4,8,12,15}, then verify De Morgan's Laws for complementation.

9. If $U=\{x:-4\leq x\leq 4,\ x\in\mathbb{Z}\}$, $A=\{x:-4<x\leq 2, x\in\mathbb{Z}\}$ and $B=\{x:-2\leq x\leq 3, x\in\mathbb{Z}\}$, then verify De Morgan's laws for complementation.

10. Verify $(A\cap B)'=A'\cup B'$ using Venn diagrams.

ICT Corner

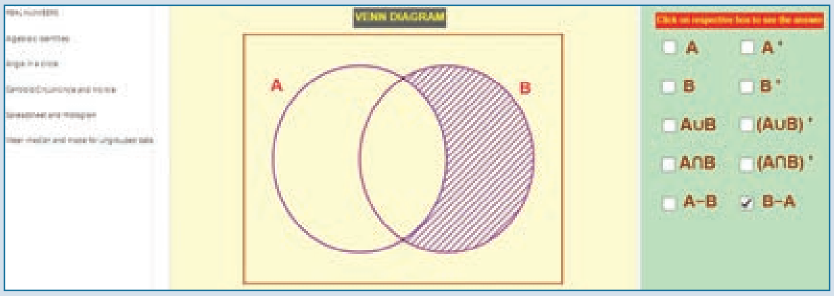
Expected Result is shown in this picture



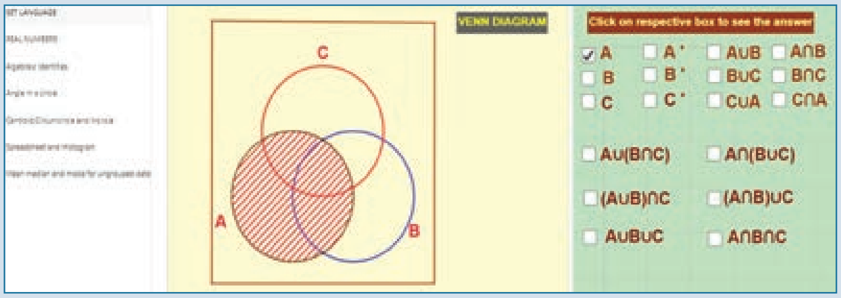
Step – 1 : Open the Browser, type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Set Language" will open. In the work sheet there are two activities. 1. Venn Diagram for two sets and 2. Venn Diagram for three sets.
In the first activity Click on the boxes on the right side to see the respective shading and analyse.

Step - 2 : Do the same for the second activity for three sets.

Step 1



Step 2



Browse in the link

Set Language: https://ggbm.at/BrG952dw or Scan the QR Code.

B466_MAT_9_T2_EM

Thinking Corner

$(A - B)'$ is equal to

(i) $B - A$ (ii) $B - A'$ (iii) $A' \cup B$ (iv) $A' \cap B$

1.4 Cardinality and Practical Problems on Set Operations

In the first term, we have learnt to solve problems involving two sets using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. Suppose we have three sets, we can apply this formula to get a similar formula for three sets.

For any three finite sets A, B and C

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

Note

Let us consider the following results which will be useful in solving problems using Venn diagram. Let three sets A, B and C represent the students. From the Venn diagram,

Number of students in only set $A = a$, only set $B = b$, only set $C = c$.

- Total number of students in only one set $= (a + b + c)$
- Total number of students in only two sets $= (x + y + z)$
- Number of students exactly in three sets $= r$
- Total number of students in atleast two sets (two or more sets)
- Total number of students in 3 sets = $(a + b + c + x + y + z + r)$

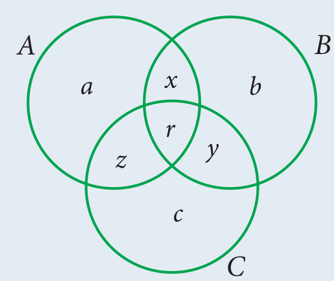


Fig.1.4

Activity - 1

In the following table A, B, C, D, E, F, G, H, I, J represents the players :

Player	A	B	C	D	E	F	G	H	I	J
Gender	Male	Female	Male	Female	Male	Female	Male	Female	Male	Female
Age	7	15	11	10	13	12	24	17	25	28
Game	Tennis	Cricket	Kabaddi	Cricket	Tennis	Tennis	Tennis	Tennis	Hockey	Kabaddi

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Using the above information, complete the following Venn diagram with given alphabets. Also answer the following questions:

(i) The number of players in only one set = _______.

(ii) The number of players in only two sets= _______.

(iii) The number of players in exactly three sets= _______.

(iv) The number of female players who are not playing tennis =_______.

(v) The number of players in $(X \cup Y \cup Z)'$ =_______.

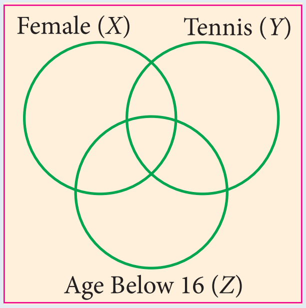


Fig.1.5

Example 1.10

In a college, 240 students play cricket, 180 students play football, 164 students play hockey, 42 play both cricket and football, 38 play both football and hockey, 40 play both cricket and hockey and 16 play all the three games. If each student participate in atleast one game, then find (i) the number of students in the college (ii) the number of students who play only one game.

Solution

Let C, F and H represent sets of students who play Cricket, Football and Hockey respectively.

Then , $n(C) = 240$, $n(F) = 180$, $n(H) = 164$, $n(C \cap F) = 42$,
$n(F \cap H) = 38$, $n(C \cap H) = 40$, $n(C \cap F \cap H) = 16$.

Let us represent the given data in a Venn diagram.

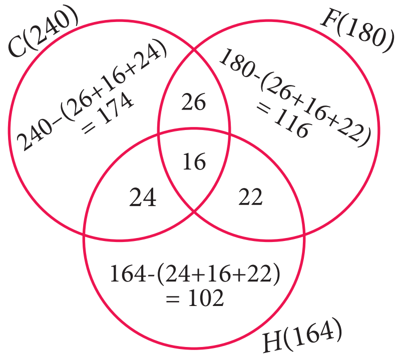


Fig.1.6

(i) The number of students in the college

$= 174+26+116+22+102+24+16 = 480$

(ii) The number of students who play only one game

$= 174+116+102 = 392$

Example 1.11

In a residential area with 600 families $\frac{3}{5}$ owned scooter, $\frac{1}{3}$ owned car, $\frac{1}{4}$ owned bicycle, 120 families owned scooter and car, 86 owned car and bicylce while 90 families owned scooter and bicylce. If $\frac{2}{15}$ of families owned all the three types of vehicles, then find (i) the number of families owned atleast two types of vehicle. (ii) the number of families owned no vehicle.

Solution

Let S, C and B represent sets of families who owned Scooter, Car and Bicycle respectively.

Given, $n(\text{U}) = 600$ $n(S) = \frac{3}{5} \times 600 = 360$

$n(C) = \frac{1}{3} \times 600 = 200 \quad n(B) = \frac{1}{4} \times 600 = 150$

$n(S \cap C \cap B) = \frac{2}{15} \times 600 = 80$

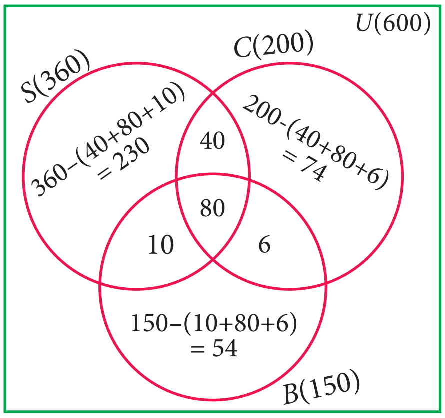


Fig.1.7

From Venn diagram,

(i) The number of families owned atleast two types of vehicles = 40+6+10+80 = 136

(ii) The number of families owned no vehicle

= 600 – (owned atleast one vehicle)

$= 600 - (230 + 40 + 74 + 6 + 54 + 10 + 80)$

$= 600 - 494 = 106$

Example 1.12 In a group of 100 students, 85 students speak Tamil, 40 students speak English, 20 students speak French, 32 speak Tamil and English, 13 speak English and French and 10 speak Tamil and French. If each student knows atleast any one of these languages, then find the number of students who speak all these three languages.

Solution

Let A, B and C represent sets of students who speak Tamil, English and French respectively.

Given, $n(A \cup B \cup C) = 100$, $n(A) = 85$, $n(\text{B}) = 40$, $n(\text{C}) = 20$,

$n(\text{A}\cap B) = 32$, $n(\text{B}\cap C) = 13$, $n(\text{A}\cap C) = 10$.

We know that,

$$n(A \cup B \cup C) = n(A) + n(B) + n(\text{C}) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$100 = 85 + 40 + 20 - 32 - 13 - 10 + n(A \cap B \cap C)$$

Then, $n(A \cap B \cap C) = 100 - 90 = 10$

Therefore, 10 students speak all the three languages.

Example 1.13 A survey was conducted among 200 magazine subscribers of three different magazines A, B and C. It was found that 75 members do not subscribe magazine A, 100 members do not subscribe magazine B, 50 members do not subscribe magazine C and 125 subscribe atleast two of the three magazines. Find

(i) Number of members who subscribe exactly two magazines.

(ii) Number of members who subscribe only one magazine.

Solution

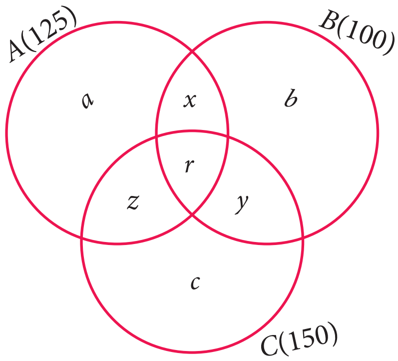


Fig.1.8

Total number of subscribers = 200

Magazine	Do not subscribe	Subscribe
A	75	125
B	100	100
C	50	150

From the Venn diagram,

Number of members who subscribe only one magazine $= a + b + c$

Number of members who subscribe exactly two magazines $= x + y + z$

and 125 members subscribe atleast two magazines.

That is, $x + y + z + r = 125$ ... (1)

Now, $n(A \cup B \cup C) = 200$, $n(A) = 125$, $n(B) = 100$, $n(C) = 150$, $n(A \cap B) = x + r$

$n(B \cap C) = y + r$, $n(A \cap C) = z + r$, $n(A \cap B \cap C) = r$

We know that,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$200 = 125+100+150-x-r-y-r-z-r+r$$

$$= 375-(x+y+z+r)-r$$

$$= 375-125-r \quad \left[\because x + y + z + r = 125\right]$$

$$200 = 250-r \quad \Rightarrow r = 50$$

From (1) $x + y + z + 50 = 125$

We get, $x + y + z = 75$

Therefore, number of members who subscribe exactly two magazines = 75.

From Venn diagram,

$$(a + b + c) + (x + y + z + r) = 200 \quad \text{... (2)}$$

substitute (1) in (2),

$$a + b + c + 125 = 200$$

$$a + b + c = 75$$

Therefore, number of members who subscribe only one magazine = 75.

Exercise 1.4

1. Verify $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C)$ $+ n(A \cap B \cap C)$ for the following sets.

   (i) $A = \{a, c, e, f, h\}$, $B = \{c, d, e, f\}$ and $C = \{a, b, c, f\}$

   (ii) $A = \{1, 3, 5\}$, $B = \{2, 3, 5, 6\}$ and $C = \{1, 5, 6, 7\}$

2. In a colony, 275 families buy Tamil newspaper, 150 families buy English newspaper, 45 families buy Hindi newspaper, 125 families buy Tamil and English newspapers, 17 families buy English and Hindi newspapers, 5 families buy Tamil and Hindi newspapers and 3 families buy all the three newspapers. If each family buy atleast one of these newspapers then find

   (i) Number of families buy only one newspaper

   (ii) Number of families buy atleast two newspapers

   (iii) Total number of families in the colony.

3. A soap company interviewed 800 people in a city. It was found out that $\frac{3}{8}$ use brand A soap, $\frac{1}{5}$ use brand B soap, $\frac{1}{2}$ use brand C soap, 70 use brand A and B soap, 55 use brand B and C soap, 60 use brand A and C soap and $\frac{1}{40}$ use all the three brands.

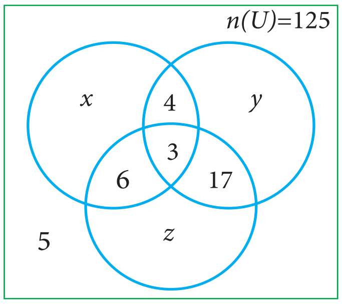
   Find,

   (i) Number of people who use exactly two branded soaps,

   (ii) Number of people who use atleast one branded soap,

   (iii) Number of people who do not use any one of these brands.

4. A survey of 1000 farmers found that 600 grew paddy, 350 grew ragi, 280 grew corn, 120 grew paddy and ragi, 100 grew ragi and corn, 80 grew paddy and corn. If each farmer grew atleast any one of the above three, then find the number of farmers who grew all the three.



5. In the adjacent diagram, if $n(\mathrm{U}) = 125$, y is two times of x and z is 10 more than x, then find the value of x, y and z.

6. Each student in a class of 35 plays atleast one game among chess, carrom and table tennis. 22 play chess, 21 play carrom, 15 play table tennis, 10 play chess and table tennis, 8 play carrom and table tennis and 6 play all the three games. Find the number of students who play

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(i) chess and carrom but not table tennis (ii) only chess (iii) only carrom (**Hint:** Use Venn diagram)

7. In a class of 50 students, each one come to school by bus or by bicycle or on foot. 25 by bus, 20 by bicycle, 30 on foot and 10 students by all the three. Now how many students come to school exactly by two modes of transport?

Exercise 1.5

Multiple Choice Questions

1. If U $= \{x : x \in \mathbb{N} \text{ and } x < 10\}$, $A = \{1,2,3,5,8\}$ and $B = \{2,5,6,7,9\}$, then $n\left[(A \cup B)'\right]$ is

   (1) 1 (2) 2 (3) 4 (4) 8

2. For any three sets P, Q and R, $P - (Q \cap R)$ is

   (1) $P - (Q \cup R)$ (2) $(P \cap Q) - R$

   (3) $(P - Q) \cup (P - R)$ (4) $(P - Q) \cap (P - R)$

3. Which of the following is true?

   (1) $A - B = A \cap B$ (2) $A - B = B - A$

   (3) $(A \cup B)' = A' \cup B'$ (4) $(A \cap B)' = A' \cup B'$

4. If $n(A \cup B \cup C) = 40$, $n(A) = 30$, $n(B) = 25$, $n(C) = 20$, $n(A \cap B) = 12$, $n(B \cap C) = 18$ and $n(A \cap C) = 15$, then $n(A \cap B \cap C)$ is

   (1) 5 (2) 10 (3) 15 (4) 20

5. If $n(A \cup B \cup C) = 100$, $n(A) = 4x$, $n(B) = 6x$, $n(C) = 5x$, $n(A \cap B) = 20$, $n(B \cap C) = 15$, $n(A \cap C) = 25$ and $n(A \cap B \cap C) = 10$, then the value of x is

   (1) 10 (2) 15 (3) 25 (4) 30

6. For any three sets A, B and C, $(A - B) \cap (B - C)$ is equal to

   (1) A only (2) B only (3) C only (4) ϕ

7. If J = Set of three sided shapes, K = Set of shapes with two equal sides and L = Set of shapes with right angle, then $J \cap K \cap L$ is

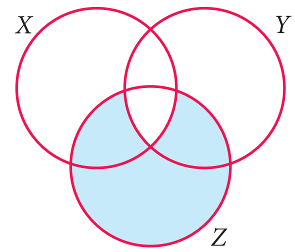
   (1) Set of isoceles triangles (2) Set of equilateral triangles

   (3) Set of isoceles right triangles (4) Set of right angled triangles

8. If A and B are two non-empty sets, then $(A - B) \cup (A \cap B)$ is

(1) A　　(2) B　　(3) ϕ　　(4) U

9. The shaded region in the Venn diagram is



(1) $Z - (X \cup Y)$　　(2) $(X \cup Y) \cap Z$

(3) $Z - (X \cap Y)$　　(4) $Z \cup (X \cap Y)$

10. In a city, 40% people like only one fruit, 35% people like only two fruits, 20% people like all the three fruits. How many percentage of people do not like any one of the above three fruits?

(1) 5　　(2) 8　　(3) 10　　(4) 15

Project

1. Collect the following data from 20 houses of your friends and neighbours. The number of houses which owned,

(i) Bicylce　　(ii) Motorbike

(iii) Laptop　　(iv) Both Bicycle and Motorbike

(v) Both Motorbike and Laptop　　(vi) Both Bicycle and Laptop

(vii) All the three things

2. Represent this information using Venn diagrams and answer the following questions. How many houses,

(i) owned only one of these?　　(ii) owned motorbike only?

(iii) not owned Laptop?　　(iv) not owned Bicycle?

(v) owned exactly two of these?

Points to Remember

- **Commutative Property**

  For any two sets A and B,

  $A \cup B = B \cup A$; $A \cap B = B \cap A$

- **Associative Property**

  For any three sets A, B and C

  $A \cup (B \cup C) = (A \cup B) \cup C$; $A \cap (B \cap C) = (A \cap B) \cap C$

- **Distributive Property**

  For any three sets A, B and C

  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [Intersection over union]

  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [Union over intersection]

- **De Morgan's Laws for Set Difference**

  For any three sets A, B and C

  $A - (B \cup C) = (A - B) \cap (A - C)$

  $A - (B \cap C) = (A - B) \cup (A - C)$

- **De Morgan's Laws for Complementation**

  Consider an Universal set and A, B are two subsets, then

  $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$

- **Cardinality of Sets**

  If A and B are any two sets, then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

  If A, B and C are three sets, then

  $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

2 REAL NUMBERS

"When I consider what people generally want in calculating,
I found that it always is a number"

- Al-Khwarizmi

BBQEVH

Al-Khwarizmi
(780 - 850 A.D. (CE))

Al-Khwarizmi, a Persian scholar, is credited with identification of surds as something noticeable in mathematics. He referred to the irrational numbers as 'inaudible', which was later translated to the Latin word 'surdus' (meaning 'deaf' or 'mute'). In mathematics, a surd came to mean a root (radical) that cannot be expressed (spoken) as a rational number.

Learning Outcomes

- To recall indices and their laws.
- To identify surds.
- To carry out basic operations of addition, subtraction, multiplication and division using surds.
- To rationalise denominators of surds.
- To understand the scientific notation.

2.1 Introduction

You know that $3\times3\times3\times3$ is shortly written as 3^4; thus $81 = 3^4$ where 3 is known as the base and 4 is its exponent. Another name for 'exponent' is 'index'. When one writes

$$x^n = \underbrace{x\times x\times x\times \ldots \times x}_{n \text{ factors}} \quad \text{(where } n \text{ is a positive integer)}$$

x is the base and n is the index (Plural for 'index' is indices'). What is x^{-n}? It is the multiplicative inverse of x^n (with the understanding that $x^n \times x^{-n} = x^o = 1$). Thus we write

$$x^{-n} = \frac{1}{x^n}$$

So, can we write $\frac{1}{2}$ as 2^{-1}? Yes, of course! and, is not 4^{-3} same as $\frac{1}{4^3}$?

Example 2.1

Evaluate : (i) 10^{-3} (ii) $\left(\frac{1}{3}\right)^{-2}$ (iii) $(0.1)^{-4}$

Solution

(i) $10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$

(ii) $\left(\frac{1}{3}\right)^{-2} = \frac{1}{\left(\frac{1}{3}\right)^2} = \frac{1}{\left(\frac{1}{9}\right)} = 9$

(iii) $(0.1)^{-4} = \left(\frac{1}{10}\right)^{-4} = \frac{1}{\left(\frac{1}{10}\right)^4} = \frac{1}{\left(\frac{1}{10000}\right)} = 10000$

Thinking Corner

- Is 2^{10} equal to 1024?
- Can we write $2^{-1} < 1$?
- Is -2^3 same as $(-2)^3$?
- What could be $x^{-5} \times x^{-2} = ?$
- What is $x^{-m} \times x^{-n} = ?$

Progress Check

1. Simplify by giving your answer without negative indices:

(i) $a^5 \times a^3$	(ii) $a^5 \times a^{-3}$	(iii) $a^5 \times a^{-5}$
(iv) $3x^5 \times x^{-1}$	(v) $6x^{-4} \times 7x^5$	(vi) $10m^{-1} \times 8m^{-1}$
(vii) $x^5 \div x^{-2}$	(viii) $x^{-5} \div x^{-2}$	(ix) $x^{-5} \div x^2$
(x) $24p^{-2} \div 6p^3$	(xi) $(a^{-5})^2$	(xii) $(a^{-5})^{-2}$
(xiii) $(a^5)^{-2}$	(xiv) $(2x^2)^{-3}$	(xv) $(2x^2y^3)^{-4}$

2. Evaluate: (i) 9^{-2} (ii) $2^4 \times 4^{-2}$ (iii) $25^{-1} \times 5^{-2}$ (iv) $(0.2)^5$ (v) $\left(\frac{1}{5}\right)^{-4}$
3. If $x = 3$, $y = -1$ find the value of $(xy^2)^{-1}$.
4. Simplify: $\frac{2^n \times 3^{3n} \times 6^n}{2^{3n} \times 3^{2n} \times 30^n}$

2.2 Radical Notation

Let n be a positive integer and r be a real number. If $r^n = x$, then r is called the n^{th} root of x and we write

$$\sqrt[n]{x} = r$$

The symbol $\sqrt[n]{}$ (read as n^{th} root) is called a radical; n is the index of the radical (hitherto we named it as exponent); and x is called the radicand.

Note

It is worth spending some time on the concepts of the 'square root' and the 'cube root', for better understanding of surds.

What happens when $n = 2$? Then we get $r^2 = x$, so that r is $\sqrt[2]{x}$, our good old friend, the square root of x. Thus $\sqrt[2]{16}$ is written as $\sqrt{16}$, and when $n = 3$, we get the cube root of x, namely $\sqrt[3]{x}$. For example, $\sqrt[3]{8}$ is cube root of 8, giving 2. (Is not $8 = 2^3$?)

How many square roots are there for 4? Since $(+2)\times(+2) = 4$ and also $(-2)\times(-2) = 4$, we can say that both +2 and –2 are square roots of 4. But it is incorrect to write that $\sqrt{4} = \pm 2$. This is because, when n is even, it is an accepted convention to reserve the symbol $\sqrt[n]{x}$ for the positive n^{th} root and to denote the negative n^{th} root by $-\sqrt[n]{x}$. Therefore we need to write $\sqrt{4} = 2$ and $-\sqrt{4} = -2$.

When n is odd, for any value of x, there is exactly one real n^{th} root. For example, $\sqrt[3]{8} = 2$ and $\sqrt[5]{-32} = -2$.

Thinking Corner

Which one of the following is false?

(a) The square root of 9 is 3 or –3.

(b) $\sqrt{9} = 3$

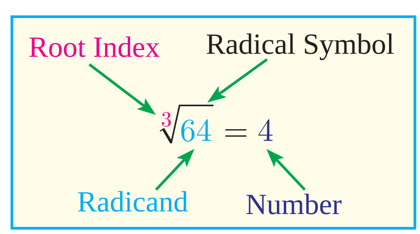
(c) $-\sqrt{9} = -3$

(d) $\sqrt{9} = \pm 3$

2.2.1 Fractional Index

Consider again results of the form $r = \sqrt[n]{x}$.

In the adjacent notation, the index of the radical (namely n which is 3 here) tells you how many times the answer (that is 4) must be multiplied with itself to yield the radicand.



To express the powers and roots, there is one more way of representation. It involves the use of fractional indices.

$$\text{We write } \sqrt[n]{x} \text{ as } x^{\frac{1}{n}}.$$

With this notation, for example

$$\sqrt[3]{64} \text{ is } 64^{\frac{1}{3}} \text{ and } \sqrt{25} \text{ is } 25^{\frac{1}{2}}.$$

Observe in the following table just some representative patterns arising out of this new acquaintance:

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Power	Radical Notation	Index Notation	Read as
$2^6 = 64$	$2 = \sqrt[6]{64}$	$2 = 64^{\frac{1}{6}}$	2 is the 6^{th} root of 64
$2^5 = 32$	$2 = \sqrt[5]{32}$	$2 = 32^{\frac{1}{5}}$	2 is the 5^{th} root of 32
$2^4 = 16$	$2 = \sqrt[4]{16}$	$2 = 16^{\frac{1}{4}}$	2 is the 4^{th} root of 16
$2^3 = 8$	$2 = \sqrt[3]{8}$	$2 = 8^{\frac{1}{3}}$	2 is the cube root of 8 meaning 2 is the 3^{rd} root of 8
$2^2 = 4$	$2 = \sqrt[2]{4}$ or simply $2 = \sqrt{4}$	$2 = 4^{\frac{1}{2}}$	2 is the square root of 4 meaning 2 is the 2^{nd} root of 4

Example 2.2

Express the following in the form 2^n :

(i) 8 (ii) 32 (iii) $\frac{1}{4}$ (iv) $\sqrt{2}$ (v) $\sqrt{8}$.

Solution

(i) $8 = 2\times2\times2$; therefore $8 = 2^3$

(ii) $32 = 2\times2\times2\times2\times2 = 2^5$

(iii) $\frac{1}{4} = \frac{1}{2\times2} = \frac{1}{2^2} = 2^{-2}$

(iv) $\sqrt{2} = 2^{1/2}$

(v) $\sqrt{8} = \sqrt{2}\times\sqrt{2}\times\sqrt{2} = \left(2^{\frac{1}{2}}\right)^3$ which may be written as $2^{\frac{3}{2}}$

2.2.2 Meaning of $x^{\frac{m}{n}}$, (where m and n are Positive Integers)

We interpret $x^{\frac{m}{n}}$ either as the n^{th} root of the m^{th} power of x or as the m^{th} power of the n^{th} root of x.

In symbols, $x^{\frac{m}{n}} = (x^m)^{\frac{1}{n}}$ or $(x^{\frac{1}{n}})^m = \sqrt[n]{x^m}$ or $(\sqrt[n]{x})^m$

Example 2.3

Find the value of (i) $81^{\frac{5}{4}}$ (ii) $64^{\frac{-2}{3}}$

Solution

(i) $81^{\frac{5}{4}} = \left(\sqrt[4]{81}\right)^5 = \left(\sqrt[4]{3^4}\right)^5 = 3^5 = 3\times3\times3\times3\times3 = 243$

(ii) $64^{\frac{-2}{3}} = \frac{1}{64^{\frac{2}{3}}} = \frac{1}{\left(\sqrt[3]{64}\right)^2} = \frac{1}{4^2}$ (How?) $= \frac{1}{16}$

Exercise 2.1

1. Write the following in the form of 5^n:

   (i) 625 (ii) $\frac{1}{5}$ (iii) $\sqrt{5}$ (iv) $\sqrt{125}$

2. Write the following in the form of 4^n:

   (i) 16 (ii) 8 (iii) 32

3. Find the value of

   (i) $(49)^{\frac{1}{2}}$ (ii) $(243)^{\frac{2}{5}}$ (iii) $(8)^{\frac{5}{3}}$ (iv) $9^{\frac{-3}{2}}$

   (v) $\left(\frac{1}{27}\right)^{\frac{-2}{3}}$ (vi) $\left(\frac{64}{125}\right)^{\frac{-2}{3}}$

4. Use a fractional index to write:

   (i) $\sqrt{5}$ (ii) $\sqrt[2]{7}$ (iii) $\left(\sqrt[3]{49}\right)^5$ (iv) $\left(\frac{1}{\sqrt[3]{100}}\right)^7$

5. Find the 5^{th} root of

   (i) 32 (ii) 243 (iii) 100000 (iv) $\frac{1024}{3125}$

2.3 Surds

Having familiarized with the concept of Real numbers, representing them on the number line and manipulating them, we now learn about surds, a distinctive way of representing certain approximate values.

Can you simplify $\sqrt{4}$ and remove the $\sqrt{\ }$ symbol? Yes; one can replace $\sqrt{4}$ by the number 2. How about $\sqrt{\frac{1}{9}}$? It is easy; without $\sqrt{\ }$ symbol, the answer is $\frac{1}{3}$. What about $\sqrt{0.01}$?. This is also easy and the solution is 0.1

In the cases of $\sqrt{4}$, $\sqrt{\frac{1}{9}}$ and $\sqrt{0.01}$, you can resolve to get a solution and make sure that the symbol $\sqrt{\ }$ is not seen in your solution. Is this possible at all times?

Consider $\sqrt{18}$. Can you evaluate it and also remove the radical symbol? Surds are unresolved radicals, such as square root of 2, cube root of 5, etc.

They are irrational roots of equations with rational coefficients.

A surd is an irrational root of a rational number. $\sqrt[n]{a}$ is a surd, provided $n \in \mathbb{N}$, $n > 1$, 'a' is rational.

Examples : $\sqrt{2}$ is a surd. It is an irrational root of the equation $x^2 = 2$. (Note that $x^2 - 2 = 0$ is an equation with rational coefficients. $\sqrt{2}$ is irrational and may be shown as 1.4142135… a non-recurring, non-terminating decimal).

$\sqrt[3]{3}$ (which is same as $3^{\frac{1}{3}}$) is a surd since it is an irrational root of the equation $x^3 - 3 = 0$. ($\sqrt{3}$ is irrational and may be shown as 1.7320508… a non-recurring, non-terminating decimal).

You will learn solving (quadratic) equations like $x^2 - 6x + 7 = 0$ in your next class. This is an equation with rational coefficients and one of its roots is $3+\sqrt{2}$, which is a surd.

Is $\sqrt{\frac{1}{25}}$ a surd? No; it can be simplified and written as rational number $\frac{1}{5}$. How about $\sqrt[4]{\frac{16}{81}}$? It is not a surd because it can be simplified as $\frac{2}{3}$.

The famous irrational number π is not a surd! Though it is irrational, it cannot be expressed as a rational number under the $\sqrt{\ }$ symbol. (In other words, it is not a root of any equation with rational co-efficients).

Why surds are important? For calculation purposes we assume approximate value as $\sqrt{2} = 1.414, \sqrt{3} = 1.732$ and so on.

$$\left(\sqrt{2}\right)^2 = \left(1.414\right)^2 = 1.99936 \neq 2 \ ; \qquad \left(\sqrt{3}\right)^2 = \left(1.732\right)^2 = 3.999824 \neq 3$$

Hence, we observe that $\sqrt{2}$ and $\sqrt{3}$ represent the more accurate and precise values than their assumed values. Engineers and scientists need more accurate values while constructing the bridges and for architectural works. Thus it becomes essential to learn surds.

Progress Check

1. Which is the odd one out? Justify your answer.

   (i) $\sqrt{36}$, $\sqrt{\frac{50}{98}}$, $\sqrt{1}$, $\sqrt{1.44}$, $\sqrt[5]{32}$, $\sqrt{120}$

   (ii) $\sqrt{7}$, $\sqrt{48}$, $\sqrt[3]{36}$, $\sqrt{5}+\sqrt{3}$, $\sqrt{1.21}$, $\sqrt{\frac{1}{10}}$

2. Are all surds irrational numbers? - Discuss with your answer.
3. Are all irrational numbers surds? Verify with some examples.

2.3.1 Order of a Surd

The order of a surd is the index of the root to be extracted. The order of the surd $\sqrt[n]{a}$ is n. What is the order of $\sqrt[5]{99}$? It is 5.

2.3.2 Types of Surds

Surds can be classified in different ways:

(i) Surds of same order : Surds of same order are surds for which the index of the root to be extracted is same. (They are also called equiradical surds).

For example, $\sqrt{x}$, $a^{\frac{3}{2}}$, $\sqrt[2]{m}$ are all 2nd order (called quadratic) surds .

$\sqrt[3]{5}$, $\sqrt[3]{(x-2)}$, $(ab)^{\frac{1}{3}}$ are all 3rd order (called cubic) surds.

$\sqrt{3}$, $\sqrt[3]{10}$, $\sqrt[4]{6}$ and $8^{\frac{2}{5}}$ are surds of different order.

Example 2.4

Can you reduce the following numbers to surds of same order :

(i) $\sqrt{3}$ (ii) $\sqrt[4]{3}$ (iii) $\sqrt[3]{3}$

Solution

(i) $\sqrt{3} = 3^{\frac{1}{2}} = 3^{\frac{6}{12}} = \sqrt[12]{3^6} = \sqrt[12]{729}$

(ii) $\sqrt[4]{3} = 3^{\frac{1}{4}} = 3^{\frac{3}{12}} = \sqrt[12]{3^3} = \sqrt[12]{27}$

(iii) $\sqrt[3]{3} = 3^{\frac{1}{3}} = 3^{\frac{4}{12}} = \sqrt[12]{3^4} = \sqrt[12]{81}$

The last row has surds of same order.

(ii) Simplest form of a surd : A surd is said to be in simplest form, when it is expressed as the product of a rational factor and an irrational factor. In this form the surd has

(a) the smallest possible index of the radical sign.

(b) no fraction under the radical sign.

(c) no factor is of the form a^n, where a is a positive integer under index n.

For example

(i) Rationalising the Denominator

$$\sqrt[3]{\frac{18}{25}} = \sqrt[3]{\frac{18\times 5}{25\times 5}}$$

$$= \sqrt[3]{\frac{90}{125}}$$

$$= \sqrt[3]{\frac{90}{5^3}} = \frac{\sqrt[3]{90}}{5}$$

(ii) Rationalising the numerator

$$\sqrt[3]{\frac{18}{25}} = \sqrt[3]{\frac{18\times 12}{25\times 12}}$$

$$= \sqrt[3]{\frac{216}{300}}$$

$$= \sqrt[3]{\frac{6^3}{300}} = \frac{6}{\sqrt[3]{300}}$$

Simplifying the given surd

$$\sqrt[3]{\frac{18}{125}} = \sqrt[3]{\frac{18}{5^3}} = \frac{\sqrt[3]{18}}{5}$$

Will it also be rationalisation? Discuss - your views.

Example 2.5

1. Express the surds in the simplest form: i) $\sqrt{8}$ ii) $\sqrt[3]{192}$

2. Show that $\sqrt[3]{7} > \sqrt[4]{5}$.

Solution

1. (i) $\sqrt{8} = \sqrt{4\times 2} = 2\sqrt{2}$

 (ii) $\sqrt[3]{192} = \sqrt[3]{4\times 4\times 4\times 3} = 4\sqrt[3]{3}$

2. $\sqrt[3]{7} = \sqrt[12]{7^4} = \sqrt[12]{2401}$

 $\sqrt[4]{5} = 5^{\frac{1}{4}} = 5^{\frac{3}{12}} = \sqrt[12]{5^3} = \sqrt[12]{125}$

 $\sqrt[12]{2401} > \sqrt[12]{125}$

 Therefore, $\sqrt[3]{7} > \sqrt[4]{5}$.

(iii) Pure and Mixed Surds : A surd is called a pure surd if its coefficient in its simplest form is 1. For example, $\sqrt{3}$, $\sqrt[3]{6}$, $\sqrt[4]{7}$, $\sqrt[5]{49}$ are pure surds.

A surd is called a mixed surd if its co-efficient in its simplest form is other than 1. For example, $5\sqrt{3}$, $2\sqrt[4]{5}$, $3\sqrt[4]{6}$ are mixed surds.

(iv) **Simple and Compound Surds :** A surd with a single term is said to be a simple surd. For example, $\sqrt{3}$, $2\sqrt{5}$ are simple surds.

The algebraic sum of two (or more) surds is called a compound surd. For example, $\sqrt{5}+3\sqrt{2}, \sqrt{3}-2\sqrt{7}, \sqrt{5}-7\sqrt{2}+6\sqrt{3}$ are compound surds.

(v) **Binomial Surd :** A binomial surd is an algebraic sum (or difference) of 2 terms both of which could be surds or one could be a rational number and another a surd. For example, $\frac{1}{2}-\sqrt{19}, 5+3\sqrt{2}, \sqrt{3}-2\sqrt{7}$ are binomial surds.

Example 2.6 Arrange in ascending order: $\sqrt[3]{2}, \sqrt[2]{4}, \sqrt[4]{3}$

Solution

The order of the surds $\sqrt[3]{2}, \sqrt[2]{4}$ and $\sqrt[4]{3}$ are 3, 2, 4.

L.C.M. of 3, 2, 4 = 12.

$$\sqrt[3]{2} = \left(2^{\frac{1}{3}}\right) = \left(2^{\frac{4}{12}}\right) = \sqrt[12]{2^4} = \sqrt[12]{16} \quad ; \qquad \sqrt[2]{4} = \left(4^{\frac{1}{2}}\right) = \left(4^{\frac{6}{12}}\right) = \sqrt[12]{4^6} = \sqrt[12]{4096}$$

$$\sqrt[4]{3} = \left(3^{\frac{1}{4}}\right) = \left(3^{\frac{3}{12}}\right) = \sqrt[12]{3^3} = \sqrt[12]{27}$$

The ascending order of the surds $\sqrt[3]{2}, \sqrt[4]{3}, \sqrt[2]{4}$ is $\sqrt[12]{16} < \sqrt[12]{27} < \sqrt[12]{4096}$ that is, $\sqrt[3]{2}, \sqrt[4]{3}, \sqrt[2]{4}$.

2.3.3 Laws of Radicals

For positive integers m, n and positive rational numbers a and b, it is worth remembering the following properties of radicals:

S.No.	Radical Notation	Index Notation
1.	$\left(\sqrt[n]{a}\right)^n = a = \sqrt[n]{a^n}$	$\left(a^{\frac{1}{n}}\right)^n = a = \left(a^n\right)^{\frac{1}{n}}$
2.	$\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$	$a^{\frac{1}{n}} \times b^{\frac{1}{n}} = (ab)^{\frac{1}{n}}$
3.	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$	$\left(a^{\frac{1}{n}}\right)^{\frac{1}{m}} = a^{\frac{1}{mn}} = \left(a^{\frac{1}{m}}\right)^{\frac{1}{n}}$
4.	$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$	$\frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} = \left(\frac{a}{b}\right)^{\frac{1}{n}}$

We shall now discuss certain problems which require the laws of radicals for simplifying.

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Example 2.7 Express each of the following surds in its simplest form (i) $\sqrt[3]{108}$ (ii) $\sqrt[3]{(1024)^{-2}}$ and find its order, radicand and coefficient.

Solution

(i)

$$\sqrt[3]{108} = \sqrt[3]{27\times 4}$$
$$= \sqrt[3]{3^3\times 4}$$
$$= \sqrt[3]{3^3}\times\sqrt[3]{4} \text{ (Laws of radicals - ii)}$$
$$= 3\times\sqrt[3]{4} \text{ (Laws of radicals- i)}$$

2	108
2	54
3	27
3	9
3	3
	1

order = 3; radicand = 4; coefficient = 3

(ii)

$$\sqrt[3]{(1024)^{-2}} = \left[\sqrt[3]{\left(2^3\times 2^3\times 2^3\times 2\right)^{-2}}\right]$$
$$= \left[\sqrt[3]{\left(2^3\times 2^3\times 2^3\times 2\right)}\right]^{-2} \text{ [Laws of radicals - (i)]}$$
$$= \left[\sqrt[3]{2^3}\times\sqrt[3]{2^3}\times\sqrt[3]{2^3}\times\sqrt[3]{2}\right]^{-2} \text{ [Laws of radicals – (ii)]}$$
$$= \left[2\times 2\times 2\times\sqrt[3]{2}\right]^{-2} \text{ [Laws of radicals – (i)]}$$
$$= \left[8\times\sqrt[3]{2}\right]^{-2} = \left[\frac{1}{8}\right]^2\times\left(\frac{1}{\sqrt[3]{2}}\right)^2$$
$$= \frac{1}{64}\sqrt[3]{\frac{1}{4}}$$

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

order $= 3$; radicand $= \frac{1}{4}$; coefficient $= \frac{1}{64}$

(These results can also be obtained using index notation).

Note

Consider the numbers 5 and 6. As $5 = \sqrt{25}$ and $6 = \sqrt{36}$

Therefore, $\sqrt{26}$, $\sqrt{27}$, $\sqrt{28}$, $\sqrt{29}$, $\sqrt{30}$, $\sqrt{31}$, $\sqrt{32}$, $\sqrt{33}$, $\sqrt{34}$, and $\sqrt{35}$ are surds between 5 and 6.

Consider $3\sqrt{2} = \sqrt{3^2\times 2} = \sqrt{18}$, $2\sqrt{3} = \sqrt{2^2\times 3} = \sqrt{12}$

Therefore, $\sqrt{17}$, $\sqrt{15}$, $\sqrt{14}$, $\sqrt{13}$ are surds between $2\sqrt{3}$ and $3\sqrt{2}$.

2.3.4 Four Basic Operations on Surds

(i) **Addition and subtraction of surds :** Like surds can be added and subtracted using the following rules:

$$a\sqrt[n]{b} \pm c\sqrt[n]{b} = (a \pm c)\sqrt[n]{b}, \text{ where } b > 0.$$

Example 2.8 (i) Add $3\sqrt{7}$ and $5\sqrt{7}$. Check whether the sum is rational or irrational. (ii) Subtract $4\sqrt{5}$ from $7\sqrt{5}$. Is the answer rational or irrational?

Solution

(i) $3\sqrt{7} + 5\sqrt{7} = (3+5)\sqrt{7} = 8\sqrt{7}$. The answer is irrational.

(ii) $7\sqrt{5} - 4\sqrt{5} = (7-4)\sqrt{5} = 3\sqrt{5}$. The answer is irrational.

Example 2.9 Simplify the following:

(i) $\sqrt{63} - \sqrt{175} + \sqrt{28}$ (ii) $2\sqrt[3]{40} + 3\sqrt[3]{625} - 4\sqrt[3]{320}$

Solution

(i) $\sqrt{63} - \sqrt{175} + \sqrt{28} = \sqrt{9\times7} - \sqrt{25\times7} + \sqrt{4\times7}$

$$= 3\sqrt{7} - 5\sqrt{7} + 2\sqrt{7}$$

$$= \left(3\sqrt{7} + 2\sqrt{7}\right) - 5\sqrt{7}$$

$$= 5\sqrt{7} - 5\sqrt{7}$$

$$= 0$$

(ii) $2\sqrt[3]{40} + 3\sqrt[3]{625} - 4\sqrt[3]{320}$

$$= 2\sqrt[3]{8\times5} + 3\sqrt[3]{125\times5} - 4\sqrt[3]{64\times5}$$

$$= 2\sqrt[3]{2^3\times5} + 3\sqrt[3]{5^3\times5} - 4\sqrt[3]{4^3\times5}$$

$$= 2\times2\sqrt[3]{5} + 3\times5\sqrt[3]{5} - 4\times4\sqrt[3]{5}$$

$$= 4\sqrt[3]{5} + 15\sqrt[3]{5} - 16\sqrt[3]{5}$$

$$= (4+15-16)\sqrt[3]{5} = 3\sqrt[3]{5}$$

(ii) Multiplication and division of surds

Like surds can be multiplied or divided by using the following rules:

Multiplication property of surds	Division property of surds
(i) $\sqrt[n]{a}\times\sqrt[n]{b}=\sqrt[n]{ab}$	(iii) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\dfrac{a}{b}}$
(ii) $a\sqrt[n]{b}\times c\sqrt[n]{d}=ac\sqrt[n]{bd}$ where $b,d>0$	(iv) $\dfrac{a\sqrt[n]{b}}{c\sqrt[n]{d}}=\dfrac{a}{c}\sqrt[n]{\dfrac{b}{d}}$ where $b,d>0$

Example 2.10 Multiply $\sqrt[3]{40}$ and $\sqrt[3]{16}$.

Solution

$$\sqrt[3]{40}\times\sqrt[3]{16}=\left(\sqrt[3]{2\times2\times2\times5}\right)\times\left(\sqrt[3]{2\times2\times2\times2}\right)$$

$$=\left(2\times\sqrt[3]{5}\right)\times\left(2\times\sqrt[3]{2}\right)=4\times\left(\sqrt[3]{2}\times\sqrt[3]{5}\right)=4\times\sqrt[3]{2\times5}$$

$$=4\sqrt[3]{10}$$

Example 2.11 Compute and give the answer in the simplest form: $2\sqrt{72}\times5\sqrt{32}\times3\sqrt{50}$

Solution

$$2\sqrt{72}\times5\sqrt{32}\times3\sqrt{50}=\left(2\times6\sqrt{2}\right)\times\left(5\times4\sqrt{2}\right)\times\left(3\times5\sqrt{2}\right)$$

$$=2\times5\times3\times6\times4\times5\times\sqrt{2}\times\sqrt{2}\times\sqrt{2}$$

$$=3600\times2\sqrt{2}$$

$$=7200\sqrt{2}$$

Let us simplify:

$\sqrt{72}=\sqrt{36\times2}=6\sqrt{2}$

$\sqrt{32}=\sqrt{16\times2}=4\sqrt{2}$

$\sqrt{50}=\sqrt{25\times2}=5\sqrt{2}$

Example 2.12 Divide $\sqrt[9]{8}$ by $\sqrt[6]{6}$.

Solution

$$\frac{\sqrt[9]{8}}{\sqrt[6]{6}}=\frac{8^{\frac{1}{9}}}{6^{\frac{1}{6}}}\text{ (Note that 18 is the LCM of 6 and 9)}$$

$$=\frac{8^{\frac{2}{18}}}{6^{\frac{3}{18}}}\text{ (How?)}$$

$$=\left(\frac{8^2}{6^3}\right)^{\frac{1}{18}}\text{ (How ?)}=\left(\frac{8\times8}{6\times6\times6}\right)^{\frac{1}{18}}$$

$$=\left(\frac{8}{27}\right)^{\frac{1}{18}}=\left[\left(\frac{2}{3}\right)^3\right]^{\frac{1}{18}}=\left(\frac{2}{3}\right)^{\frac{1}{6}}=\sqrt[6]{\frac{2}{3}}$$

Activity - 1

Is it interesting to see this pattern ?

$\sqrt{4\dfrac{4}{15}}=4\sqrt{\dfrac{4}{15}}$ and $\sqrt{5\dfrac{5}{24}}=5\sqrt{\dfrac{5}{24}}$

Verify it. Can you frame 4 such new surds?

Exercise 2.2

1. Simplify the following using addition and subtraction properties of surds:

   (i) $5\sqrt{3}+18\sqrt{3}-2\sqrt{3}$ (ii) $4\sqrt[3]{5}+2\sqrt[3]{5}-3\sqrt[3]{5}$

   (iii) $3\sqrt{75}+5\sqrt{48}-\sqrt{243}$ (iv) $5\sqrt[3]{40}+2\sqrt[3]{625}-3\sqrt[3]{320}$

2. Simplify the following using multiplication and division properties of surds:

   (i) $\sqrt{3}\times\sqrt{5}\times\sqrt{2}$ (ii) $\sqrt{35}\div\sqrt{7}$ (iii) $\sqrt[3]{27}\times\sqrt[3]{8}\times\sqrt[3]{125}$

   (iv) $\left(7\sqrt{a}-5\sqrt{b}\right)\left(7\sqrt{a}+5\sqrt{b}\right)$ (v) $\left[\sqrt{\frac{225}{729}}-\sqrt{\frac{25}{144}}\right]\div\sqrt{\frac{16}{81}}$

3. If $\sqrt{2}=1.414$, $\sqrt{3}=1.732$, $\sqrt{5}=2.236$, $\sqrt{10}=3.162$, then find the values of the following correct to 3 places of decimals.

   (i) $\sqrt{40}-\sqrt{20}$ (ii) $\sqrt{300}+\sqrt{90}-\sqrt{8}$

4. Arrange surds in descending order : (i) $\sqrt[3]{5},\sqrt[9]{4},\sqrt[6]{3}$ (ii) $\sqrt[2]{\sqrt[3]{5}},\sqrt[3]{\sqrt[4]{7}},\sqrt{\sqrt{3}}$

5. Can you get a pure surd when you find

   (i) the sum of two surds (ii) the difference of two surds

   (iii) the product of two surds (iv) the quotient of two surds

   Justify each answer with an example.

6. Can you get a rational number when you compute

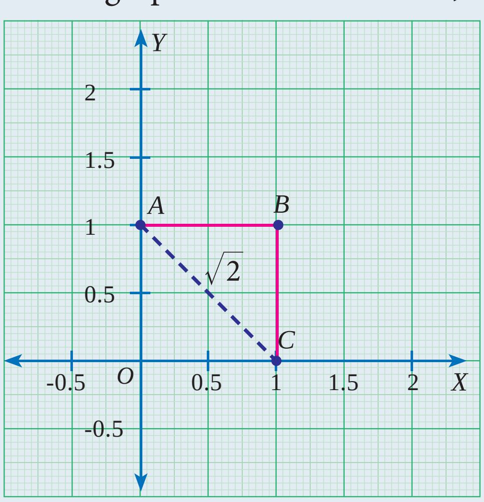
   (i) the sum of two surds (ii) the difference of two surds

   (iii) the product of two surds (iv) the quotient of two surds

   Justify each answer with an example.

Activity - 2

Take a graph sheet and mark O, A, B, C as follows.



In the square $OABC$,

$$OA = AB = BC = OC = 1\text{unit}$$

Consider right angled ΔOAC

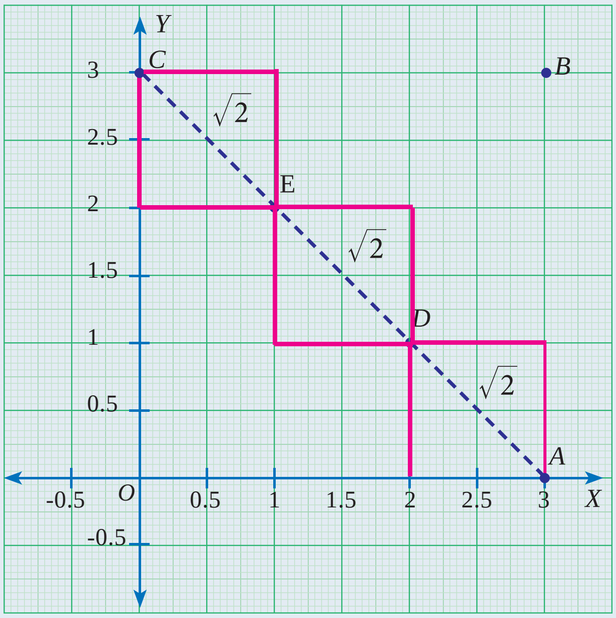
$$AC=\sqrt{1^2+1^2}$$

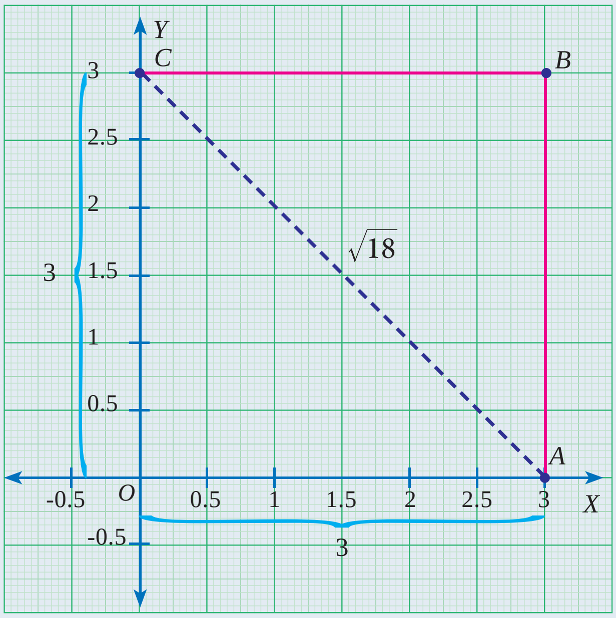
$$=\sqrt{2}\text{ unit [By Pythagoras theorem]}$$

The length of the diagonal (hypotenuse)

$AC=\sqrt{2}$, which is a surd.

Consider the following graphs:





Let us try to find the length of AC in two different ways :

$AC = AD + DE + EC$

(diagonals of units squares)

$= \sqrt{2} + \sqrt{2} + \sqrt{2}$

$AC = 3\sqrt{2}$ units

$AC = \sqrt{OA^2 + OC^2} = \sqrt{3^2 + 3^2}$

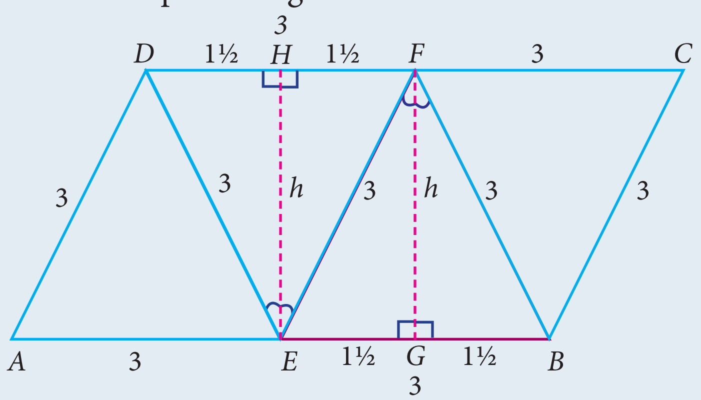
$= \sqrt{9+9}$

$AC = \sqrt{18}$ units

Are they equal? Discuss. Can you verify the same by taking different squares of different lengths?

Activity-3

Consider a parallelogram with sides 6 cm and 3 cm as follows.



$A =$ {Area of parallellogram $ABCD$}

$=$ {sum of areas of equilateral triangles AED, EDF, EFB, BFC}

$= 9\sqrt{3}\ cm^2$ (verify)

Note: (i) Area of parallellogram $= A = b \times h$ sq. units, $b = AE + EB = 3 + 3 = 6\,\text{cm}$

$$h = FG = \sqrt{EF^2 - EG^2} = \sqrt{3^2 - \left(1\frac{1}{2}\right)^2} = 3\frac{\sqrt{3}}{2}\ \text{cm}$$

(ii) Area of equilateral triangle $= A = \frac{\sqrt{3}}{4}a^2$ sq. units, $a = 3$ cm

You can extend the number of triangles (size of parallelogram) and verify the results.

2.4 Rationalisation of Surds

Rationalising factor is a term with which a term is multiplied or divided to make the whole term rational.

Examples:

(i) $\sqrt{3}$ is a rationalising factor of $\sqrt{3}$ (since $\sqrt{3} \times \sqrt{3}$ = the rational number 3)

(ii) $\sqrt[7]{5^4}$ is a rationalising factor of $\sqrt[7]{5^3}$ (since their product $= \sqrt[7]{5^7} = 5$, a rational)

Thinking Corner

1. In the example (i) above, can $\sqrt{12}$ also be a rationalising factor? Can you think of any other number as a rationalising factor for $\sqrt{3}$?
2. Can you think of any other number as a rationalising factor for $\sqrt[7]{5^3}$ in example (ii) ?
3. If there can be many rationalising factors for an expression containing a surd, is there any advantage in choosing the smallest among them for manipulation?

Progress Check

Identify a rationalising factor for each one of the following surds and verify the same in each case:

(i) $\sqrt{18}$ (ii) $5\sqrt{12}$ (iii) $\sqrt[3]{49}$ (iv) $\dfrac{1}{\sqrt{8}}$

2.4.1 Conjugate Surds

Can you guess a rationalising factor for $3+\sqrt{2}$? This surd has one rational part and one radical part. In such cases, the rationalising factor has an interesting form.

A rationalising factor for $3+\sqrt{2}$ is $3-\sqrt{2}$. You can very easily check this.

$$\begin{aligned}\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right) &= 3^2 - \left(\sqrt{2}\right)^2 \\ &= 9-2 \\ &= 7, \text{ a rational.}\end{aligned}$$

What is the rationalising factor for $a+\sqrt{b}$ where a and b are rational numbers? Is it $a-\sqrt{b}$? Check it. What could be the rationalising factor for $\sqrt{a}+\sqrt{b}$ where a and b are rational numbers? Is it $\sqrt{a}-\sqrt{b}$? Or, is it $-\sqrt{a}+\sqrt{b}$? Investigate.

Surds like $a+\sqrt{b}$ and $a-\sqrt{b}$ are called conjugate surds. What is the conjugate of $\sqrt{b}+a$? It is $-\sqrt{b}+a$. You would have perhaps noted by now that a conjugate is usually obtained by changing the sign in front of the surd!

Example 2.13 Rationalise the denominator of (i) $\frac{7}{\sqrt{14}}$ (ii) $\frac{5+\sqrt{3}}{5-\sqrt{3}}$

Solution

(i) Multiply both numerator and denominator by the rationalising factor $\sqrt{14}$.

$$\frac{7}{\sqrt{14}} = \frac{7}{\sqrt{14}} \times \frac{\sqrt{14}}{\sqrt{14}} = \frac{7\sqrt{14}}{14} = \frac{\sqrt{14}}{2}$$

(ii) $$\frac{5+\sqrt{3}}{5-\sqrt{3}} = \frac{\left(5+\sqrt{3}\right)}{\left(5-\sqrt{3}\right)} \times \frac{\left(5+\sqrt{3}\right)}{\left(5+\sqrt{3}\right)} = \frac{\left(5+\sqrt{3}\right)^2}{5^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{5^2+\left(\sqrt{3}\right)^2+2\times 5\times\sqrt{3}}{25-3}$$

$$= \frac{25+3+10\sqrt{3}}{22} = \frac{28+10\sqrt{3}}{22} = \frac{2\times[14+5\sqrt{3}]}{22}$$

$$= \frac{14+5\sqrt{3}}{11}$$

Exercise 2.3

1. Rationalise the denominator

   (i) $\frac{1}{\sqrt{50}}$ (ii) $\frac{5}{3\sqrt{5}}$ (iii) $\frac{\sqrt{75}}{\sqrt{18}}$ (iv) $\frac{3\sqrt{5}}{\sqrt{6}}$

2. Rationalise the denominator and simplify

   (i) $\frac{\sqrt{48}+\sqrt{32}}{\sqrt{27}-\sqrt{18}}$ (ii) $\frac{5\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

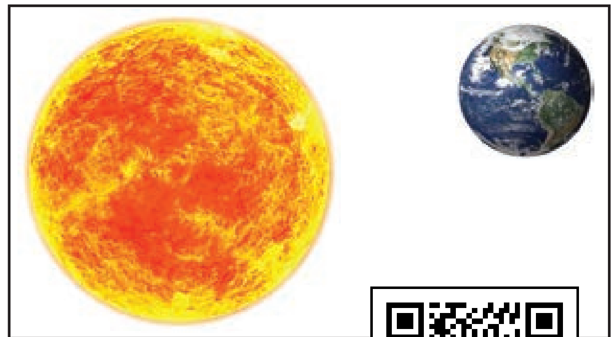
   (iii) $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$ (iv) $\frac{\sqrt{5}}{\sqrt{6}+2} - \frac{\sqrt{5}}{\sqrt{6}-2}$

3. Find the value of a and b if $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7}+b$

4. If $x = \sqrt{5}+2$, then find the value of $x^2 + \frac{1}{x^2}$

5. Given $\sqrt{2} = 1.414$, find the value of $\frac{8-5\sqrt{2}}{3-2\sqrt{2}}$ (to 3 places of decimals).

2.5 Scientific Notation



Suppose you are told that the diameter of the Sun is 13,92,000 km and that of the Earth is 12,740 km, it would seem to be a daunting task to compare them. In contrast, if 13,92,000 is written as 1.392×10^6 and 12,740 as 1.274×10^4, one will feel comfortable. This sort of representation is known as scientific notation.

Since $\dfrac{1.392 \times 10^6}{1.274 \times 10^4} \approx \dfrac{14}{13} \times 10^2 \approx 108$.

You can imagine 108 Earths could line up across the face of the sun.

Scientific notation is a way of representing numbers that are too large or too small, to be conveniently written in decimal form. It allows the numbers to be easily recorded and handled.

2.5.1 Writing a Number in Scientific Notation

Here are steps to help you to represent a number in scientific form:

(i) Move the decimal point so that there is only one non-zero digit to its left.

(ii) Count the number of digits between the old and new decimal point. This gives 'n', the power of 10.

(iii) If the decimal is shifted to the left, the exponent n is positive. If the decimal is shifted to the right, the exponent n is negative.

Expressing a number N in the form of $N = a \times 10^n$ where, $1 \le a < 10$ and 'n' is an integer is called as Scientific Notation.

The following table of base 10 examples may make things clearer:

Decimal notation	Scientific notation	Decimal notation	Scientific notation
100	1×10^2	0.01	1×10^{-2}
1,000	1×10^3	0.001	1×10^{-3}
10,000	1×10^4	0.0001	1×10^{-4}
1,00,000	1×10^5	0.00001	1×10^{-5}
10,00,000	1×10^6	0.000001	1×10^{-6}
1,00,00,000	1×10^7	0.0000001	1×10^{-7}

Let us look into few more examples.

Example 2.14 Express in scientific notation (i) 9768854 (ii) 0.04567891 (iii) 72006865.48

Solution

(i) 9 7 6 8 8 5 4 . 0 $= 9.768854 \times 10^{6}$

6 5 4 3 2 1

The decimal point is to be moved six places to the left. Therefore $n = 6$.

(ii) 0 . 0 4 5 6 7 8 9 1 $= 4.567891 \times 10^{-2}$

1 2

The decimal point is to be moved two places to the right. Therefore $n = -2$.

(iii) 7 2 0 0 6 8 6 5 . 4 8 $= 7.200686548 \times 10^{7}$

7 6 5 4 3 2 1

The decimal point is to be moved seven places to the left. Therefore $n = 7$.

2.5.2 Converting Scientific Notation to Decimal Form

The reverse process of converting a number in scientific notation to the decimal form is easily done when the following steps are followed:

(i) Write the decimal number.

(ii) Move the decimal point by the number of places specified by the power of 10, to the right if positive, or to the left if negative. Add zeros if necessary.

(iii) Rewrite the number in decimal form.

Example 2.15 Write the following numbers in decimal form:

(i) 6.34×10^{4} (ii) 2.00367×10^{-5}

Solution

(i) 6.34×10^{4}

$\Rightarrow$ 6 . 3 4 0 0 $= 63400$

1 2 3 4

(ii) 2.00367×10^{-5}

0 0 0 0 0 2 . 0 0 3 6 7

5 4 3 2 1

$= 0.0000200367$

2.5.3 Arithmetic of Numbers in Scientific Notation

(i) If the indices in the scientific notation of two numbers are the same, addition (or subtraction) is easily performed.

Example 2.16 The mass of the Earth is 5.97×10^{24} kg and that of the Moon is 0.073×10^{24} kg. What is their total mass?

Solution

$$\text{Total mass} = 5.97 \times 10^{24} \text{ kg} + 0.073 \times 10^{24} \text{ kg}$$
$$= (5.97 + 0.073) \times 10^{24} \text{ kg}$$
$$= 6.043 \times 10^{24} \text{ kg}$$

(ii) The product or quotient of numbers in scientific notation can be easily done if we make use of the laws of radicals appropriately.

Example 2.17 Write the following in scientific notation :

(i) $(50000000)^4$ (ii) $(0.00000005)^3$

(iii) $(300000)^3 \times (2000)^4$ (iv) $(4000000)^3 \div (0.00002)^4$

Solution

(i) $(50000000)^4 = (5.0 \times 10^7)^4$

$= (5.0)^4 \times (10^7)^4$

$= 625.0 \times 10^{28}$

$= 6.25 \times 10^2 \times 10^{28}$

$= 6.25 \times 10^{30}$

(ii) $(0.00000005)^3 = (5.0 \times 10^{-8})^3$

$= (5.0)^3 \times (10^{-8})^3$

$= (125.0) \times (10)^{-24}$

$= 1.25 \times 10^2 \times 10^{-24}$

$= 1.25 \times 10^{-22}$

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(iii) $(300000)^3 \times (2000)^4$

$= (3.0\times10^5)^3 \times (2.0\times10^3)^4$

$= (3.0)^3 \times (10^5)^3 \times (2.0)^4 \times (10^3)^4$

$= (27.0)\times(10^{15})\times(16.0)\times(10^{12})$

$= (2.7\times10^1)\times(10^{15})\times(1.6\times10^1)\times(10^{12})$

$= 2.7\times1.6\times10^1\times10^{15}\times10^1\times10^{12}$

$= 4.32\times10^{1+15+1+12} = 4.32\times10^{29}$

(iv) $(4000000)^3 \div (0.00002)^4$

$= (4.0\times10^6)^3 \div (2.0\times10^{-5})^4$

$= (4.0)^3 \times (10^6)^3 \div (2.0)^4 \times (10^{-5})^4$

$= \dfrac{64.0\times10^{18}}{16.0\times10^{-20}}$

$= 4\times10^{18}\times10^{+20}$

$= 4.0\times10^{38}$

Thinking Corner

1. Write two numbers in scientific notation whose product is 2. 83104.
2. Write two numbers in scientific notation whose quotient is 2. 83104.

Exercise 2.4

1. Represent the following numbers in the scientific notation:
   (i) 569430000000 (ii) 2000.57 (iii) 0.0000006000 (iv) 0.0009000002
2. Write the following numbers in decimal form:
   (i) 3.459×10^6 (ii) 5.678×10^4
   (iii) 1.00005×10^{-5} (iv) 2.530009×10^{-7}
3. Represent the following numbers in scientific notation:
   (i) $(300000)^2\times(20000)^4$ (ii) $(0.000001)^{11}\div(0.005)^3$
   (iii) $\{(0.00003)^6\times(0.00005)^4\}\div\{(0.009)^3\times(0.05)^2\}$
4. Represent the following information in scientific notation:
   (i) The world population is nearly 7000,000,000.
   (ii) One light year means the distance 9460528400000000 km.
   (iii) Mass of an electron is
   0.000 000 000 000 000 000 000 000 000 00091093822 kg.
5. Simplify: (i) $(2.75\times10^7) + (1.23\times10^8)$ (ii) $(1.598\times10^{17}) - (4.58\times10^{15})$
   (iii) $(1.02\times10^{10})\times(1.20\times10^{-3})$ (iv) $(8.41\times10^4)\div(4.3\times10^5)$

Activity - 4

The following list shows the mean distance of the planets of the solar system from the Sun. Complete the following table. Then arrange in order of magnitude starting with the distance of the planet closest to the Sun.

Planet	Decimal form (in Km)	Scientific Notation (in Km)
Jupiter		7.78×10^8
Mercury	58000000	
Mars		2.28×10^8
Uranus	2870000000	
Venus	108000000	
Neptune	4500000000	
Earth		1.5×10^8
Saturn		1.43×10^8

Exercise 2.5

Multiple Choice Questions

1. Which of the following statement is false?

   (1) The square root of 25 is 5 or −5 (3) $\sqrt{25} = 5$

   (2) $-\sqrt{25} = -5$ (4) $\sqrt{25} = \pm 5$

2. Which one of the following is not a rational number?

   (1) $\sqrt{\frac{8}{18}}$ (2) $\frac{7}{3}$ (3) $\sqrt{0.01}$ (4) $\sqrt{13}$

3. In simplest form, $\sqrt{640}$ is

   (1) $8\sqrt{10}$ (2) $10\sqrt{8}$ (3) $2\sqrt{20}$ (4) $4\sqrt{5}$

4. $\sqrt{27} + \sqrt{12} =$

   (1) $\sqrt{39}$ (2) $5\sqrt{6}$ (3) $5\sqrt{3}$ (4) $3\sqrt{5}$

5. If $\sqrt{80} = k\sqrt{5}$, then $k =$

   (1) 2 (2) 4 (3) 8 (4) 16

6. $4\sqrt{7} \times 2\sqrt{3} =$

   (1) $6\sqrt{10}$ (2) $8\sqrt{21}$ (3) $8\sqrt{10}$ (4) $6\sqrt{21}$

7. When written with a rational denominator, the expression $\frac{2\sqrt{3}}{3\sqrt{2}}$ can be simplified as

(1) $\frac{\sqrt{2}}{3}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{\sqrt{6}}{3}$ (4) $\frac{2}{3}$

8. When $\left(2\sqrt{5}-\sqrt{2}\right)^2$ is simplified, we get

(1) $4\sqrt{5}+2\sqrt{2}$ (2) $22-4\sqrt{10}$ (3) $8-4\sqrt{10}$ (4) $2\sqrt{10}-2$

9. $\frac{\sqrt[3]{18}}{\sqrt[3]{2}}$ is same as

(1) 3 (2) $\sqrt[3]{9}$ (3) 9 (4) $\sqrt[3]{3}$

10. $\left(0.000729\right)^{\frac{-3}{4}} \times \left(0.09\right)^{\frac{-3}{4}}$ = ______

(1) $\frac{10^3}{3^3}$ (2) $\frac{10^5}{3^5}$ (3) $\frac{10^2}{3^2}$ (4) $\frac{10^6}{3^6}$

11. If $\sqrt{9^x} = \sqrt[3]{9^2}$, then x = ______

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{1}{3}$ (4) $\frac{5}{3}$

12. Which is the best example of a number written in scientific notation?

(1) 0.5×10^5 (2) 0.1254 (3) 5.367×10^{-3} (4) 12.5×10^2

13. What is 5.92×10^{-3} written in decimal form?

(1) 0.000592 (2) 0.00592 (3) 0.0592 (4) 0.592

14. The length and breadth of a rectangular plot are 5×10^5 and 4×10^4 metres respectively. Its area is ______.

(1) $9\times10^1\ m^2$ (2) $9\times10^9\ m^2$ (3) $2\times10^{10}\ m^2$ (4) $20\times10^{20}\ m^2$

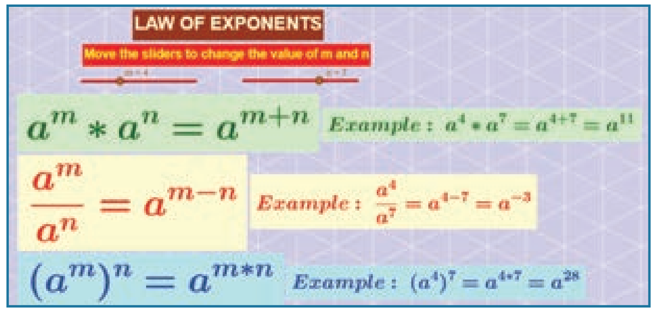
Points to Remember

- If 'a' is a positive rational number, 'n' is a positive integer and if $\sqrt[n]{a}$ is an irrational number, then $\sqrt[n]{a}$ is called as a surd.
- If 'm', 'n' are positive integers and a, b are positive rational numbers, then

(i) $\left(\sqrt[n]{a}\right)^n = a = \sqrt[n]{a^n}$ (ii) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ (iii) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$ (iv) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

- The process of multiplying a surd by another surd to get a rational number is called Rationalisation.
- Expressing a number N in the form of $N = a\times10^n$ where, $1 \le a < 10$ and 'n' is an integer is called as Scientific Notation.

ICT Corner



Expected Result is shown in this picture

Step – 1

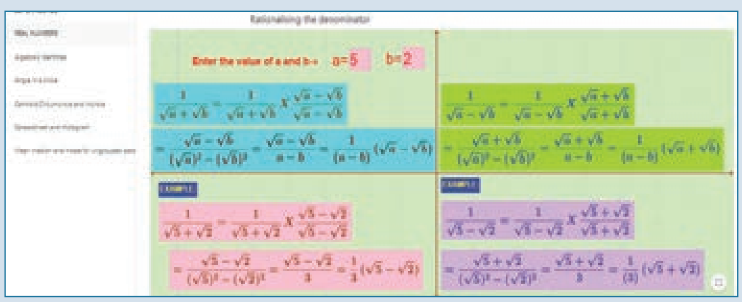
Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Real Numbers" will open. In the work sheet there are two activities. 1. Rationalising the denominator for surds and 2. Law of exponents.

In the first activity procedure for rationalising the denominator is given. Also, example is given under. To change the values of a and b enter the value in the input box given.

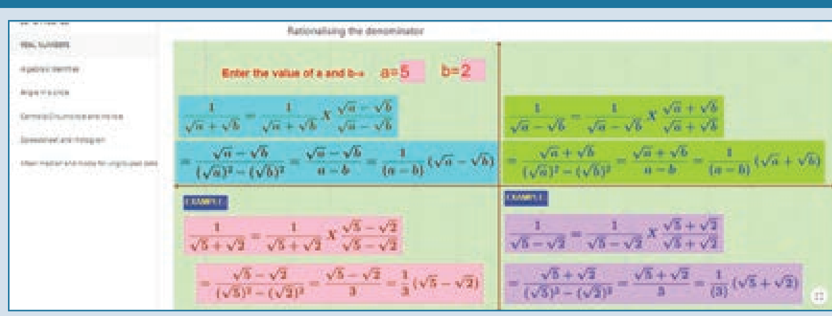
Step - 2

In the second activity law of exponents is given. Also, example is given on right side. To change the value of m and n move the sliders and check the answers.

Step 1



Step 2



Browse in the link

Real Numbers: https://ggbm.at/BYEWDpHU or Scan the QR Code.

B466_MAT_9_T2_EM

3 ALGEBRA

" Algebra is generous. She often gives more than is asked of her"

– D'Alambert

BC96YS

Paolo Ruffini
(1765 -1822, Italy)

Paolo Ruffini was an Italian mathematician and philosopher. By 1781 he had been awarded university degree of philosophy, medicine, surgery and mathematics. An elegant way of dividing polynomial by a linear polynomial with the help of the coefficients involved was introduced by him in 1809. His method is known as synthetic division method. He found the answer to the number of solutions for a given polynomial equations which is derived from the fundamental theorem of algebra based on mathematical beauty of having 'n' solutions for 'n' degree equations.

Learning Objectives

- To understand and apply Factor theorem.
- To use Algebraic Identities in factorisation.
- To factorise a polynomial.
- To find GCD.
- To factorise a quadratic polynomial of the type $ax^2 + bx + c \ (a \neq 0)$
- To factorise a cubic polynomial.
- To use synthetic division to factorise a polynomial.

3.1 Introduction

In the first term, we learnt about Polynomials, classification of polynomials based on degrees and number of terms, zeros of polynomial, basic operations on polynomials, remainder theorem and their applications.

Polynomial

A polynomial is an expression consisting of variables and constants that involves four fundamental arithmetic operations and non-negative integer exponents of variables.

Classification of polynomial

According to number of terms		
Number of terms	**Name**	**Example**
one term	Monomial	$5, -5x, 7x^2$
two terms	Binomial	$(x+5), x^2-75$
three terms	Trinomial	$(a+b+c), a^3+b^2+c^5$
4 terms	Quadrinomial	$x^6+y^4+2+m^3$

According to degree of the polynomial		
Degree	**Name**	**Example**
zero	Constant Polynomial	$5, -7, -5, 25$
one	Linear polynomial	$x+5, y-5, 7x, -25x$
two	Quadratic polynomial	x^2-5x, x^2+5x+6
three	Cubic polynomial	$x^3+x+7, 7x^3-5x^2+3x+4$

Zero of a polynomial or root of a polynomial

If $p(a)=0$ we say that,

'a' is zero of a polynomial $p(x)$ or

'a' is a root of a polynomial equation $p(x)=0$.

Note

Degree of constant polynomial = 0. For example, $5 = 5\times1 = 5\times x^0 = 5x^0$

Thinking Corner

(1) Is $2x^2+\dfrac{5x}{1+x}$ a polynomial ? (2) Is $5x^{-3}+2x^2-x-9$ a polynomial?

(3) Is $4x+3\sqrt{x}-1$ a polynomial? Give reasons for your answers.

Remainder theorem

If a polynomial $p(x)$ is divided by $(x-a)$, then $p(a)$ is the remainder.

If (x^2-6x+8) is divided by $(x-3)$, then by using remainder theorem, the remainder is $p(3)$ $p(3)=3^2-6(3)+8$ $=9-18+8$ $=-1$ $p(3)\neq 0$ Therefore $(x-3)$ is not a factor of $p(x)$.	To find the zero of $x-3$, put $x-3=0$ we get, $x=3$

If the above polynomial is divided by $(x-4)$, then by using remainder theorem, the remainder is $p(4)$; $p(4) = 4^2 - 6(4) + 8$ $= 16 - 24 + 8$ $p(4) = 0$ Therefore $(x-4)$ is a factor of $p(x)$.	To find the zero of $x-4$, put $x-4=0$ we get $x=4$

Thus, if $p(x)$ is divided by $(x-a)$ with the remainder $p(a)=0$, then $(x-a)$ is a factor of $p(x)$. Remainder theorem leads to Factor theorem.

3.2 Factor Theorem

If $p(x)$ is a polynomial of degree $n \geq 1$ and 'a' is any real number then

(i) $p(a)=0$ implies $(x-a)$ is a factor of $p(x)$.

(ii) $(x-a)$ is a factor of $p(x)$ implies $p(a)=0$.

Significance of Factor Theorem

It enables us to find whether the given linear polynomial is a factor or not without actually following the process of long division.

Proof

If $p(x)$ is the dividend and $(x-a)$ is a divisor, then by division algorithm we write, $p(x)=(x-a)q(x)+p(a)$ where $q(x)$ is the quotient and $p(a)$ is the remainder.

(i) If $p(a)=0$, we get $p(x)=(x-a)\,q(x)$ which shows that $(x-a)$ is a factor of $p(x)$.

(ii) Since $(x-a)$ is a factor of $p(x)$

$p(x)=(x-a)g(x)$ for some polynomial $g(x)$.

In this case

$$p(a) = (a-a)\,g(a) = 0 \times g(a) = 0$$

Hence, $p(a)=0$, when $(x-a)$ is a factor of $p(x)$.

Thinking Corner

For any two integers a $(a \neq 0)$ and b, a divides b if $b = ax$, for some integer x.

Note

- $(x-a)$ is a factor of $p(x)$, if $p(a)=0$ $\quad (\because x-a=0,\ x=a)$
- $(x+a)$ is a factor of $p(x)$, if $p(-a)=0$ $\quad (\because x+a=0,\ x=-a)$
- $(ax+b)$ is a factor of $p(x)$, if $p\left(-\frac{b}{a}\right)=0$ $\quad \left(\because ax+b=0,\ ax=-b,\ x=-\frac{b}{a}\right)$
- $(ax-b)$ is a factor of $p(x)$, if $p\left(\frac{b}{a}\right)=0$ $\quad \left(\because ax-b=0,\ ax=b,\ x=\frac{b}{a}\right)$
- $(x-a)(x-b)$ is a factor of $p(x)$, if $p(a)=0$ and $p(b)=0$ $\quad \left(\because \begin{array}{ccc} x-a=0 & \text{or} & x-b=0 \\ x=a & \text{or} & x=b \end{array}\right)$

Example 3.1 Show that $(x+2)$ is a factor of $x^3-4x^2-2x+20$

Solution

Let $p(x)=x^3-4x^2-2x+20$

By factor theorem, $(x+2)$ is factor of $p(x)$, *if* $p(-2)=0$

$$p(-2)=(-2)^3-4(-2)^2-2(-2)+20$$
$$=-8-4(4)+4+20$$
$$p(-2)=0$$

Therefore, $(x+2)$ is a factor of $x^3-4x^2-2x+20$

To find the zero of $x+2$;
put $x+2=0$
we get $x=-2$

Example 3.2 Is $(3x-2)$ a factor of $3x^3+x^2-20x+12$?

Solution

Let $p(x)=3x^3+x^2-20x+12$

By factor theorem, $(3x-2)$ is a factor, if $p\left(\frac{2}{3}\right)=0$

$$p\left(\frac{2}{3}\right)=3\left(\frac{2}{3}\right)^3+\left(\frac{2}{3}\right)^2-20\left(\frac{2}{3}\right)+12$$
$$=3\left(\frac{8}{27}\right)+\left(\frac{4}{9}\right)-20\left(\frac{2}{3}\right)+12$$
$$=\frac{8}{9}+\frac{4}{9}-\frac{120}{9}+\frac{108}{9}$$
$$=\frac{(120-120)}{9}$$
$$p\left(\frac{2}{3}\right)=0$$

Therefore, $(3x-2)$ is a factor of $3x^3+x^2-20x+12$

To find the zero of $3x-2$;
put $3x-2=0$
$3x=2$
we get $x=\frac{2}{3}$

Progress Check

1. $(x+3)$ is a factor of $p(x)$, if $p(\_\_)=0$
2. $(3-x)$ is a factor of $p(x)$, if $p(\_\_)=0$
3. $(y-3)$ is a factor of $p(y)$, if $p(\_\_)=0$
4. $(-x-b)$ is a factor of $p(x)$, if $p(\_\_)=0$
5. $(-x+b)$ is a factor of $p(x)$, if $p(\_\_)=0$

Example 3.3 Find the value of m, if $(x-2)$ is a factor of the polynomial $2x^3-6x^2+mx+4$.

Solution

Let $p(x)=2x^3-6x^2+mx+4$

By factor theorem, $(x-2)$ is a factor of $p(x)$, if $p(2)=0$

To find the zero of $x-2$;
put $x-2=0$
we get $x=2$

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$$
\begin{aligned}
p(2) &= 0\\
2(2)^3 - 6(2)^2 + m(2) + 4 &= 0\\
2(8) - 6(4) + 2m + 4 &= 0\\
-4 + 2m &= 0\\
m &= 2
\end{aligned}
$$

Exercise 3.1

1. Determine whether $(x-1)$ is a factor of the following polynomials:

   i) $x^3 + 5x^2 - 10x + 4$ *ii)* $x^4 + 5x^2 - 5x + 1$

2. Determine whether $(x+2)$ is a factor of $2x^4 + x^3 + 4x^2 - x - 7$.
3. Using factor theorem, show that $(x-5)$ is a factor of the polynomial $2x^3 - 5x^2 - 28x + 15$
4. Determine the value of m, if $(x+3)$ is a factor of $x^3 - 3x^2 - mx + 24$.
5. If both $(x-2)$ and $\left(x-\frac{1}{2}\right)$ are the factors of $ax^2 + 5x + b$, then show that $a = b$.
6. Is $(2x-3)$ a factor of $p(x) = 2x^3 - 9x^2 + x + 12$?
7. If $(x-1)$ divides the polynomial $kx^3 - 2x^2 + 25x - 26$ without remainder, then find the value of k.
8. Check if $(x+2)$ and $(x-4)$ are the sides of a rectangle whose area is $x^2 - 2x - 8$ by using factor theorem.

3.3 Algebraic Identities

An identity is an equality that remains true regardless of the values chosen for its variables.

We have already learnt about the following identities:

(1) $(a+b)^2 \equiv a^2 + 2ab + b^2$ (2) $(a-b)^2 \equiv a^2 - 2ab + b^2$

(3) $(a+b)(a-b) \equiv a^2 - b^2$ (4) $(x+a)(x+b) \equiv x^2 + (a+b)x + ab$

Note

(i) $a^2 + b^2 = (a+b)^2 - 2ab$ (ii) $a^2 + b^2 = (a-b)^2 + 2ab$

(iii) $a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$ (iv) $a^2 + \frac{1}{a^2} = \left(a - \frac{1}{a}\right)^2 + 2$

Example 3.4

Expand the following using identities: (i) $(3x+4y)^2$

(ii) $(2a-3b)^2$ (iii) $(5x+4y)(5x-4y)$ (iv) $(m+5)(m-8)$

Solution

(i) $(3x+4y)^2$ $\left[\text{we have } (a+b)^2 = a^2+2ab+b^2\right]$

$(3x+4y)^2 = (3x)^2 + 2(3x)(4y) + (4y)^2$ put $[a = 3x,\ b = 4y]$

$= 9x^2 + 24xy + 16y^2$

(ii) $(2a-3b)^2$ $\left[\text{we have } (a-b)^2 = a^2-2ab+b^2\right]$

$(2a-3b)^2 = (2a)^2 - 2(2a)(3b) + (3b)^2$ put $[a = 2a,\ b = 3b]$

$= 4a^2 - 12ab + 9b^2$

(iii) $(5x+4y)(5x-4y)$ $\left[\text{we have } (a+b)(a-b) = a^2-b^2\right]$

$(5x+4y)(5x-4y) = (5x)^2 - (4y)^2$ put $[a = 5x\ \ b = 4y]$

$= 25x^2 - 16y^2$

(iv) $(m+5)(m-8)$ $\left[\text{we have } (x+a)(x-b) = x^2+(a-b)x-ab\right]$

$(m+5)(m-8) = m^2 + (5-8)m - (5)(8)$ put $[x = m,\ a = 5,\ b = 8]$

$= m^2 - 3m - 40$

3.3.1 Expansion of Trinomial $(a+b+c)^2$

We know that $(x+y)^2 = x^2 + 2xy + y^2$

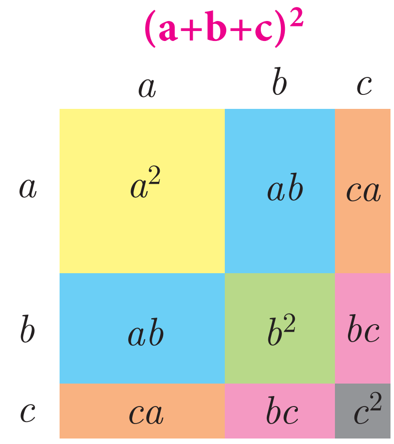
Put $x = a+b,\ y = c$

Then, $(a+b+c)^2 = (a+b)^2 + 2(a+b)(c) + c^2$

$= a^2 + 2ab + b^2 + 2ac + 2bc + c^2$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Thus, $(a+b+c)^2 \equiv a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$



Example 3.5

Expand $(a-b+c)^2$

Solution

Replacing 'b' by '$-b$' in the expansion of

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$(a+(-b)+c)^2 = a^2+(-b)^2+c^2+2a(-b)+2(-b)c+2ca$$
$$= a^2+b^2+c^2-2ab-2bc+2ca$$

Progress Check

Expand the following and verify :

$(a+b+c)^2 = (-a-b-c)^2$

$(-a+b+c)^2 = (a-b-c)^2$

$(a-b+c)^2 = (-a+b-c)^2$

$(a+b-c)^2 = (-a-b+c)^2$

Example 3.6 Expand $(2x+3y+4z)^2$

Solution

We know that,

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

Substituting, $a=2x, b=3y$ and $c=4z$

$$(2x+3y+4z)^2 = (2x)^2+(3y)^2+(4z)^2+2(2x)(3y)+2(3y)(4z)+2(4z)(2x)$$
$$= 4x^2+9y^2+16z^2+12xy+24yz+16xz$$

Example 3.7 Find the area of square whose side length is $3m+2n-4l$

Solution

Area of square $=$ side $\times$ side

$= (3m+2n-4l)\times(3m+2n-4l)$

$= (3m+2n-4l)^2$

We know that, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

substituting
$a=3m$,
$b=2n$
$c=-4l$

$$\left[3m+2n+(-4l)\right]^2 = (3m)^2+(2n)^2+(-4l)^2+2(3m)(2n)+2(2n)(-4l)+2(-4l)(3m)$$
$$= 9m^2+4n^2+16l^2+12mn-16ln-24lm$$

Therefore, Area of square $= [9m^2+4n^2+16l^2+12mn-16ln-24lm]$ sq.units.

3.3.2 Identities involving Product of Three Binomials

$$(x+a)(x+b)(x+c) = [(x+a)\ (x+b)]\ (x+c)$$
$$= [x^2+(a+b)x+ab](x+c)$$
$$= x^2(x)+(a+b)(x)(x)+abx+x^2c+(a+b)(x)c+abc$$
$$= x^3+ax^2+bx^2+abx+cx^2+acx+bcx+abc$$
$$= x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$$

Thus, $(x+a)(x+b)(x+c) \equiv x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$

Example 3.8 Expand the following:

(i) $(x+5)(x+6)(x+4)$

(ii) $(b+3)(b+4)(b-5)$

(iii) $(2a+3)(2a+4)(2a+5)$

(iv) $(3x-1)(3x+2)(3x-4)$

Solution

We know that $(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$ --(1)

(i) $(x+5)(x+6)(x+4)$

$= x^3 + (5+6+4)x^2 + (30+24+20)x + (5)(6)(4)$

$= x^3 + 15x^2 + 74x + 120$

Replace *a* by 5, *b* by 6, *c* by 4 in (1)

(ii) $(2a+3)(2a+4)(2a+5)$

$= (2a)^3 + (3+4+5)(2a)^2 + (12+20+15)(2a) + (3)(4)(5)$

$= 8a^3 + (12)(4a^2) + (47)(2a) + 60$

$= 8a^3 + 48a^2 + 94a + 60$

Replace *x* by 2*a*, *a* by 3, *b* by 4, *c* by 5 in (1)

(iii) $(b+3)(b+4)(b-5)$

$= b^3 + (3+4-5)b^2 + (12-20-15)b + (3)(4)(-5)$

$= b^3 + (2)b^2 + (-23)b + (-60)$

$= b^3 + 2b^2 - 23b - 60$

Replace *x* by *b*, *a* by 3, *b* by 4, *c* by –5 in (1)

(iv) $(3x-1)(3x+2)(3x-4)$

$= (3x)^3 + (-1+2-4)(3x)^2 + (-2-8+4)(3x) + (-1)(2)(-4)$

$= 27x^3 + (-3)9x^2 + (-6)(3x) + 8$

$= 27x^3 - 27x^2 - 18x + 8$

Replace *x* by 3*x*, *a* by –1, *b* by 2, *c* by –4 in (1)

3.3.3 Expansion of $(x+y)^3$ and $(x-y)^3$

$$(x+a)(x+b)(x+c) \equiv x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$$

substituting $a = b = c = y$ in the identity

we get, $(x+y)(x+y)(x+y) = x^3 + (y+y+y)x^2 + (yy+yy+yy)x + yyy$

$= x^3 + (3y)x^2 + (3y^2)x + y^3$

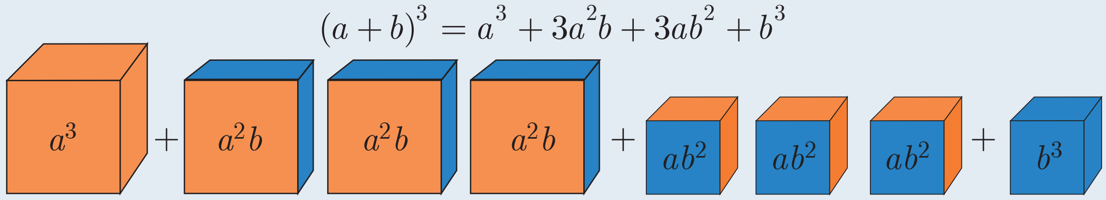
Thus, $(x+y)^3 \equiv x^3 + 3x^2y + 3xy^2 + y^3$ (or) $(x+y)^3 \equiv x^3 + y^3 + 3xy(x+y)$

by replacing y by $-y$, we get

$(x-y)^3 \equiv x^3 - 3x^2y + 3xy^2 - y^3$ (or) $(x-y)^3 \equiv x^3 - y^3 - 3xy(x-y)$

Activity - 1

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$



Use cube and cuboids like the ones seen above here to make a model of $(a-b)^3$

Example 3.9 Expand $(2x+3y)^3$

Solution

We know that, $(x+y)^3 = x^3+3x^2y+3xy^2+y^3$

$$\begin{aligned}(2x+3y)^3 &= (2x)^3+3(2x)^2(3y)+3(2x)(3y)^2+(3y)^3\\ &= 8x^3+3(4x^2)(3y)+3(2x)(9y^2)+27y^3\\ &= 8x^3+36x^2y+54xy^2+27y^3\end{aligned}$$

Example 3.10 Expand $(5a-3b)^3$

Solution

We know that, $(x-y)^3 = x^3-3x^2y+3xy^2-y^3$

$$\begin{aligned}(5a-3b)^3 &= (5a)^3-3(5a)^2(3b)+3(5a)(3b)^2-(3b)^3\\ &= 125a^3-3(25a^2)(3b)+3(5a)(9b^2)-(3b)^3\\ &= 125a^3-225a^2b+135ab^2-27b^3\end{aligned}$$

The following identity is also used:

$$x^3+y^3+z^3-3xyz \equiv (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$

We can check this by performing the multiplication on the right hand side.

Note

(i) If $(x+y+z)=0$ then $x^3+y^3+z^3=3xyz$

Some identities involving sum, difference and product are stated without proof

(i) $x^3+y^3 \equiv (x+y)^3-3xy(x+y)$ (ii) $x^3-y^3 \equiv (x-y)^3+3xy(x-y)$

Example 3.11 Find the product of

$(2x+3y+4z)(4x^2+9y^2+16z^2-6xy-12yz-8zx)$

Solution

We know that, $(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = a^3+b^3+c^3-3abc$

$$\begin{aligned}&(2x+3y+4z)(4x^2+9y^2+16z^2-6xy-12yz-8zx)\\ &= (2x)^3+(3y)^3+(4z)^3-3(2x)(3y)(4z)\\ &= 8x^3+27y^3+64z^3-72xyz\end{aligned}$$

Example 3.12 Evaluate $10^3 - 15^3 + 5^3$

Solution

Replace a by 10, b by –15, c by 5

We know that, if $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Here, $a+b+c \;=10-15+5=0$

$$\text{Therefore, } 10^3+(-15)^3+5^3 = 3(10)(-15)(5)$$

$$10^3-15^3+5^3 = -2250$$

Example 3.13 Prove $a^2+b^2+c^2-ab-bc-ca=\frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

Solution

$$a^2+b^2+c^2-ab-bc-ca = \frac{2[a^2+b^2+c^2-ab-bc-ca)}{2} \quad \text{(Multiply and divide by 2)}$$

$$= \frac{1}{2}\left[2a^2+2b^2+2c^2-2ab-2bc-2ca\right]$$

$$= \frac{1}{2}[(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ca)]$$

$$= \frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]$$

$$\therefore a^2+b^2+c^2-ab-bc-ca = \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right].$$

Exercise 3.2

1. Expand the following:
   (i) $(2x+3y+4z)^2$ (ii) $(2a-3b+4c)^2$
   (iii) $(-p+2q+3r)^2$ (iv) $\left(\frac{a}{4}+\frac{b}{3}+\frac{c}{2}\right)^2$

2. Find the expansion of the following:
   (i) $(x+4)(x+5)(x+6)$ (ii) $(2p+3)(2p-4)(2p-5)$
   (iii) $(3a+1)(3a-2)(3a+4)$ (iv) $(5+4m)(4m+4)(-5+4m)$

3. Using algebraic identity, find the coefficients of x^2, x and constant term without actual expansion.
   (i) $(x+5)(x+6)(x+7)$ (ii) $(2x+3)(2x-5)(2x-6)$

4. If $(x+a)(x+b)(x+c)=x^3+14x^2+59x+70$, find the value of
   (i) $a+b+c$ (ii) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ (iii) $a^2+b^2+c^2$ (iv) $\frac{a}{bc}+\frac{b}{ac}+\frac{c}{ab}$

5. Expand:
   (i) $(2a+3b)^3$ (ii) $(3a-4b)^3$ (iii) $\left(x+\frac{1}{y}\right)^3$ (iv) $\left(a+\frac{1}{a}\right)^3$
6. Evaluate the following by using identities:
   (i) 98^3 (ii) 103^3 (iii) 99^3 (iv) 1001^3
7. If $(x+y+z)=9$ and $(xy+yz+zx)=26$, then find the value of $x^2+y^2+z^2$.
8. Find $27a^3+64b^3$, if $3a+4b=10$ and $ab=2$.
9. Find x^3-y^3, if $x-y=5$ and $xy=14$.
10. If $a+\frac{1}{a}=6$, then find the value of $a^3+\frac{1}{a^3}$.
11. If $x^2+\frac{1}{x^2}=23$, then find the value of $x+\frac{1}{x}$ and $x^3+\frac{1}{x^3}$.
12. If $\left(y-\frac{1}{y}\right)^3=27$, then find the value of $y^3-\frac{1}{y^3}$.
13. Simplify: (i) $(2a+3b+4c)(4a^2+9b^2+16c^2-6ab-12bc-8ca)$
    (ii) $(x-2y+3z)(x^2+4y^2+9z^2+2xy+6yz-3xz)$
14. By using identity evaluate the following:
    (i) $7^3-10^3+3^3$ (ii) $729-216-27$
    (iii) $\left(\frac{1}{3}\right)^3+\left(\frac{1}{2}\right)^3-\left(\frac{5}{6}\right)^3$ (iv) $1+\frac{1}{8}-\frac{27}{8}$
15. Simplify : $\left[\frac{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}{(x-y)^3+(y-z)^3+(z-x)^3}\right]$ by using identity.
16. If $a=4, b=5$ and $c=6$, then find the value of $\left[\frac{(ab+bc+ca-a^2-b^2-c^2)}{(3abc-a^3-b^3-c^3)}\right]$.
17. Verify $x^3+y^3+z^3-3xyz=\frac{1}{2}[x+y+z][(x-y)^2+(y-z)^2+(z-x)^2]$.
18. If $2x-3y-4z=0$, then find $8x^3-27y^3-64z^3$.

3.4 Factorisation

Factorisation is the reverse of multiplication.

Example 1 : Multiply 3 and 5; we get product 15.
Factorise 15; we get factors 3 and 5.

Example 2 : Multiply $(x+2)$ and $(x+3)$; we get product x^2+5x+6.
Factorise x^2+5x+6 ; we get factors $(x+2)$ and $(x+3)$.

Multiply ↓	Factorise ↑
$(x+3)(x+2)$	$(x+3)(x+2)$
$x(x+2)+3(x+2)$	$x(x+2)+3(x+2)$
$x^2+2x+3x+6$	$x^2+2x+3x+6$
x^2+5x+6	x^2+5x+6

Thus, the process of converting the given higher degree polynomial as the product of factors of its lower degree, which cannot be further factorised is called factorisation.

Two important ways of factorisation are :

(i) By taking common factor

$ab+ac$

$a\cdot b+a\cdot c$

$a(b+c)$ factored form

(ii) By grouping them

$a+b-pa-pb$

$(a+b)-p(a+b)$ group in pairs

$(a+b)(1-p)$ factored form

When a polynomial is factored, we "factored out" the common factor.

Example 3.14 Factorise the following:

(i) $am+bm+cm$ (ii) a^3-a^2b (iii) $5a-10b-4bc+2ac$ (iv) $x+y-1-xy$

Solutions

(i) $am+bm+cm$

$am+bm+cm$

$m(a+b+c)$ factored form

(ii) a^3-a^2b

$a^2\cdot a-a^2\cdot b$ group in pairs

$a^2\times(a-b)$ factored form

(iii) $5a-10b-4bc+2ac$

$5a-10b+2ac-4bc$

$5(a-2b)+2c(a-2b)$

$(a-2b)(5+2c)$

(iv) $x+y-1-xy$

$x-1+y-xy$

$(x-1)+y(1-x)$

$(x-1)-y(x-1)$

$(x-1)(1-y)$

$(a-b)=-(b-a)$

3.4.1 Greatest Common Divisor (GCD)

The Greatest Common Divisor, abbreviated as GCD, of two or more polynomials is a polynomial, of the highest common possible degree, that is a factor of the given two or more polynomials. It is also known as the Highest Common Factor (HCF).

This concept is similar to the greatest common divisor of two integers.

For example : Consider the numbers 24 and 36. Their common divisors are 1, 2, 3, 4, 6 and 12. Among these, 12 is the largest.

24 = 1, 2, 3, 4, 6, 8, 12, 24

36 = 1, 2, 3, 4, 6, 9, 12, 18, 36

Therefore, the GCD of 24 and 36 is 12.

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For example : Consider the expressions $14xy^2$ and $42xy$. The common divisors of 14 and 42 are 2, 7 and 14. Their GCD is thus 14. The only common divisors of xy^2 and xy *are* x, y and xy; their GCD is thus xy.

$$14xy^2 = 1 \cdot 2 \cdot 7 \cdot x \cdot y \cdot y$$
$$42xy = 1 \cdot 2 \cdot 3 \cdot 7 \cdot x \cdot y$$

Therefore the requried GCD of $14xy^2$ and $42xy$ is $14xy$.

To find the GCD by Factorisation

(i) Each expression is to be resolved into factors first.

(ii) The product of factors having the highest common powers in those factors will be the GCD.

(iii) If the expression have numerical coefficient, find their GCD separately and then prefix it as a coefficient to the GCD for the given expressions.

Example 3.15 Find GCD of the following:

(i) $16x^3y^2$, $24xy^3z$ (ii) (y^3+1) and (y^2-1)

(iii) $2x^2-18$ and x^2-2x-3 (iv) $(a-b)^2$, $(b-c)^3$, $(c-a)^4$

Solutions

(i) $16x^3y^2 = 2\times2\times2\times2\times x^3y^2 = 2^4\times x^3\times y^2 = 2^3\times2\times x^2\times x\times y^2$

$24xy^3z = 2\times2\times2\times3\times x\times y^3\times z = 2^3\times3\times x\times y^3\times z = 2^3\times3\times x\times y\times y^2\times z$

Therefore, $GCD = 2^3xy^2$

(ii) $y^3+1 = y^3+1^3 = (y+1)(y^2-y+1)$

$y^2-1 = y^2-1^2 = (y+1)(y-1)$

Therefore, $GCD = (y+1)$

(iii) $2x^2-18 = 2(x^2-9) = 2(x^2-3^2) = 2(x+3)(x-3)$

$$x^2-2x-3 = x^2-3x+x-3$$
$$= x(x-3)+1(x-3)$$
$$= (x-3)(x+1)$$

Therefore, $GCD = (x-3)$

(iv) $(a-b)^2$, $(b-c)^3$, $(c-a)^4$

There is no common factor other than one.

Therefore, $GCD = 1$

Exercise 3.3

1. Find the GCD for the following:

(i) p^5, p^{11}, p^9 (ii) $4x^3, y^3, z^3$

(iii) $9a^2b^2c^3, 15a^3b^2c^4$ (iv) $64x^8, 240x^6$

(v) $ab^2c^3, a^2b^3c, a^3bc^2$ (vi) $35x^5y^3z^4, 49x^2yz^3, 14xy^2z^2$

(vii) $25ab^3c, 100a^2bc, 125ab$ (viii) $3abc, 5xyz, 7pqr$

2. Find the GCD of the following:

(i) $(2x+5), (5x+2)$ (ii) $a^{m+1}, a^{m+2}, a^{m+3}$

(iii) $2a^2+a, 4a^2-1$ (iv) $3a^2, 5b^3, 7c^4$

(v) x^4-1, x^2-1 (vi) $a^3-9ax^2, (a-3x)^2$

3.4.2 Factorisation using Identity

(i) $a^2+2ab+b^2 \equiv (a+b)^2$ (ii) $a^2-2ab+b^2 \equiv (a-b)^2$

(iii) $a^2-b^2 \equiv (a+b)(a-b)$ (iv) $a^2+b^2+c^2+2ab+2bc+2ca \equiv (a+b+c)^2$

(v) $a^3+b^3 \equiv (a+b)(a^2-ab+b^2)$ (vi) $a^3-b^3 \equiv (a-b)(a^2+ab+b^2)$

(vii) $a^3+b^3+c^3-3abc \equiv (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Note

$(a+b)^2+(a-b)^2 = 2(a^2+b^2)$; $a^4-b^4 = (a^2+b^2)(a+b)(a-b)$

$(a+b)^2-(a-b)^2 = 4ab$; $a^6-b^6 = (a+b)(a-b)(a^2-ab+b^2)(a^2+ab+b^2)$

Progress Check

Prove: (i) $\left(a+\frac{1}{a}\right)^2+\left(a-\frac{1}{a}\right)^2 = 2\left(a^2+\frac{1}{a^2}\right)$ (ii) $\left(a+\frac{1}{a}\right)^2-\left(a-\frac{1}{a}\right)^2 = 4$

Example 3.16 Factorise the following:

(i) $9x^2+12xy+4y^2$ (ii) $25a^2-10a+1$ (iii) $36m^2-49m^2$

(iv) x^3-x (v) x^4-16 (vi) $x^2+4y^2+9z^2-4xy+12yz-6xz$

Solution

(i) $9x^2 + 12xy + 4y^2 = (3x)^2 + 2(3x)(2y) + (2y)^2 [\because a^2 + 2ab + b^2 = (a+b)^2]$

$= (3x + 2y)^2$

(ii) $25a^2 - 10a + 1 = (5a)^2 - 2(5a)(1) + 1^2$

$= (5a - 1)^2 \quad \left[\because a^2 - 2ab + b^2 = (a-b)^2\right]$

(iii) $36m^2 - 49n^2. = (6m)^2 - (7n)^2$

$= (6m + 7n)(6m - 7n) \quad \left[\because a^2 - b^2 = (a+b)(a-b)\right]$

(iv) $x^3 - x = x(x^2 - 1)$

$= x(x^2 - 1^2)$

$= x(x + 1)(x - 1)$

(v) $x^4 - 16 = x^4 - 2^4 \quad \left[\because a^4 - b^4 = (a^2 + b^2)(a+b)(a-b)\right]$

$= (x^2 + 2^2)(x^2 - 2^2)$

$= (x^2 + 4)(x + 2)(x - 2)$

(vi) $x^2 + 4y^2 + 9z^2 - 4xy + 12yz - 6xz$

$= (-x)^2 + (2y)^2 + (3z)^2 + 2(-x)(2y) + 2(2y)(3z) + 2(3z)(-x)$

$= (-x + 2y + 3z)^2$ (or) $(x - 2y - 3z)^2$

Example 3.17 Factorise the following:

(i) $27x^3 + 125y^3$ (ii) $216m^3 - 343m^3$

(iii) $2x^4 - 16xy^3$ (iv) $8x^3 + 27y^3 + 642^3 - 72xy^2$

Solution

(i) $27x^3 + 125y^3 = (3x)^3 + (5y)^3 \quad \left[\because (a^3 + b^3) = (a+b)(a^2 - ab + b^2)\right]$

$= (3x + 5y)\left((3x)^2 - (3x)(5y) + (5y)^2\right)$

$= (3x + 5y)(9x^2 - 15xy + 25y^2)$

(ii) $216m^3 - 343m^3 = (6m)^3 - (7n)^3 \quad \left[\because (a^3 - b^3) = (a-b)(a^2 + ab + b^2)\right]$

$= (6m - 7n)\left((6m)^2 + (6m)(7n) + (7n)^2\right)$

$= (6m - 7n)(36m^2 + 42mn + 49n^2)$

(iii) $2x^4 - 16xy^3 = 2x(x^3 - 8y^3)$

$$= 2x\left(x^3 - (2y)^3\right) \quad \left[\because (a^3 - b^3) = (a-b)(a^2 + ab + b^2)\right]$$

$$= 2x\left((x - 2y)(x^2 + (x)(2y) + (2y)^2)\right)$$

$$= 2x(x - 2y)(x^2 + 2xy + 4y^2)$$

(iv) $8x^3 + 27y^3 + 64z^3 - 72xyz$

$$= (2x)^3 + (3y)^3 + (4z)^3 - 3(2x)(3y)(4z)$$

$$= (2x + 3y + 4z)(4x^2 + 9y^2 + 16z^2 - 6xy - 12yz - 8xz)$$

Example 3.18

Find : (i) $a^3 + b^3$ if $a + b = 6,\ ab = 5$ (ii) $x^3 - y^3$, if $x - y = 4,\ xy = 5$

Solution

(i) Given, $a + b = 6,\ ab = 5$

$$a^3 + b^3 = (a+b)^3 - 3ab(a+b)$$

$$= (6)^3 - 3(5)(6)$$

$$= 126$$

(ii) Given, $x - y = 4,\ xy = 5$

$$x^3 - y^3 = (x-y)^3 + 3xy(x-y)$$

$$= 4^3 + 3(5)(4)$$

$$= 124$$

Example 3.19

If $\left(y - \frac{1}{y}\right)^3 = 729$, then find the value of $y - \frac{1}{y}$ and $y^3 - \frac{1}{y^3}$

Solution

Given, $\left(y - \frac{1}{y}\right)^3 = 729$

Take cube root on both sides

$$\sqrt[3]{\left(y - \frac{1}{y}\right)^3} = \sqrt[3]{729} = \sqrt[3]{9^3}$$

Therefore, $y - \frac{1}{y} = 9$

$$y^3 - \frac{1}{y^3} = \left(y - \frac{1}{y}\right)^3 + 3\left(y - \frac{1}{y}\right) \quad \left[\because a^3 - b^3 = (a-b)^3 + 3ab(a-b)\right]$$

$$= 9^3 + 3(9)$$

$$y^3 - \frac{1}{y^3} = 756$$

Thinking Corner

Check 15 divides the following

(i) $2017^3 + 2018^3$

(ii) $2018^3 - 1973^3$

Exercise 3.4

1. Factorise the following expressions:
   (i) $2a^2 + 4a^2b + 8a^2c$ (ii) $ab - ac - mb + mc$
   (iii) $pr + qr + pq + p^2$ (iv) $2y^3 + y^2 - 2y - 1$
2. Factorise the following:
   (i) $x^2 + 4x + 4$ (ii) $3a^2 - 24ab + 48b^2$ (iii) $x^5 - 16x$
   (iv) $m^2 + \frac{1}{m^2} - 23$ (v) $6 - 216x^2$ (vi) $a^2 + \frac{1}{a^2} - 18$
   (vii) $m^4 - 7m^2 + 1$ (viii) $x^{2n} + 2x^n + 1$ (ix) $\frac{1}{3}a^2 - 2a + 3$
   (x) $a^4 + a^2b^2 + b^4$ (xi) $x^4 + 4y^4$
3. Factorise the following:
   (i) $4x^2 + 9y^2 + 25z^2 + 12xy + 30yz + 20xz$ (ii) $1 + x^2 + 9y^2 + 2x - 6xy - 6y$
   (iii) $25x^2 + 4y^2 + 9z^2 - 20xy + 12yz - 30xz$ (iv) $\frac{1}{x^2} + \frac{4}{y^2} + \frac{9}{z^2} + \frac{4}{xy} + \frac{12}{yz} + \frac{6}{xz}$
4. Factorise the following:
   (i) $8x^3 + 125y^3$ (ii) $a^3 - 729$ (iii) $27x^3 - 8y^3$
   (iv) $m^3 + 512$ (v) $a^3 + 3a^2b + 3ab^2 + 2b^3$ (vi) $a^6 - 64$
5. Factorise the following:
   (i) $x^3 + 8y^3 + 27z^3 - 18xyz$ (ii) $a^3 + b^3 - 3ab + 1$
   (iii) $x^3 + 8y^3 + 6xy - 1$ (iv) $l^3 - 8m^3 - 27n^3 - 18lmn$

3.4.3 Factorising the Quadratic Polynomial (Trinomial) of the type $ax^2 + bx + c,\ a \neq 0$

The linear factors of $ax^2 + bx + c$ will be in the form $(kx + m)$ and $(lx + n)$

Thus, $ax^2 + bx + c = (kx + m)(lx + n) = klx^2 + (lm + kn)x + mn$

Comparing coefficients of x^2, x and constant term c on both sides.
We have, $a = kl$, $b = (lm + kn)$ and $c = mn$, where ac is the product of kl and mn that is, equal to the product of lm and kn which are the coefficient of x. Therefore $(kl \times mn) = (lm \times kn)$.

Steps to be followed to factorise $ax^2 + bx + c$:

Step 1 : Multiply the coefficient of x^2 and constant term, that is ac.

Step 2 : Split ac into two factors whose sum and product is equal to b and ac respectively.

Step 3 : The terms are grouped into two pairs and factorise.

Example 3.20

Factorise $2x^2+15x+27$

Solution

Compare with ax^2+bx+c

we get, $a=2,\ b=15,\ c=27$

product $ac=2\times 27=54$ and sum $b=15$

We find the pair 6, 9 only satisfies "$b = 15$" and also "$ac = 54$".

$\therefore$ we split the middle term as $6x$ and $9x$

Product of factors $ac = 54$	Sum of factors $b = 15$	Product of factors $ac = 54$	Sum of factors $b = 15$
1×54	55	-1×-54	-55
2×27	29	-2×-27	-29
3×18	21	-3×-18	-21
6×9	**15**	-6×-9	-15
The required factors are **6** and **9**			

$$2x^2+15x+27 = 2x^2+6x+9x+27$$
$$= 2x(x+3)+9(x+3)$$
$$= (x+3)(2x+9)$$

Therefore, $(x+3)$ and $(2x+9)$ are the factors of $2x^2+15x+27$.

Example 3.21

Factorise $2x^2-15x+27$

Solution

Compare with ax^2+bx+c

$a=2,\ b=-15,\ c=27$

product $ac = 2\times 27 = 54$, sum $b=-15$

$\therefore$ we split the middle term as $-6x$ and $-9x$

Product of factors $ac = 54$	Sum of factors $b = -15$	Product of factors $ac = 54$	Sum of factors $b = -15$
1×54	55	-1×-54	-55
2×27	29	-2×-27	-29
3×18	21	-3×-18	-21
6×9	15	**-6×-9**	**-15**
The required factors are **-6** and **-9**			

$$2x^2-15x+27 = 2x^2-6x-9x+27$$
$$= 2x(x-3)-9(x-3)$$
$$= (x-3)(2x-9)$$

Therefore, $(x-3)$ and $(2x-9)$ are the factors of $2x^2-15x+27$.

Example 3.22

Factorise $2x^2+15x-27$

Solution :

Compare with ax^2+bx+c

Here, $a=2,\ b=15,\ c=-27$

product $ac = 2\times -27 = -54$, sum $b=15$

$\therefore$ we split the middle term as $18x$ and $-3x$

Product of factors $ac = -54$	Sum of factors $b = 15$	Product of factors $ac = -54$	Sum of factors $b = 15$
-1×54	53	1×-54	-53
-2×27	25	2×-27	-25
-3×18	**15**	3×-18	-15
-6×9	3	6×-9	-3
The required factors are **-3** and **18**			

$$2x^2+15x-27 = 2x^2+18x-3x-27$$
$$= 2x(x+9)-3(x+9)$$
$$= (x+9)(2x-3)$$

Therefore, $(x+9)$ and $(2x-3)$ are the factors of $2x^2+15x-27$.

Example 3.23 Factorise $2x^2 - 15x - 27$

Solution

Compare with $ax^2 + bx + c$

Here, $a = 2,\ b = -15,\ c = -27$

product $ac = 2 \times -27 = -54$, sum $b = -15$

$\therefore$ we split the middle term as $-18x$ and $3x$

$$2x^2 - 15x - 27 = 2x^2 - 18x + 3x - 27$$
$$= 2x(x-9) + 3(x-9)$$
$$= (x-9)(2x+3)$$

Product of factors $ac = -54$	Sum of factors $b = -15$	Product of factors $ac = -54$	Sum of factors $b = -15$
-1×54	53	1×-54	-53
-2×27	25	2×-27	-25
-3×18	15	**3×-18**	**-15**
-6×9	3	6×-9	-3
The required factors are **3** and **-18**			

Therefore, $(x-9)$ and $(2x+3)$ are the factors of $2x^2 - 15x - 27$

Example 3.24 Factorise $(x+y)^2 + 9(x+y) + 20$

Solution

Let $x + y = p$, we get $p^2 + 9p + 20$

Compare with $ax^2 + bx + c$,

We get $a = 1,\ b = 9,\ c = 20$

product $ac = 1 \times 20 = 20$, sum $b = 9$

$\therefore$ we split the middle term as $4p$ and $5p$

$$p^2 + 9p + 20 = p^2 + 4p + 5p + 20$$
$$= p(p+4) + 5(p+4)$$
$$= (p+4)(p+5)$$

Product of factors $ac = 20$	Sum of factors $b = 9$	Product of factors $ac = 20$	Sum of factors $b = 9$
1×20	21	-1×-20	-21
2×10	12	-2×-10	-12
4×5	**9**	-4×-5	-9
The required factors are **4** and **5**			

Put, $p = x + y$ we get, $(x+y)^2 + 9(x+y) + 20 = (x+y+4)(x+y+5)$

Exercise 3.5

1. Factorise the following:

(i) $x^2 + 10x + 24$ (ii) $x^2 - 2x - 99$ (iii) $z^2 + 4z - 12$

(iv) $x^2 + 14x - 15$ (v) $p^2 - 6p - 16$ (vi) $t^2 + 72 - 17t$

(vii) $x^2 - 8x + 15$ (viii) $y^2 - 16y - 80$ (ix) $a^2 + 10a - 600$

2. Factorise the following:

(i) $2a^2 + 9a + 10$ (ii) $11 + 5m - 6m^2$ (iii) $4x^2 - 20x + 25$

(iv) $32 + 8x - 60x^2$ (v) $5x^2 - 29xy - 42y^2$ (vi) $9 - 18x + 8x^2$

(vii) $6x^2 + 16xy + 8y^2$ (viii) $9 + 3x - 12x^2$ (ix) $10 - 7a - 3a^2$

(x) $12x^2 + 36x^2y + 27y^2x^2$ (xi) $(a+b)^2 + 9(a+b) + 18$

3. Factorise the following:

(i) $(p-q)^2 - 6(p-q) - 16$ (ii) $9(2x-y)^2 - 4(2x-y) - 13$

(iii) $m^2 + 2mn - 24n^2$ (iv) $\sqrt{5}a^2 + 2a - 3\sqrt{5}$ (v) $a^4 - 3a^2 + 2$

(vi) $8m^3 - 2m^2n - 15mn^2$ (vii) $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ (viii) $a^4 - 7a^2 + 1$

(ix) $a^2 + \frac{1}{a^2} - 18$ (x) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{2}{xy}$ (xi) $\frac{3}{x^2} + \frac{8}{xy} + \frac{4}{y^2}$

Activity - 1

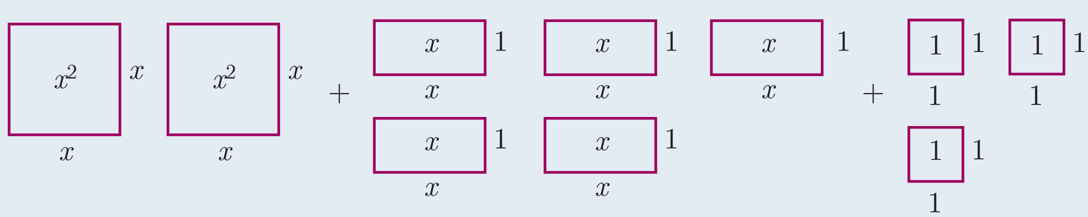
(1) **Objective :** To know the factorisation of polynomials using paper cuttings.

Required material : Cut out a paper into three types of sheets as given below.

Type 1	**Type 2**	**Type 3**
(square x by x)	(rectangle x by 1)	(square 1 by 1)
Square sheets each of area x^2 sq.units	Rectangular sheets each of area x sq. units	Sheets of units squares (1 sq. unit)

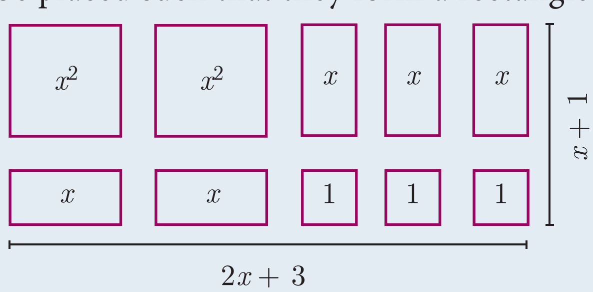
Procedure : For example, to factorise $2x^2 + 5x + 3$, the students need to take two x^2 sheets, five x sheets and three unit sheets.

The sheets selected are given below



$2x^2$ + $5x$ + 3

The sheets are to be placed such that they form a rectangle.



The sides of the rectangle are $(2x+3)$ and $(x+1)$

(2) Factorise the following by using paper cuttings:

(i) $x^2 + 5x + 6$ (ii) $4x^2 + 8x + 3$ (iii) $3x^2 + 4x + 1$

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3.5 Synthetic Division

Synthetic Division is a shortcut method of polynomial division in the special case of dividing by a linear factor. It is used in dividing out factors; its main use is in finding roots of polynomials. We learn about the process now.

We will first solve by the traditional method and then narrate the process of synthetic division.

Given, dividend : $p(x) = (3x^3 - 2x^2 - 5 + 7x)$

divisor : $d(x) = x + 3$

To find : The quotient and remainder

Traditional Method (Long Division Method)

Write the polynomial in descending order (standard form) and then start dividing.

$$
\begin{array}{r|l}
 & 3x^2 - 11x + 40 \\
\hline
x+3 & 3x^3 - 2x^2 + 7x - 5 \\
 & 3x^3 + 9x^2 \\
 & (-) \quad (-) \\
\hline
 & -11x^2 + 7x - 5 \\
 & -11x^2 - 33x \\
 & (+) \quad (+) \\
\hline
 & 40x - 5 \\
 & 40x + 120 \\
 & (-) \quad (-) \\
\hline
 & -125 \\
\hline
\end{array}
$$

To find the quotient:

$\frac{3x^3}{x} = 3x^2$

$\frac{-11x^2}{x} = -11x$

$\frac{40x}{x} = 40$

Here quotient is $3x^2 - 11x + 40$ and remainder is -125.

Note that $3x^3 - 2x^2 + 7x - 5 = 3x^2 - 11x + 40 - \frac{125}{x+3}$

Let us explain the above example by the method of synthetic division.

Let, $p(x) = (3x^3 - 2x^2 - 5 + 7x)$ be the dividend and $d(x) = x + 3$ be the divisor. We shall find the quotient and the remainder r by proceeding as follows.

Step 1 Arrange dividend and the divisor in standard form.

$3x^3 - 2x^2 + 7x - 5$ (standard form of dividend)

$x + 3$ (standard form of divisor)

Write the coefficients of dividend in the first row. Put '0' for missing term(s).

3 –2 7 –5 (first row)

Step 2 Find out the zero of the divisor.

$x+3=0$ implies $x=-3$

Step 3 Write the zero of divisor in front of dividend in the first row. Put '0' in the first column of second row.

–3	3	–2	7	–5	(first row)
	0				(second row)

Step 4 Complete the second row and third row as shown below.

–3	3		–2		7		–5	(first row)
	0	–3×3 →	–9	–3×–11 →	33	–3×40 →	–120	(second row)
	3		–11		40		–125	(third row)

All the entries except the last one in the third row are the coefficients of the quotient.

Then quotient is $3x^2-11x+40$ and remainder is -125.

Example 3.25 Find the quotient and remainder when $(3x^3-4x^2-5)$ is divided by $(3x+1)$ using synthetic division.

Solution

To find the zero of $3x+1$;
put $3x+1=0$
we get $3x=-1$
$x=-\frac{1}{3}$

$p(x)=3x^3-4x^2-5,\ d(x)=(3x+1)$

Standard form of $p(x)=3x^3-4x^2+0x-5$ and $d(x)=3x+1$

$\frac{-1}{3}$	3	–4	0	–5	
	0	–1	$\frac{5}{3}$	$\frac{-5}{9}$	
	3	–5	$\frac{5}{3}$	$\frac{-50}{9}$	(remainder)

$$3x^3-4x^2-5=\left(x+\frac{1}{3}\right)\left(3x^2-5x+\frac{5}{3}\right)-\frac{50}{9}$$

$$=\frac{(3x+1)}{3}\times 3\left(x^2-\frac{5}{3}x+\frac{5}{9}\right)-\frac{50}{9}$$

$$3x^3-4x^2-5=(3x+1)\left(x^2-\frac{5}{3}x+\frac{5}{9}\right)-\left(\frac{50}{9}\right) \qquad \text{(since, } p(x)=d(x)q(x)+r\text{)}$$

Hence the quotient is $\left(x^2-\frac{5}{3}x+\frac{5}{9}\right)$ and remainder is $\frac{-50}{9}$

Example 3.26 If the quotient on dividing $x^4+10x^3+35x^2+50x+29$ by $(x+4)$ is x^3-ax^2+bx+6, then find the value of a, b and also remainder.

Solution

To find the zero of x+4;
put $x+4=0$
we get $x=-4$

Let $p(x)=x^4+10x^3+35x^2+50x+29$

Standard form $=x^4+10x^3+35x^2+50x+29$

Coefficient are 1 10 35 50 29

–4	1	10	35	50	29	
	0	–4	–24	–44	–24	
	1	6	11	6	5	(remainder)

quotient $x^3+6x^2+11x+6$ is compared with given quotient x^3-ax^2+bx+6

coefficient of x^2 is $6=-a$ and coefficient of x is $11=b$

Therefore, $a=-6$, $b=11$ and remainder$=5$.

Exercise 3.6

1. Find the quotient and remainder for the following using synthetic division:

   (i) $(x^3+x^2-7x-3)\div(x-3)$ (ii) $(x^3+2x^2-x-4)\div(x+2)$

   (iii) $(x^3+4x^2+16x+61)\div(x-4)$ (iv) $(3x^3-2x^2+7x-5)\div(x+3)$

   (v) $(3x^3-4x^2-10x+8)\div(3x-2)$ (vi) $(8x^4-2x^2+6x+5)\div(4x+1)$

2. If the quotient obtained on dividing $(8x^4-2x^2+6x-7)$ by $(2x+1)$ is $(4x^3+px^2-qx+3)$, then find p, q and also the remainder.

3. If the quotient obtained on dividing $3x^3+11x^2+34x+106$ by $x-3$ is $3x^2+ax+b$, then find a, b and also the remainder.

3.5.1 Factorisation using Synthetic Division

In this section, we use the synthetic division method that helps to factorise a cubic polynomial into linear factors. If we identify one linear factor of cubic polynomial $p(x)$, then using synthetic division we can get the quadratic factor of $p(x)$. Further if possible one can factorise the quadratic factor into linear factors.

Note

- For any non constant polynomial $p(x)$, $x=a$ is zero if and only if $p(a)=0$
- $x-a$ is a factor for $p(x)$ if and only if $p(a)=0$ (Factor theorem)

To identify $(x-1)$ and $(x+1)$ are the factors of a polynomial

- $(x-1)$ is a factor of $p(x)$ if and only if the sum of coefficients of $p(x)$ is 0.
- $(x+1)$ is a factor of $p(x)$ if and only if the sum of the coefficients of even power of x, including constant is equal to the sum of the coefficients of odd powers of x

Example 3.27 (i) Prove that $(x-1)$ is a factor of $x^3-7x^2+13x-7$

(ii) Prove that $(x+1)$ is a factor of $x^3+7x^2+13x+7$

Solution

(i) Let $p(x)=x^3-7x^2+13x-7$

Sum of coefficients $=1-7+13-7=0$

Thus $(x-1)$ is a factor of $p(x)$

(ii) Let $q(x)=x^3+7x^2+13x+7$

Sum of coefficients of even powers of x with constant = 7 + 7 = 14

Sum of coefficients of odd powers of x = 1 + 13 = 14

Hence, $(x+1)$ is a factor of $q(x)$

Example 3.28 Factorise $x^3+13x^2+32x+20$ into linear factors.

Solution

Let, $p(x)=x^3+13x^2+32x+20$

Sum of all the coefficients $=1+13+32+20=66\neq 0$

Hence, $(x-1)$ is not a factor.

Sum of coefficients of even powers with constant = 13 + 20 = 33

Sum of coefficients of odd powers = 1 + 32 = 33

Hence, $(x+1)$ is a factor of $p(x)$

Now we use synthetic division to find the other factors

Method I

–1	1	13	32	20	
	0	–1	–12	–20	
–2	1	12	20	0	(remainder)
	0	–2	–20		
	1	10	0	(remainder)	

$p(x)=(x+1)(x+2)(x+10)$

Hence,

$x^3+13x^2+32x+20$

$=(x+1)(x+2)(x+10)$

Method II

–1	1	13	32	20	
	0	–1	–12	–20	
	1	12	20	0	(remainder)

Then $p(x)=(x+1)(x^2+12x+20)$

Now $x^2+12x+20=x^2+10x+2x+20$

$=x(x+10)+2(x+10)$

$=(x+2)(x+10)$

Hence, $x^3+13x^2+32x+20$

$=(x+1)(x+2)(x+10)$

Example 3.29 Factorise $x^3 - 5x^2 - 2x + 24$

Solution

Let $p(x) = x^3 - 5x^2 - 2x + 24$

When $x = 1, p(1) = 1 - 5 - 2 + 24 = 18 \neq 0$ $(x-1)$ is not a factor.

When $x = -1, p(-1) = -1 - 5 + 2 + 24 = 20 \neq 0$ $(x+1)$ is not a factor.

Therefore, we have to search for different values of x by trial and error method.

When $x = 2$

$$p(2) = 2^3 - 5(2)^2 - 2(2) + 24$$
$$= 8 - 20 - 4 + 24$$
$$= 8 \neq 0$$

Hence, $(x-2)$ is not a factor

When $x = -2$

$$p(-2) = (-2)^3 - 5(-2)^2 - 2(-2) + 24$$
$$= -8 - 20 + 4 + 24$$
$$p(-2) = 0$$

Hence, $(x+2)$ is a factor

Note

Check whether 3 is a zero of $x^2 - 7x + 12$. If it is not, then check for –3 or 4 or –4 and so on.

–2	1	–5	–2	24	
	0	–2	+14	–24	
3	1	–7	12	0	(remainder)
	0	3	–12		
	1	–4	0	(remainder)	

Thus, $(x+2)(x-3)(x-4)$ are the factors.

Therefore, $x^3 - 5x^2 - 2x^2 + 24 = (x+2)(x-3)(x-4)$

Exercise 3.7

1. Factorise each of the following polynomials using synthetic division:

(i) $x^3 - 3x^2 - 10x + 24$ (ii) $2x^3 - 3x^2 - 3x + 2$

(iii) $4x^3 - 5x^2 + 7x - 6$ (iv) $-7x + 3 + 4x^3$

(v) $x^3 + x^2 - 14x - 24$ (vi) $x^3 - 7x + 6$

(vii) $x^3 - 10x^2 - x + 10$ (viii) $x^3 - 5x + 4$

Exercise 3.8

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Multiple Choice Questions

1. If $p(a)=0$ then $(x-a)$ is a ___________ of $p(x)$

   (1) divisor (2) quotient (3) remainder (4) factor

2. Zeros of $(2-3x)$ is ___________

   (1) 3 (2) 2 (3) $\frac{2}{3}$ (4) $\frac{3}{2}$

3. Which of the following has $x-1$ as a factor?

   (1) $2x-1$ (2) $3x-3$ (3) $4x-3$ (4) $3x-4$

4. If $x-3$ is a factor of $p(x)$, then the remainder is

   (1) 3 (2) –3 (3) $p(3)$ (4) $p(-3)$

5. $(x+y)(x^2-xy+y^2)$ is equal to

   (1) $(x+y)^3$ (2) $(x-y)^3$ (3) x^3+y^3 (4) x^3-y^3

6. If one of the factors of $x^2-6x-16$ is $x-8$ then the other factor is

   (1) $(x+6)$ (2) $(x-2)$ (3) $(x+2)$ (4) $(x-16)$

7. $(a+b-c)^2$ is equal to ___________

   (1) $(a-b+c)^2$ (2) $(-a-b+c)^2$ (3) $(a+b+c)^2$ (4) $(a-b-c)^2$

8. In an expression ax^2+bx+c the sum and product of the factors respectively,

   (1) a, bc (2) b, ac (3) ac, b (4) bc, a

9. If $(x+5)$ and $(x-3)$ are the factors of ax^2+bx+c, then values of a, b and c are

   (1) 1,2,3 (2) 1,2,15 (3) 1,2, –15 (4) 1, –2,15

10. Cubic polynomial may have maximum of ___________ linear factors

    (1) 1 (2) 2 (3) 3 (4) 4

11. Degree of the constant polynomial is ___________

    (1) 3 (2) 2 (3) 1 (4) 0

12. GCD of any two prime numbers is ___________

    (1) –1 (2) 0 (3) 1 (4) 2

13. The remainder when (x^2-2x+7) is divided by $(x+4)$ is

    (1) 28 (2) 31 (3) 30 (4) 29

14. The GCD of a^k, a^{k+1}, a^{k+5} where, $k \in N$
    (1) a^k (2) a^{k+1} (3) a^{k+5} (4) 1

15. The GCD of $x^4 - y^4$ and $x^2 - y^2$ is
    (1) $x^4 - y^4$ (2) $x^2 - y^2$ (3) $(x+y)^2$ (4) $(x+y)^4$

16. If there are 36 students of class 9 and 48 students of class 10, what is the minimum number of rows to arrange them in which each row consists of same class with same number of students.
    (1) 12 (2) 144 (3) 7 (4) 72

Points to Remember

Factor Theorem

- If $p(x)$ is divided by $(x-a)$ and the remainder $p(a)=0$, then $(x-a)$ is a factor of the polynomial $p(x)$

Identities

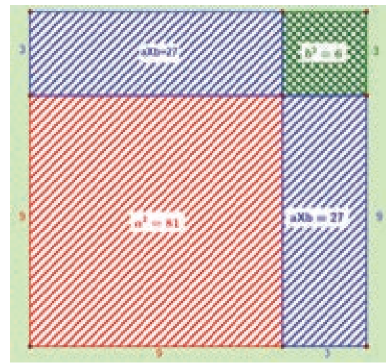
- $(a+b+c)^2 \equiv a^2+b^2+c^2+2ab+2bc+2ca$
- $(a+b)^3 \equiv a^3+b^3+3ab(a+b)$, $a^3+b^3 \equiv (a+b)^3-3ab(a+b)$
- $(a-b)^3 \equiv a^3-b^3-3ab(a-b)$, $a^3-b^3 \equiv (a-b)^3+3ab(a-b)$

Product identities

- $a^3+b^3 \equiv (a+b)(a^2-ab+b^2)$
- $a^3-b^3 \equiv (a-b)(a^2+ab+b^2)$
- $a^3+b^3+c^3-3abc \equiv (a+b+c)(a^2+b^2+c^2-ac-bc-ca)$
- $x^3+(a+b+c)x^2+(ab+bc+ca)x+abc \equiv (x+a)(x+b)(x+c)$

ICT Corner

Expected Result is shown in this picture



Step – 1

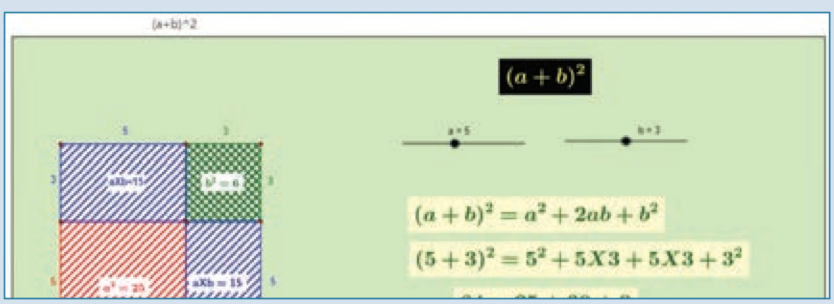
Open the Browser, type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Algebraic Identities" will open. In the work sheet, there are many activities on Algebraic Identities.

In the first activity diagrammatic approach for $(a+b)^2$ is given . Move the sliders a and b and compare the areas with the Identity given.

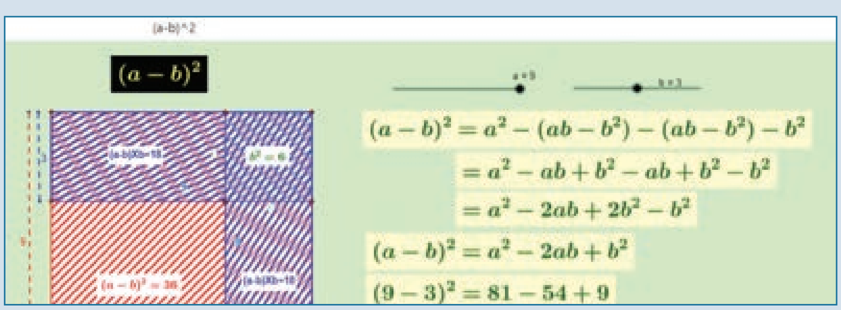
Step - 2

Similarly move the sliders a and b and compare the areas with the remaining Identities.

Step 1



Step 2



Browse in the link

Algebraic Identities: https://ggbm.at/PyUj657Y or Scan the QR Code.

B466_MAT_9_T2_EM

4 GEOMETRY

Inspiration is needed in geometry just as much as in poetry.
- Alexander Pushkin

BD1U4Q

Euclid
(325 – 265 B C (BCE))

Euclid was a Greek mathematician, often referred to as the "Father of Geometry". His 'Elements' is one of the most influential works in the history of mathematics, serving as the main textbook for teaching mathematics (especially geometry) from the time of its publication. In the 'Elements', Euclid deduced the principles of what is now called Euclidean geometry and Number theory.

Learning Outcomes

- To understand, interpret and apply theorems on the chords of a circle.
- To understand, interpret and apply theorems on the angles subtended by arcs of the circle.
- To understand, interpret and apply theorems on the cyclic quadrilaterals.
- To specifically use the theorems in problem solving.
- To construct the incircle and locate incentre of the triangle.
- To construct the medians and locate the centroid of the triangle.

4.1 Introduction

Circles are geometric shapes you can see all around you. The significance of the concept of a circle can be well understood from the fact that the wheel is one of the ground-breaking inventions in the history of mankind.

Fig. 4.1

4.2 Parts of Circle

A circle, you can describe, is the set of all points in a plane at a constant distance from a fixed point. The fixed point is the centre of the circle; the constant distance corresponds to a radius of the circle.

A line that cuts the circle in two points is called a secant of the circle.

A line segment whose end points lie on the circle is called a chord of the circle.

Note

A circle notably differs from a polygon. A polygon (for example, a quadrilateral) has edges and corners while, a circle is a 'smooth' curve.

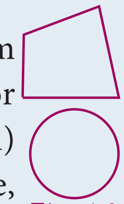


Fig. 4.2

A chord of a circle that has the centre is called a diameter of the circle. The circumference of a circle is its boundary. (We use the term perimeter in the case of polygons).

Activity - 1

1. Place a bottle top or a glass upside down on a chart.
2. Neatly trace around the boundary. Remove the object. The figure drawn is a circle. Cut out the circle.
3. Now fold the circle into two halves such that the boundry of the circle overlap. Make a crease on the fold. Open the fold and mark A and B to the end points and draw a line segment along crease line. Repeat the same process from new position and give C and D to the end points. These two line segments intersect at a point. Name the point as O. In the same way, you can fold the circle with number of other positions. What do you observe?

Think! Whether all the lines are concurrent?

From this activity, we observe that all the lines are intersecting at a point. Do you know what do we call this point? We call this as the centre of the circle.

Using a divider, measure the length of each line segments of a circle. What do you observe?. They are all equal and each of them is dividing the circle into two halves. The line segment which divides the circle into two halves is called diameter of the circle. In other words, a line segment joining any two points on the circle that passes through the centre is called the diameter. Diameter of a circle is twice of its radius.

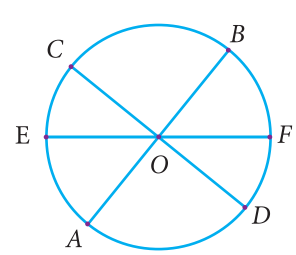


Fig. 4.3

If we fold the circular paper in all possible way including the one passing through the centre (Fig.4.4), we see that all the creases meet at two points on the circle. These creases are called the chords of the circle. So, a line segment joining any two points on the circle is called a chord of the circle. In this figure AB, PQ and RS are the chords of the circle.

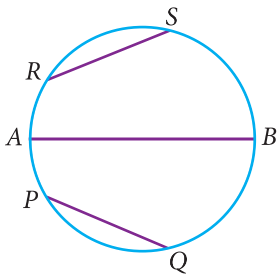


Fig. 4.4

Now place four points P, R, Q and S on the same circle (Fig.4.5), then PRQ and QSP are the continuous parts (sections) of the circle. These parts (sections) are to be denoted by $\overset{\frown}{PRQ}$ and $\overset{\frown}{QSP}$ or simply by $\overset{\frown}{PQ}$ and $\overset{\frown}{QP}$. This continuous part of a circle is called an arc of the circle. Usually the arcs are denoted in anti-clockwise direction.

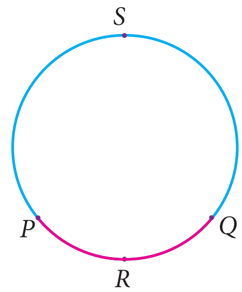


Fig. 4.5

Now consider the points P and Q in the circle (Fig.4.6). It divides the whole circle into two parts. One is longer and another is shorter. The longer one is called major arc $\overset{\frown}{QP}$ and shorter one is called minor arc $\overset{\frown}{PQ}$.

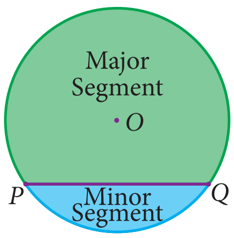


Fig. 4.6

Now in (Fig.4.6), consider the region which is surrounded by the chord PQ and major arc $\overset{\frown}{QP}$. This is called the major segment of the circle. In the same way, the segment containing the minor arc and the same chord is called the minor segment.

Note

A diameter of a circle is:

- the line segment which bisects the circle.
- the largest chord of a circle.
- a line of symmetry for the circle.
- twice in length of a radius in a circle.

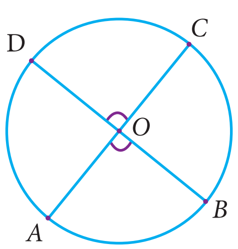


Fig. 4.7

In (Fig.4.7), if two arcs $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ of a circle subtend the same angle at the centre, they are said to be congruent arcs and we write,

$$\overset{\frown}{AB} \equiv \overset{\frown}{CD} \text{ implies } m\overset{\frown}{AB} = m\overset{\frown}{CD}$$

$$\text{implies } \angle AOB = \angle COD$$

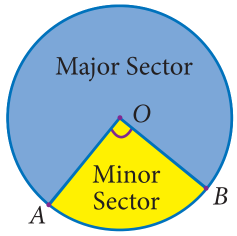


Fig. 4.8

Now, let us observe (Fig.4.8). Is there any special name for the region surrounded by two radii and arc? Yes, its name is sector. Like segment, we find that the minor arc corresponds to the minor sector and the major arc corresponds to the major sector.

Concentric Circles

Circles with the same centre but different radii are said to be concentric.

Here are some real-life examples:


An Archery target


A carom board coin


Water ripples

Fig. 4.9

Congruent Circles

Two circles are congruent if they are copies of one another or identical. That is, they have the same size. Here are some real life examples:

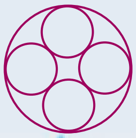

The two wheels of a bullock cart


The Olympic rings

Fig. 4.10

Thinking Corner

Draw four congruent circles as shown. What do you infer?



Position of a Point with respect to a Circle

Consider a circle in a plane (Fig.4.11). Consider any point P on the circle.Then the distance from the centre O to P be OP, then

(i) OP = radius (Point lies on the circle)

(ii) $OP <$ radius (Point lies inside the circle)

(iii) $OP >$ radius(Point lies outside the circle)

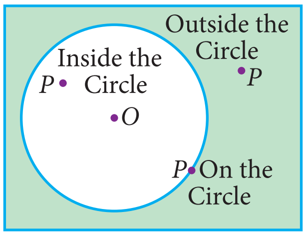


Fig. 4.11

So, a circle divides the plane on which it lies into three parts.

Progress Check

Say True or False

1. Every chord of a circle contains exactly two points of the circle.
2. All radii of a circle are of same length.
3. Every radius of a circle is a chord.
4. Every chord of a circle is a diameter.

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5. Every diameter of a circle is a chord.
6. There can be any number of diameters for a circle.
7. Two diameters cannot have the same end-point.
8. A circle divides the plane into three disjoint parts.
9. A circle can be partitioned into a major arc and a minor arc.
10. The distance from the centre of a circle to the circumference is that of a diameter

Thinking Corner

1. How many sides does a circle have ?
2. Is circle, a polygon?

Exercise 4.1

1. Fill in the blanks :
   - (i) Twice of the radius is called _______ of the circle.
   - (ii) Longest chord passes through the _______ of the circle.
   - (iii) Distance from the centre to any point on the circumference of the circle is called ________.
   - (iv) A part of a circle between any two points is called a/an ________ of the circle.
   - (v) A circle divides the plane into ________ parts.
2. Write True or False. Give reasons for your answers.
   - (i) Line segment joining any two points on the circle is called radius of the circle.
   - (ii) Point of concurrency of the diameter is the centre of the circle.
   - (iii) The boundary of the circle is called its circumference.
   - (iv) A circle has infinite number of equal chords.
   - (v) Sector is the region between the chord and its corresponding arc.

4.3 Circle Through Three Points

We have already learnt that there is one and only one line passing through two points. In the same way, we are going to see how many circles can be drawn through a given point, and through two given points. We see that in both cases there can be infinite number of circles passing through a given point P (Fig.4.12) , and through two given points A and B (Fig.4.13).

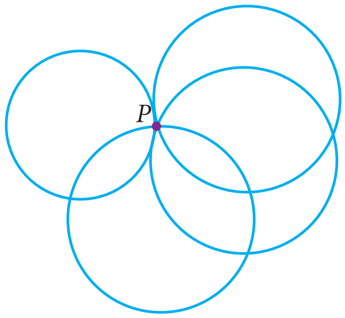


Fig. 4.12

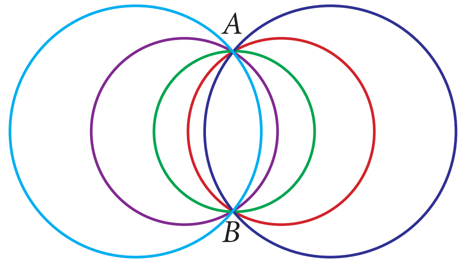


Fig. 4.13

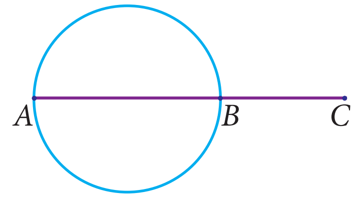


Fig. 4.14

Now consider three collinear points A, B and C (Fig.4.14). Can we draw a circle passing through these three points? Think over it. If the points are collinear, we can't?

If the three points are non collinear, they form a triangle (Fig.4.15). Recall the construction of the circumcentre. The intersecting point of the perpendicular bisector of the sides is the circumcentre and the circle is circumcircle.

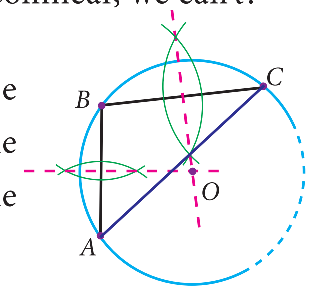


Fig. 4.15

Therefore from this we know that, there is a unique circle which passes through A, B and C. Now, the above statement leads to a result as follows.

Theorem 1 There is one and only one circle passing through three non-collinear points.

Activity - 2

Let us consider three non-collinear points A, B and C. How many circles can we draw through three non collinear points?

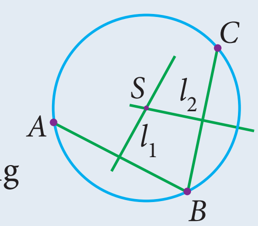


Fig. 4.16

(i) Make a line segment on a chart through AB by folding it.

(ii) Fold AB in such a way that A falls exactly on B and thereby creating a crease l_1, the perpendicular bisector of AB

(iii) Repeat the steps (i) and (ii) for the points B and C, creating a crease line l_2.

(iv) The two crease lines l_1 and l_2 meet at S.

(v) Now S as centre and SA as radius, draw a circle. Does this circle pass through the given points A, B and C ?

4.4 Properties of Chords of a Circle

In this chapter, already we come across lines, angles, triangles and quadrilaterals. Recently we have seen a new member circle. Using all the properties of these, we get some standard results one by one. Now, we are going to discuss some properties based on chords of the circle.

Considering a chord and a perpendicular line from the centre to a chord, we are going to see an interesting property.

4.4.1 Perpendicular from the Centre to a Chord

Consider a chord AB of the circle with centre O. Draw $OC \perp AB$ and join the points OA, OB. Here, easily we get two triangles ΔAOC and ΔBOC (Fig.4.17).

Can we prove these triangles are congruent? Now we try to prove this using the congruence of triangle rule which we have already learnt. $\angle OCA = \angle OCB = 90°(OC \perp AB)$ and $OA = OB$ is the radius of the circle. The side OC is common. RHS criterion tells us that ΔAOC and ΔBOC are congruent. From this we can conclude that $AC = BC$. This argument leads to the result as follows.

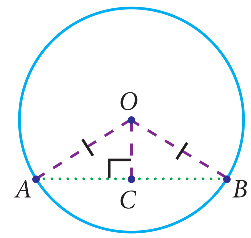


Fig. 4.17

Theorem 2 The perpendicular from the centre of a circle to a chord bisects the chord.

Converse of Theorem 2 The line joining the centre of the circle and the midpoint of a chord is perpendicular to the chord.

Activity - 3

1. Draw a circle of any radius with centre O on the chart paper / paper.
2. Cut the circle and place two points A, B on the circle.
3. Make the crease line segment AB by folding along the line joining A and B.
4. Move B along BA till it falls on A and make a crease line.
5. This crease line segment l is the perpendicular bisector of AB and meet at C.
6. Are AC and BC equal? What do you conclude?

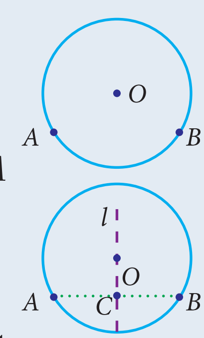


Fig. 4.18

Example 4.1 Find the length of a chord which is at a distance of $2\sqrt{11}$ cm from the centre of a circle of radius 12cm.

Solution

Let AB be the chord and C be the mid point of AB

Therefore, $OC \perp AB$

Join OA and OC.

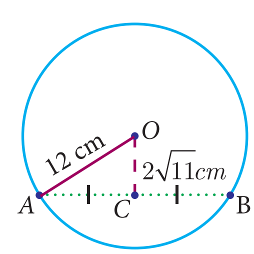


Fig. 4.19

OA is the radius

Given $OC = 2\sqrt{11}$cm and OA = 12cm

In a right ΔOAC,

using Pythagoras Theorem, we get,

$$AC^2 = OA^2 - OC^2$$
$$= 12^2 - (2\sqrt{11})^2$$
$$= 144 - 44$$
$$= 100\text{cm}$$
$$AC^2 = 100\text{cm}$$
$$AC = 10\text{cm}$$

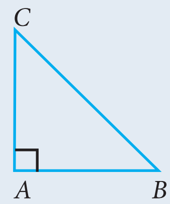
Therefore, length of the chord $AB = 2AC$

$$= 2 \times 10\text{cm} = 20\text{cm}$$

Note

Pythagoras theorem

One of the most important and well known results in geometry is Pythagoras Theorem. "In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides".



In right Δ ABC, $BC^2 = AB^2 + AC^2$.

Application of this theorem is most useful in this unit.

Example 4.2 In the concentric circles, chord AB of the outer circle cuts the inner circle at C and D as shown in the diagram. Prove that, $AB - CD = 2AC$

Solution

Given : Chord AB of the outer circle cuts the inner circle at C and D.

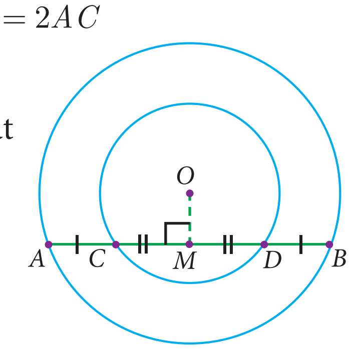


Fig. 4.20

To prove : $AB - CD = 2AC$

Construction : Draw $OM \perp AB$

Proof : Since, $OM \perp AB$ (By construction)

Also, $OM \perp CD$

Therefore, $AM = MB$ ... (1) (Perpendicular drawn from centre to chord bisect it)

$CM = MD$ ... (2)

Now, $AB - CD = 2AM - 2CM$

$= 2(AM - CM)$ from (1) and (2)

$AB - CD = 2AC$

Progress Check

1. The radius of the circle is 25 cm and the length of one of its chord is 40cm. Find the distance of the chord from the centre.
2. Draw three circles passing through the points P and Q, where PQ = 4cm.

4.4.2 Angle Subtended by Chord at the Centre

Instead of a single chord we consider two equal chords. Now we are going to discuss another property.

Let us consider two equal chords in the circle with centre O. Join the end points of the chords with the centre to get the triangles ΔAOB and ΔOCD, chord $AB=$ chord CD (because the given chords are equal). The other sides are radii, therefore $OA=OC$ and $OB=OD$. By SSS rule, the triangles are congruent, that is $\Delta OAB \equiv \Delta OCD$. This gives $m\angle AOB = m\angle COD$. Now this leads to the following result.

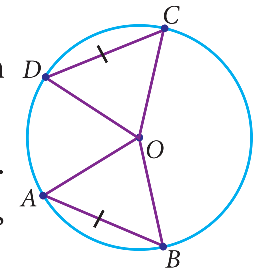


Fig. 4.21

Theorem 3 Equal chords of a circle subtend equal angles at the centre.

BDAQ6D

Activity - 4

Procedure

1. Draw a circle with centre O *and* with suitable radius.
2. Make it a semi-circle through folding. Consider the point A, B on it.
3. Make crease along AB in the semi circles and open it.
4. We get one more crease line on the another part of semi circle, name it as CD (*observe* AB = CD)
5. Join the radius to get the ΔOAB and ΔOCD.
6. Using trace paper, take the replicas of triangle ΔOAB and ΔOCD.
7. Place these triangles ΔOAB and ΔOCD one on the other.

Observation

1. What do you observe? Is $\Delta OAB \equiv \Delta OCD$?
2. Construct perpendicular line to the chords AB and CD passing through the centre O. Measure the distance from O to the chords.

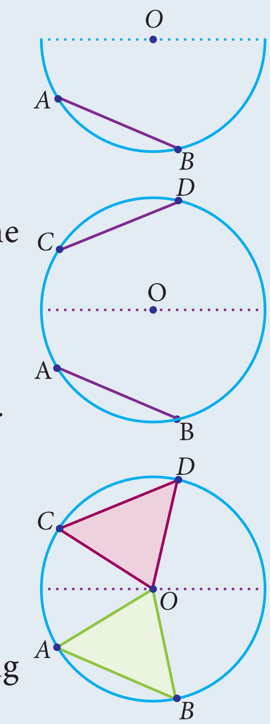


Fig. 4.22

Now we are going to find out the length of the chords AB and CD, given the angles subtended by two chords at the centre of the circle are equal. That is from theorem (3) we get $\angle AOB = \angle COD$ and all the other two sides which include the angles of the ΔAOB and ΔCOD are radii and are equal.

By SAS rule, $\Delta AOB \equiv \Delta COD$. This gives chord AB = chord CD. Now let us write the converse result as follows:

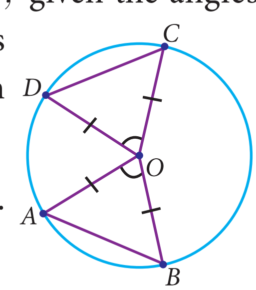


Fig. 4.23

Converse of theorem 3

If the angles subtended by two chords at the centre of a circle are equal, then the chords are equal.

In the same way we are going to discuss about the distance from the centre, when the equal chords are given. Draw the perpendicular $OL \perp AB$ and $OM \perp CD$. From theorem 2, these perpendicular divides the chords equally. So $AL = CM$. By comparing the ΔOAL and ΔOCM, the angles $\angle OLA = \angle OMC = 90°$ and $OA = OC$ are radii. By RHS rule, the $\Delta OAL \equiv \Delta OCM$. It gives the distance from the centre $OL = OM$ and write the conclusion as follows.

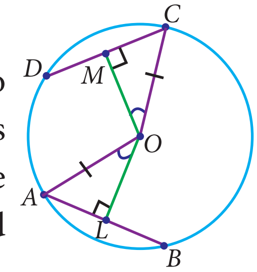


Fig. 4.24

Theorem 4 Equal chords of a circle are equidistant from the centre.

Let us know the converse of theorem 4, which is very useful in solving problems.

Converse of theorem 4

The chords of a circle which are equidistant from the centre are equal.

Exercise 4.2

1. The radius of the circle is 25cm and the length of one of its chord is 40cm. Find the distance of the chord from the centre.
2. The diameter of the circle is 52cm and the length of one of its chord is 20cm. Find the distance of the chord from the centre.
3. The chord of length 30 cm is drawn at the distance of 8cm from the centre of the circle. Find the radius of the circle
4. Find the length of the chord AC where AB and CD are the two diameters perpendicular to each other of a circle with radius $4\sqrt{2}$ cm and also find $\angle OAC$ and $\angle OCA$.
5. A chord is 12cm away from the centre of the circle of radius 15cm. Find the length of the chord.
6. In a circle, AB and CD are two parallel chords with centre O and radius 10 cm such that AB = 16 cm and CD = 12 cm determine the distance between the two chords?
7. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

4.4.3 Angle Subtended by an Arc of a Circle

Activity - 5

Procedure :

1. Draw three circles of any radius with centre O on a chart paper.
2. From these circles, cut a semi-circle, a minor segment and a major segment.

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3. Consider three points on these segment and name them as A, B and C.

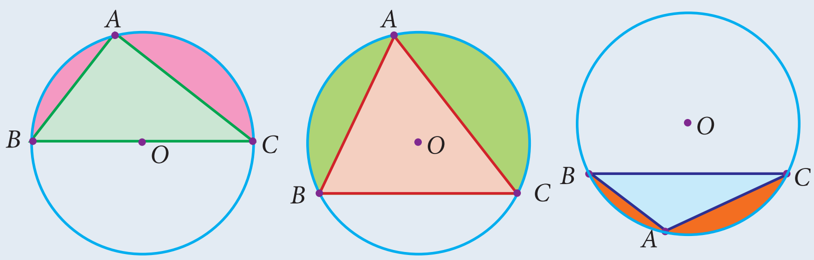


Fig. 4.25

4. (iv) Cut the triangles and paste it on the graph sheet so that the point A coincides with the origin as shown in the figure.

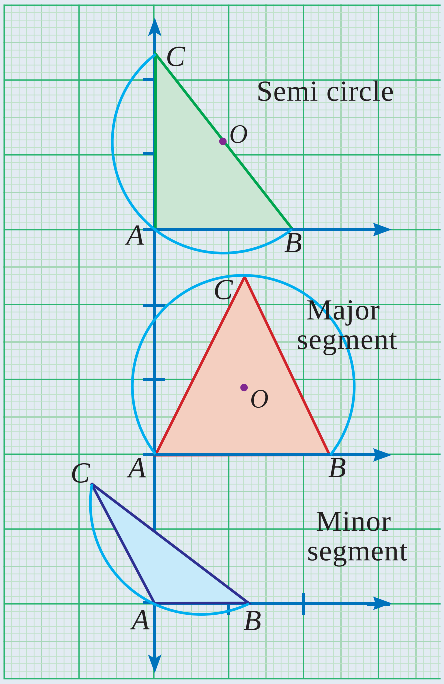


Fig. 4.26

Observation :

(i) Angle in a Semi-Circle is _______ angle.

(ii) Angle in a major segment is _____ angle.

(iii) Angle in a minor segment is _____ angle.

Now we are going to verify the relationship between the angle subtended by an arc at the centre and the angle subtended on the circumference.

4.4.4 Angle at the Centre and the Circumference

Let us consider any circle with centre O. Now place the points A, B and C on the circumference.

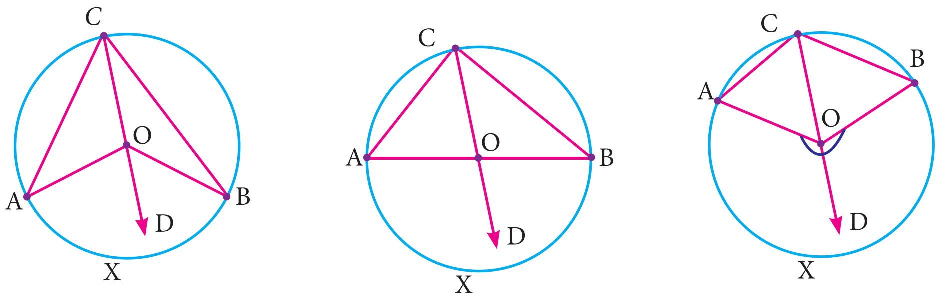


Fig. 4.27 Fig. 4.28 Fig. 4.29

Here $\overset{\frown}{AB}$ is a minor arc in Fig.4.27, a semi circle in Fig.4.28 and a major arc in Fig.4.29. The point C makes different types of angles in different positions (Fig. 4.27 to 4.29). In all these circles, $\overset{\frown}{AXB}$ subtends $\angle AOB$ at the centre and $\angle ACB$ at a point on the circumference of the circle.

We want to prove $\angle AOB = 2\angle ACB$. For this purpose extend CO to D and join CD.

$\angle OCA = \angle OAC$ since $OA = OC$ (radii)

Exterior angle = sum of two interior opposite angles.

$$\angle AOD = \angle OAC + \angle OCA$$
$$= 2\angle OCA \quad .... (1)$$

Similarly,

$$\angle BOD = \angle OBC + \angle OCB$$
$$= 2\angle OCB \quad .... (2)$$

From (1) and (2),

$$\angle AOD + \angle BOD = 2(\angle OCA + \angle OCB)$$

Finally we reach our result $\angle AOB = 2\angle ACB$.

From this we get the result as follows :

Progress Check

1. Draw the outline of different size of bangles and try to find out the centre of each using set square.
2. Trace the given cresent and complete as full moon using ruler and compass.

Fig. 4.30

Theorem 5

The angle subtended by an arc of the circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Activity - 6

1. Draw a circle with centre O and suitable radius on a chart paper (Fig. 4.31).
2. Take two points A and B on the circle to get the arc AB.
3. Join A and B to the centre O. Let P be any point on the remaining part of the circle. Join P to A and B.
4. With the help of tracing paper, make a cutout of the $\angle AOB$ portion and fold the cutout portion so that OA coincides with OB.
5. Placing the folded paper on the angle $\angle APB$ in such a way that OA lies exactly on PA. What do you observe?

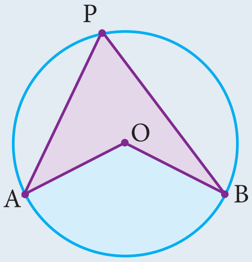


Fig. 4.31

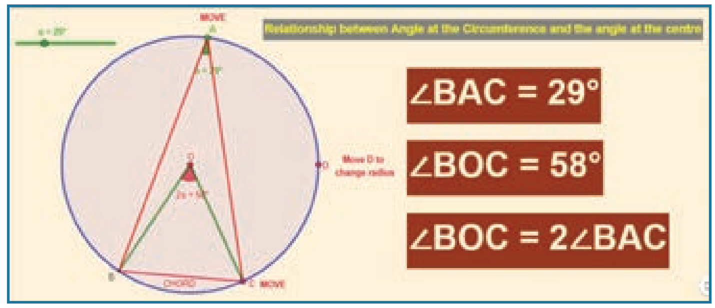
(i) Measure of $\frac{1}{2}\angle AOB =$ __________ (ii) Measure of $\angle APB =$ __________

Note

- Angle inscribed in a semicircle is a right angle.
- Equal arcs of a circle subtend equal angles at the centre.

ICT Corner

Expected Result is shown in this picture



Step – 1

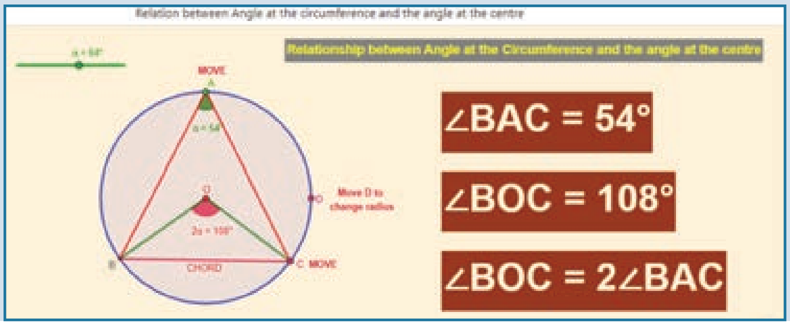
Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Angles in a circle" will open. In the work sheet there are two activities on Circles.

The first activity is the relation between Angle at the circumference and the angle at the centre. You can change the angle by moving the slider. Also, you can drag on the point *A*, *C* and *D* to change the position and the radius. Compare the angles at *A* and *O*.

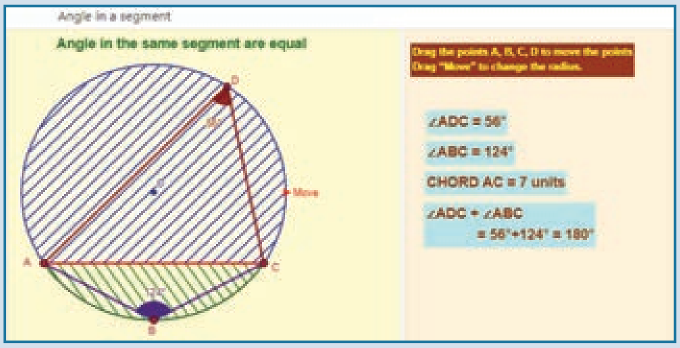
Step - 2

The second activity is "Angles in the segment of a circle". Drag the points *B* and *D* and check the angles. Also drag "Move" to change the radius and chord length of the circle.

Step 1



Step 2



Browse in the link

Angle in a circle: https://ggbm.at/yaNUhv9S or Scan the QR Code.

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Example 4.3 Find the value of $x°$ in the following figures:

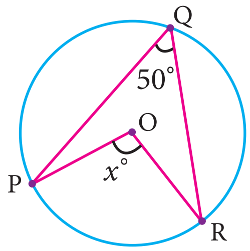
(i)


Fig. 4.32

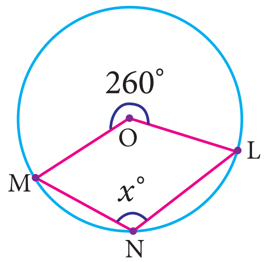
(ii)


Fig. 4.33

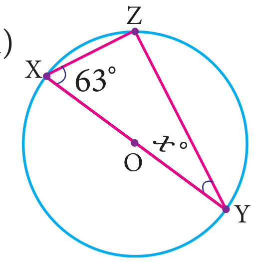
(iii)


Fig. 4.34

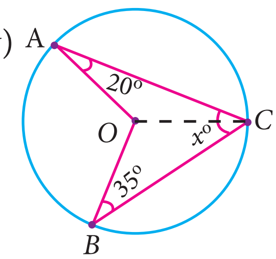
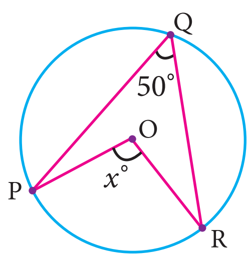
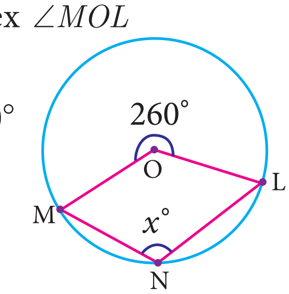
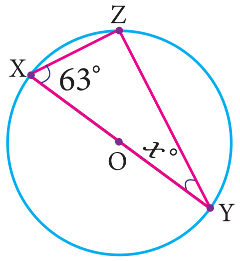
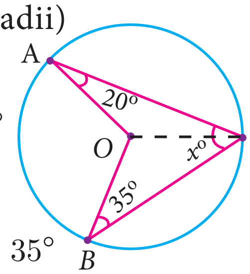
(iv)


Fig. 4.35

Solution

Using the theorem the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of a circle.

(i) $\angle POR = 2\angle PQR$ $x° = 2\times 50°$ $x° = 100°$ 	(ii) $\angle MNL = \frac{1}{2}$ Reflex $\angle MOL$ $= \frac{1}{2}\times 260°$ $x° = 130°$ 
(iii) XY is the diameter of the circle. Therefore $\angle XZY = 90°$ (Angle on a semi – circle) In right ΔXYZ $x° + 63° + 90° = 180°$ $x° = 27°$ 	(iv) $OA = OB = OC$ (Radii) In ΔOAC, $\angle OAC = \angle OCA = 20°$ In ΔOBC, $\angle OBC = \angle OCB = 35°$ (angles opposite to equal sides are equal) $\angle ACB = \angle OCA + \angle OCB$ $x° = 20° + 35°$ $x° = 55°$ 

Example 4.4 If O is the centre of the circle and $\angle ABC = 30°$ then find $\angle AOC$. (see Fig. 4.36)

Solution

Given $\angle ABC = 30°$

$\angle AOC = 2\angle ABC$ (The angle subtended by an arc at the centre is double the angle at any point on the circle)

$= 2\times 30°$

$= 60°$

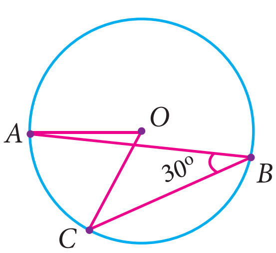


Fig. 4.36

Now we shall see, another interesting theorem. We have learnt that minor arc subtends obtuse angle, major arc subtends acute angle and semi circle subtends right angle on the circumference. If a chord AB is given and C and D are two different points on the circumference of the circle, then find $\angle ACB$ and $\angle ADB$. Is there any difference in these angles?

4.4.5 Angles at the Circumference to the same Segment

Consider the circle with centre O and chord AB. C and D are the points on the circumference of the circle in the same segment. Join the radius OA and OB.

$$\frac{1}{2}\angle AOB = \angle ACB \quad \text{(by theorem 5)}$$

and $$\frac{1}{2}\angle AOB = \angle ADB \quad \text{(by theorem 5)}$$

$$\angle ACB = \angle ADB$$

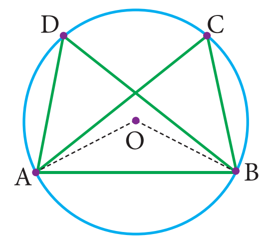


Fig. 4.37

This conclusion leads to the new result.

Theorem 6 Angles in the same segment of a circle are equal.

BDJL81

Activity - 7

Procedure :

1. Draw a circle with any radius and centre O on a chart paper.
2. Mark the points A and B on the circle and join to get a major segment AB.
3. Again mark two points C and D on the same segment of AB.
4. Draw $\angle ACB$ and $\angle ADB$
5. Trace the angles $\angle ACB$ and $\angle ADB$ on another sheet of paper and cut it.
6. Place the replica of $\angle ACB$ on $\angle ADB$

   (i) What do you see? (ii) Does $\angle ACB$ cover $\angle ADB$ exactly?

   Discuss whether it is true for different position of the point C and D on the major segment AB.

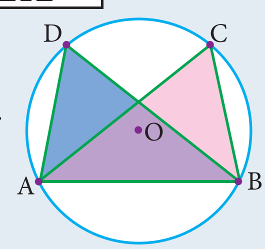


Fig. 4.38

Example 4.5 In the given figure, O is the center of the circle. If the measure of $\angle OQR = 48°$, what is the measure of $\angle P$?

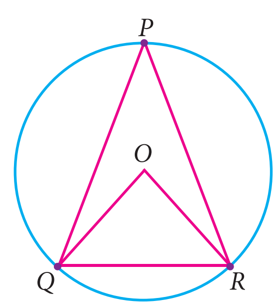


Fig. 4.39

Solution

Given $\angle OQR = 48°$.

Therefore, $\angle ORQ$ also is $48°$. (Why?__________)

$\angle QOR = 180° - (2 \times 48°) = 84°$.

The central angle made by chord QR is twice the inscribed angle at P.

Thus, measure of $\angle QPR = \frac{1}{2} \times 84° = 42°$.

Progress Check

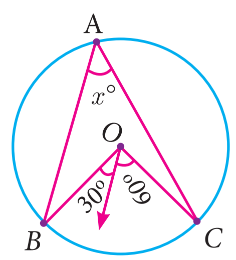
In a circle,

1. the perpendicular from the centre to a chord ________ the chord.
2. the line joining the centre to the midpoint of a chord is ________ to that chord.
3. equal chords subtend ________ angles at the centre.
4. chords which subtend equal angles at the centre are ________.
5. degree measure of a semi-circle is ________.
6. degree measure of a whole circle is ________ at the centre.
7. the angle subtended by an arc at the centre is _______ the angle subtended by it at any point on the circumference.

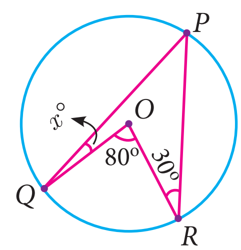
Exercise 4.3

1. Find the value of $x°$ in the following figures:

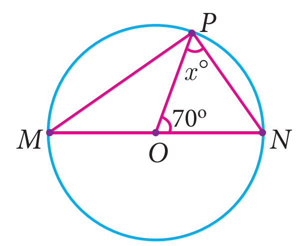
(i)



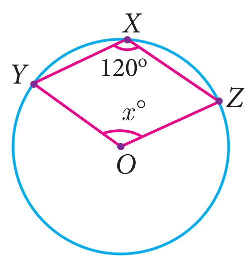
(ii)



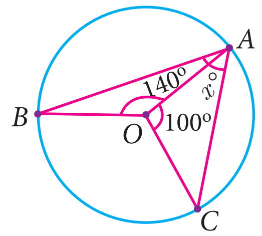
(iii)



(iv)

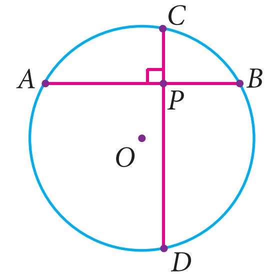


(v)



2. In the given figure, $\angle CAB = 25°$,

find $\angle BDC$, $\angle DBA$ and $\angle COB$



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4.5 Cyclic Quadrilaterals

Now we see a special quadrilateral with its properties called "Cyclic Quadrilateral". A quadrilateral is called cyclic quadrilateral if all its four vertices lie on the circumference of the circle. Now we are going to learn the special property of cyclic quadrilateral.

Consider the quadrilateral $ABCD$ whose vertices lie on a circle. We want to show that its opposite angles are supplementary. Connect the centre O of the circle with each vertex. You now see four radii OA, OB, OC and OD giving rise to four isosceles triangles OAB, OBC, OCD and ODA. The sum of the angles around the centre of the circle is $360°$. The angle sum of each isosceles triangle is $180°$

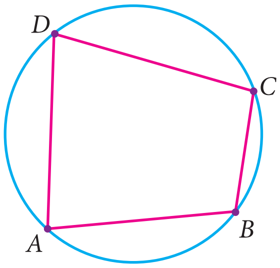


Fig. 4.40

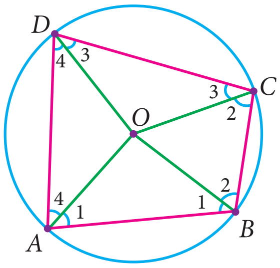


Fig. 4.41

Thus, we get from the figure,

$$2\times(\angle 1+\angle 2+\angle 3+\angle 4) + \text{Angle at centre } O = 4\times 180°$$

$$2\times(\angle 1+\angle 2+\angle 3+\angle 4) + 360° = 720°$$

Simplifying this, $(\angle 1+\angle 2+\angle 3+\angle 4) = 180°$.

You now interpret this as

(i) $(\angle 1+\angle 2) + (\angle 3+\angle 4) = 180°$ (Sum of opposite angles B and D)

(ii) $(\angle 1+\angle 4) + (\angle 2+\angle 3) = 180°$ (Sum of opposite angles A and C)

Now the result is given as follows.

Theorem 7 Opposite angles of a cyclic quadrilateral are supplementary.

Let us see the converse of theorem 7, which is very useful in solving problems

Converse of Theorem 7 If a pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is cyclic.

BDTH9N

Activity - 8

Procedure

1. Draw a circle of any radius with centre O.
2. Mark any four points A, B, C and D on the boundary. Make a cyclic quadrilateral $ABCD$ and name the angles as in Fig. 4.42
3. Make a replica of the cyclic quadrilateral $ABCD$ with the help of tracing paper.
4. Make the cutout of the angles A, B, C and D as in Fig. 4.43
5. Paste the angle cutout $\angle 1, \angle 2, \angle 3$ and $\angle 4$ adjacent to the angles opposite to A, B, C and D as in Fig. 4.44
6. Measure the angles $\angle 1 + \angle 3,$ and $\angle 2 + \angle 4$.

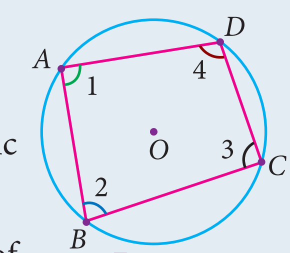


Fig. 4.42

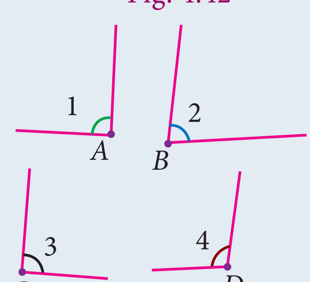


Fig. 4.43

Observe and complete the following:

1. (i) $\angle A + \angle C =$ _____ (ii) $\angle B + \angle D =$ _____

   (iii) $\angle C + \angle A =$ _____ (iv) $\angle D + \angle B =$ _____

2. Sum of opposite angles of a cyclic quadrilateral is __________.

3. A pair of opposite angles of a cyclic quadrilateral is ________.

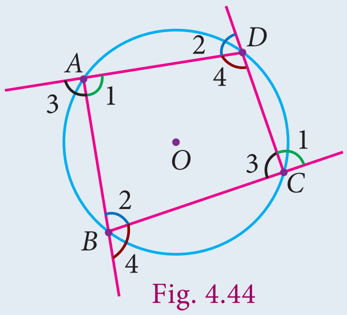


Fig. 4.44

Progress Check

1. In the given figure, find the measure of $\angle A$.
2. Is a rectangle cyclic?
3. Is any parallelogram cyclic?
4. If one angle of a cyclic quadrilateral is a right angle, what can you say about the quadrilateral?

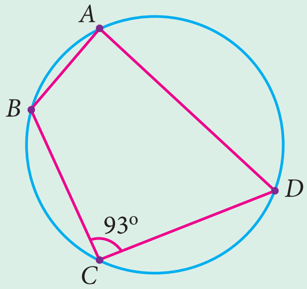


Fig. 4.45

Example 4.6 If $PQRS$ is a cyclic quadrilateral in which $\angle PSR = 70°$ and $\angle QPR = 40°$, then find $\angle PRQ$ (see Fig. 4.46).

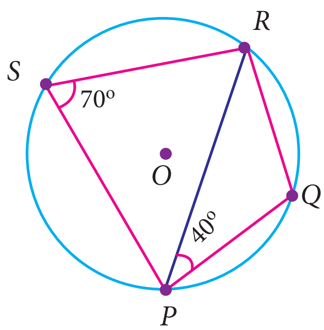


Fig. 4.46

Solution

$PQRS$ is a cyclic quadrilateral

Given $\angle PSR = 70°$

$\angle PSR + \angle PQR = 180°$ (state reason________)

$70° + \angle PQR = 180°$

$\angle PQR = 180° - 70°$

$\angle PQR = 110°$

In ΔPQR we have,

$\angle PQR + \angle PRQ + \angle QPR = 180°$ (state reason_________)

$110° + \angle PRQ + 40° = 180°$

$\angle PRQ = 180° - 150°$

$\angle PRQ = 30°$

Exterior Angle of a Cyclic Quadrilateral

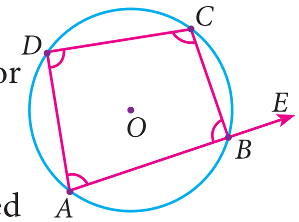


Fig. 4.47

An exterior angle of a quadrilateral is an angle in its exterior formed by one of its sides and the extension of an adjacent side.

Let the side AB of the cyclic quadrilateral $ABCD$ be extended to E. Here $\angle ABC$ and $\angle CBE$ are linear pair, their sum is $180°$ and the angles $\angle ABC$ and $\angle ADC$ are the opposite angles of a cyclic quadrilateral, and their sum is also $180°$. From this, $\angle ABC + \angle CBE = \angle ABC + \angle ADC$ and finally we get $\angle CBE = \angle ADC$. Similarly it can be proved for other angles.

Theorem 8 If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

Progress Check

1. If a pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is ______________.
2. As the length of the chord decreases, the distance from the centre ______________.
3. If one side of a cyclic quadrilateral is produced then the exterior angle is ______________ to the interior opposite angle.
4. Opposite angles of a cyclic quadrilateral are ______________.

Example 4.7 In the figure given, find the value of $x°$ and $y°$.

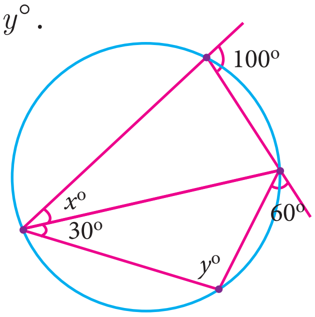


Fig. 4.48

Solution

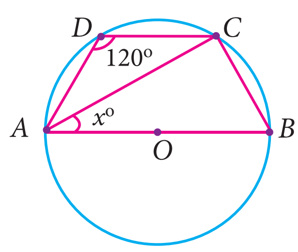
By the exterior angle property of a cyclic quadrilateral,

we get, $y° = 100°$ and

$x° + 30° = 60°$ and so $x° = 30°$

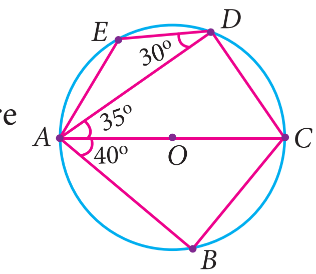
Exercise 4.4

1. Find the value of x in the given figure.

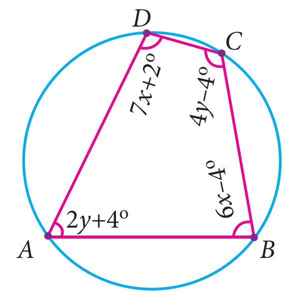


2. In the given figure, AC is the diameter of the circle with centre O. If $\angle ADE = 30°$; $\angle DAC = 35°$ and $\angle CAB = 40°$.

   Find (i) $\angle ACD$ (ii) $\angle ACB$ (iii) $\angle DAE$

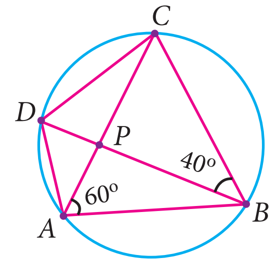


3. Find all the angles of the given cyclic quadrilateral $ABCD$ in the figure.

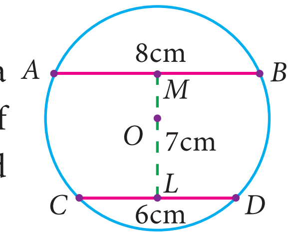


4. AB and CD are two parallel sides of a cyclic quadrilateral $ABCD$ such that AB = 10cm, CD = 24cm and the radius of the circle is 13cm. Find the shortest distance between the two sides AB and CD.

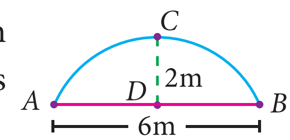
5. In the given figure, $ABCD$ is a cyclic quadrilateral where diagonals intersect at P such that $\angle DBC = 40°$ and $\angle BAC = 60°$ find (i) $\angle CAD$ (ii) $\angle BCD$



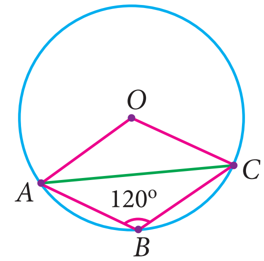
6. In the given figure, AB and CD are the parallel chords of a circle with centre O. Such that AB = 8cm and CD = 6cm. If $OM \perp AB$ and $OL \perp CD$ distance between LM is 7cm. Find the radius of the circle?



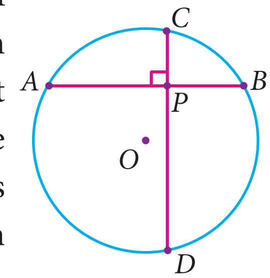
7. The arch of a bridge has dimensions as shown, where the arch measure 2m at its highest point and its width is 6m. What is the radius of the circle that contains the arch?



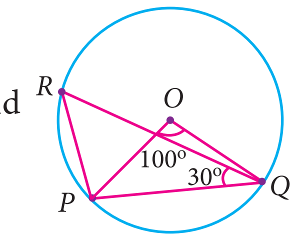
8. In figure $\angle ABC = 120^0$, where A, B and C are points on the circle with centre O. Find $\angle OAC$?



9. A school wants to conduct tree plantation programme. For this a teacher allotted a circle of radius 6m ground to nineth standard students for planting sapplings. Four students plant trees at the points A, B, C and D as shown in figure. Here AB = 8m, CD = 10m and $AB \perp CD$. If another student places a flower pot at the point P, the intersection of AB and CD, then find the distance from the centre to P.



10. In the given figure, $\angle POQ = 100^0$ and $\angle PQR = 30^0$, then find $\angle RPO$.



4.6 Constructions

In the first term we have learnt to locate circumcentre and orthocentre of a triangle. Now we are ready to locate incentre and centroid of a triangle. For this we use (i) the construction of perpendicular bisector of a line segment (ii) the construction of angle bisector of a given angle.

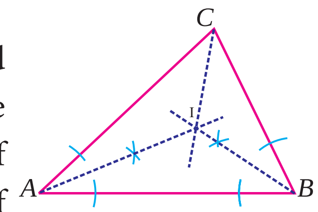


Fig. 4.49

4.6.1 Construction of the Incircle of a Triangle

Incentre

The incentre is (one of the triangle's points of concurrency formed by) the intersection of the triangle's three angle bisectors.

The incentre is the centre of the incircle ; It is usually denoted by *I*; it is the one point in the triangle whose distances to the sides are equal.

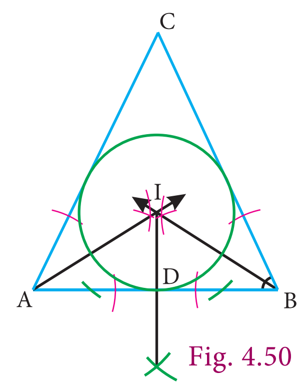


Fig. 4.50

Example 4.8 Construct the incentre of ΔABC with AB = 6 cm, $\angle B = 65°$ and $AC =$ 7 cm Also draw the incircle and measure its radius.

Rough Diagram

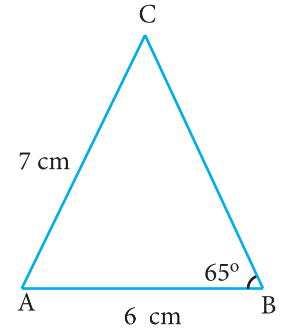


Fig. 4.51

Solution

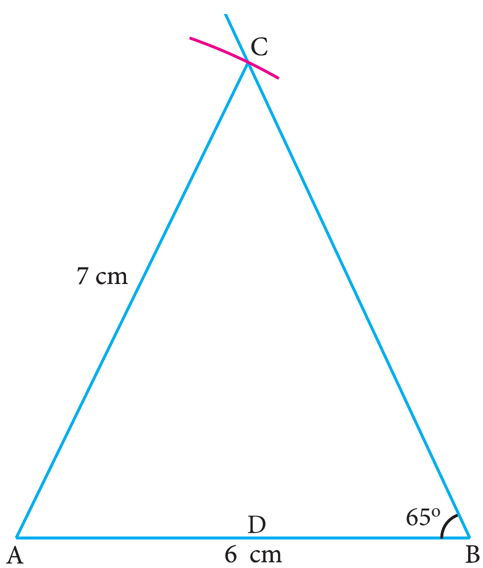


Fig. 4.52

Step 1 : Draw the ΔABC with

$AB = 6$cm, $\angle B = 65°$ and $AC = 7$cm

Step 2 : Construct the angle bisectors of any two angles (A and B) and let them meet at I. Then I is the incentre of ΔABC. Draw perpendicular from I to any one of the side (AB)to meet AB at D.

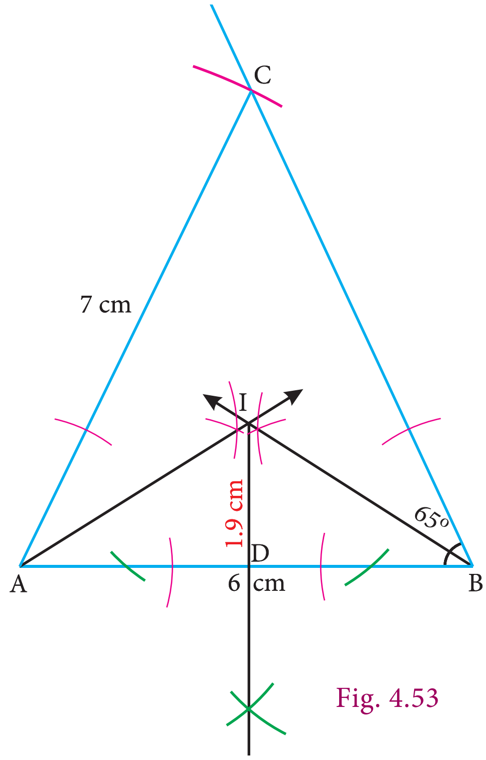


Fig. 4.53

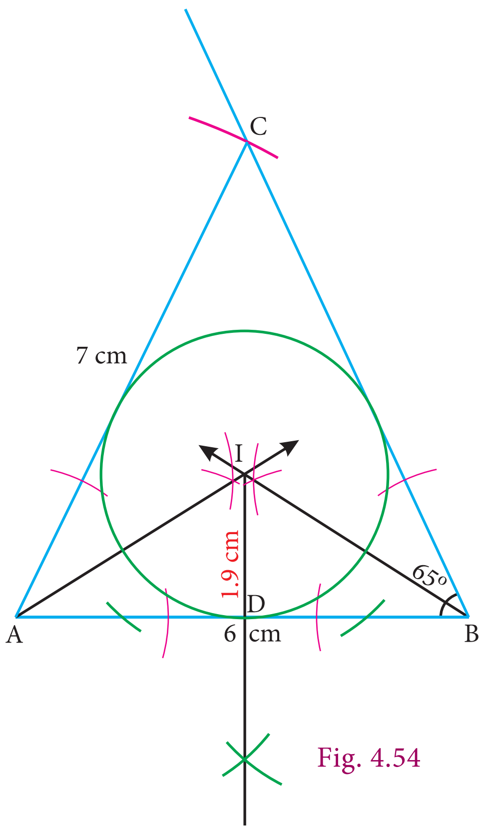


Fig. 4.54

Step 3:With I as centre and ID as radius draw the circle. This circle touches all the sides of the triangle internally.

Step 4: Measure inradius

In radius = 1.9 cm.

Note

The incentre of any triangle always lie inside the circle.

Exercise 4.5

1. Draw an equilateral triangle of side 6.5 cm and locate its incentre. Also draw the incircle.
2. Draw a right triangle whose hypotenuse is 10 cm and one of the legs is 8 cm. Locate its incentre and also draw the incircle.
3. Draw ΔABC given AB = 9 cm, $\angle CAB = 115°$ and $\Delta ABC = 40°$. Locatge its incentre and also draw the incircle. (Note: You can check from the above examples that the incentre of any triangle is always in its interior).
4. Construct ΔABC in which $AB = BC$ = 6cm and $\angle B = 80°$. Locate its incentre and draw the incircle.

4.6.2 Construction of the Centroid of a Triangle

Centroid

The point of concurrency of the medians of a triangle is called the centroid of the triangle and is usually denoted by G.

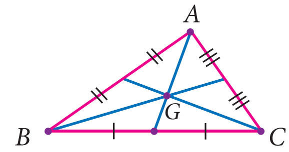


Fig. 4.55

Example 4.9 Construct the centroid of ΔPQR whose sides are $PQ = 8\text{cm}$; $QR = 6\text{cm}$; $RP = 7\text{cm}$.

Rough Diagram

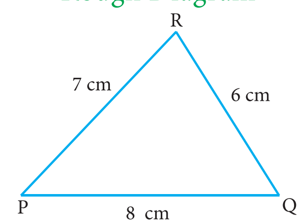


Fig. 4.56

Solution

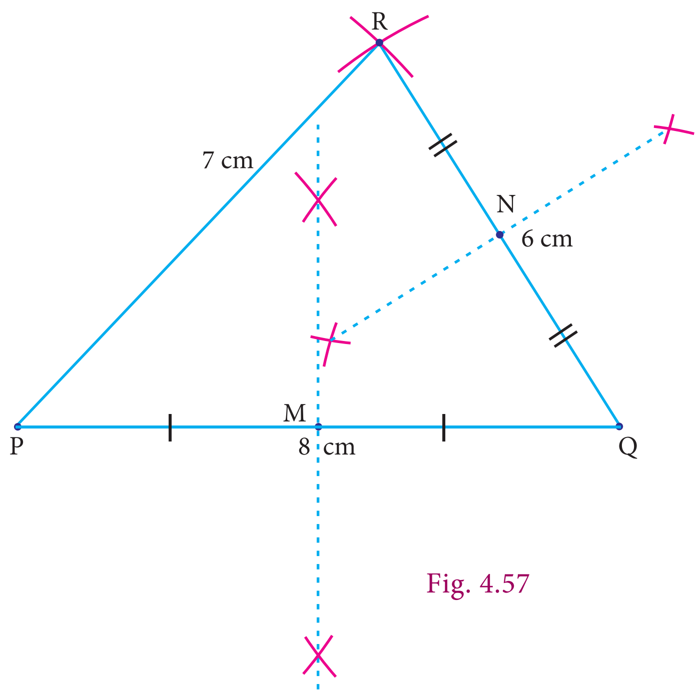


Fig. 4.57

Step 1 : Draw ΔPQR using the given measurements $PQ = 8\text{cm}$ $QR = 6\text{cm}$ and $RP = 7\text{cm}$ and construct the perpendicular bisector of any two sides (PQ and QR) to find the mid-points M and N of PQ and QR respectively.

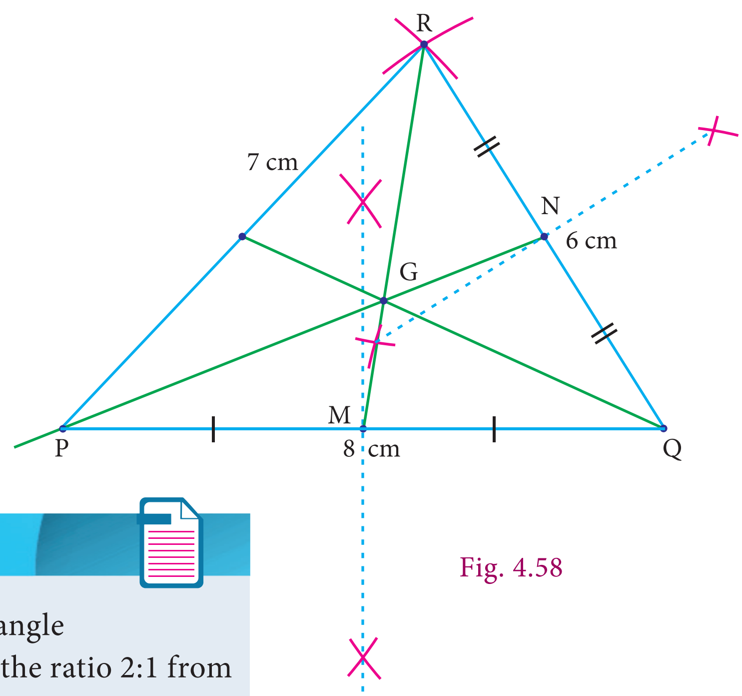


Fig. 4.58

Step 2 : Draw the medians PN and RM and let them meet at G. The point G is the centroid of the given ΔPQR.

Note

- Three medians can be drawn in a triangle
- The centroid divides each median in the ratio 2:1 from the vertex.
- The centroid of any triangle always lie inside the triangle.
- Centroid is often described as the triangle's centre of gravity (where the triangle balances evenly) and also as the barycentre.

Exercise 4.6

1. Construct the ΔLMN such that LM=7.5cm, MN=5cm and LN=8cm. Locate its centroid.
2. Draw and locate the centroid of the triangle ABC where right angle at A, AB = 4cm and AC = 3cm.
3. Draw the ΔABC, where AB = 6cm, $\angle B = 110°$ and AC = 9cm and construct the centroid.
4. Construct the ΔPQR such that PQ = 5cm, PR= 6cm and $\angle QPR = 60°$ and locate its centroid.
5. Construct an equilateral triangle of side 6cm and locate its centroid and also its incentre. What do you observe from this?

Exercise 4.7

Multiple Choice Questions

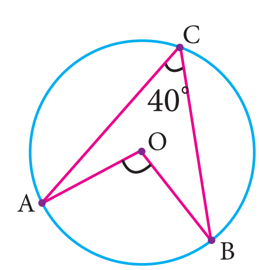
1. PQ and RS are two equal chords of a circle with centre O such that $\angle POQ = 70°$, then $\angle ORS =$

   (1) $60°$ (2) $70°$ (3) $55°$ (4) $80°$

2. A chord is at a distance of 15cm from the centre of the circle of radius 25cm. The length of the chord is

   (1) 25cm (2) 20cm (3) 40cm (4) 18cm

3. In the figure, O is the centre of the circle and $\angle ACB = 40°$ then $\angle AOB =$

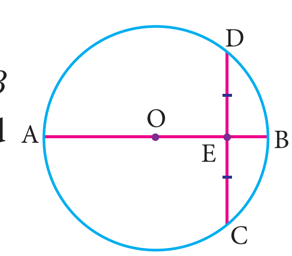
   

   (1) $80°$ (2) $85°$ (3) $70°$ (4) $65°$

4. In a cyclic quadrilaterals $ABCD$, $\angle A = 4x$, $\angle C = 2x$ the value of x is

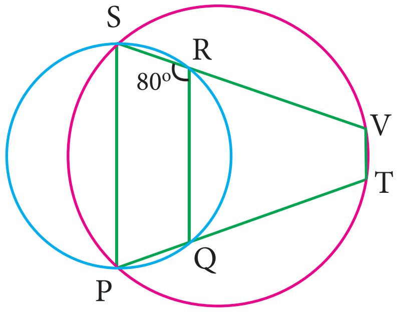
   (1) $30°$ (2) $20°$ (3) $15°$ (4) $25°$

5. In the figure, O is the centre of a circle and diameter AB bisects the chord CD at a point E such that CE=ED=8 cm and EB=4cm. The radius of the circle is

   (1) 8cm (2) 4cm (3) 6cm (4)10cm

6. In the figure, $PQRS$ and $PTVS$ are two cyclic quadrilaterals, If $\angle QRS = 80°$, then $\angle TVS =$

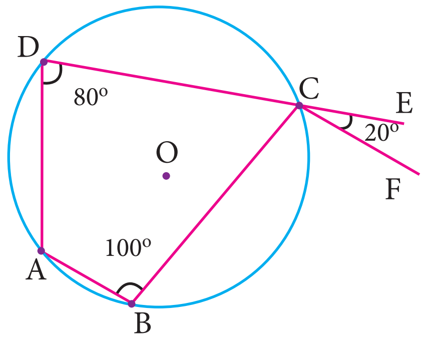


(1) 80° (2) 100° (3) 70° (4) 90°

7. If one angle of a cyclic quadrilateral is $75°$, then the opposite angle is

(1) 100° (2) 105° (3) 85° (4) 90°

8. In the figure, $ABCD$ is a cyclic quadrilateral in which DC produced to E and CF is drawn parallel to AB such that $\angle ADC = 80°$ and $\angle ECF = 20°$, then $\angle BAD = ?$



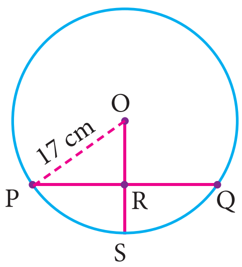
(1) 100° (2) 20°

(3) 120° (4) 110°

9. AD is a diameter of a circle and AB is a chord If $AD = 30$ cm and $AB = 24$ cm then the distance of AB from the centre of the circle is

(1) 10cm (2) 9cm (3) 8cm (4) 6cm.

10. In the given figure, If $OP = 17$cm, $PQ = 30$cm and OS is perpendicular to PQ, then RS is



(1) 10cm (2) 6cm (3) 7cm (4) 9cm.

Points to Remember

- There is one and only one circle passing through three non-collinear points.
- Equal chords of a circle subtend equal angles at the centre.
- Perpendicular from the centre of a circle to a chord bisects the chord.
- Equal chords of a circle are equidistant from the centre.
- The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.
- The angle in a semi circle is a right angle.
- Angles in the same segment of a circle are equal.
- The sum of either pair of opposite angle of a cyclic quadrilateral is 180°.
- If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

5 STATISTICS

"Lack of statistics is to hide inconvenient facts."
- Albert Bertilsson

Sir Ronald Aylmer Fisher (1890 - 1962 A.D. (CE))

Sir Ronald Aylmer Fisher was a British Statistician and Biologist. He was known as the Father of Modern Statistics and Experimental Design. Fisher did experimental agricultural research, which saved millions from starvation. He was awarded the Linnean Society of London's prestigious Darwin-Wallace Medal in 1958.

Learning Outcomes

- To interpret the term 'Statistics' and list basic ideas.
- To recall the meaning of 'average'.
- To recall different types of averages known already.
- To recall the methods of computing the Mean, Median and Mode for ungrouped data.
- To compute the Mean, Median and Mode for the grouped data.

5.1 Introduction

Statistics is the science of collecting, organising, analysing and interpreting data in order to make decisions. In everyday life, we come across a wide range of quantitative and qualitative information. These have profound impact on our lives.

Data means the facts, mostly numerical, that are gathered; statistics implies collection of data. We analyse the data to make decisions. The methods of statistics are tools to help us in this.

Cricket News

Team U19	Matches	Won	Lost	NR/Tied
India	71	52	18	0/1
Australia	67	50	15	0/2
Pakistan	69	50	19	0/0
Bangladesh	64	45	17	1/1
West Indies	71	44	27	0/0
South Africa	61	43	17	0/1
England	69	40	28	0/1
Sri Lanka	68	36	31	0/1
New Zealand	66	30	35	0/1
Zimbabwe	62	28	34	0/0
Ireland	49	16	32	1/0
Afghanistan	24	11	13	0/0
Namibia	47	9	37	1/0
Kenya	17	5	12	0/0
Canada	29	4	23	1/1
PNG	41	3	38	0/0

Customer Satisfaction Survey

Hotel Tamilnadu

Tell us how were you satisfied with our service

- Very Satisfied
- Satisfied
- Neutral
- Unsatisfied
- Annoyed

India's 2018 GDP Forecast

UN	7.2%
IMF	7.4%
World Bank	7.3%
Morgan Stanley	7.5%
Moody's	7.6%
HSBC	7%
Bank of America	7.2%
Merill Lynch	7.5%
Goldman Sachs	8%

5.2 Collection of Data

Primary data are first-hand original data that we collect ourselves. Primary data collection can be done in a variety of ways such as by conducting personal interviews (by phone, mail or face-to-face), by conducting experiments, etc.

Secondary data are the data taken from figures collected by someone else. For example, government-published statistics, available research reports etc.

Progress Check

Identify the primary data

(i) Customer surveys
(ii) Medical researches
(iii) Economic predictions
(iv) School results
(v) Political polls
(vi) Marketing details
(vii) Sales forecasts
(viii) Price index details

5.2.1 Getting the Facts Sorted Out

When data are initially collected and before it is edited and not processed for use, they are known as Raw data. It will not be of much use because it would be too much for the human eye to analyse.

Activity

Prepare an album of pictures, tables, numeric details etc that exhibit data. Discuss how they are related to daily life situations.

For example, study the marks obtained by 50 students in mathematics in an examination, given below:

61	60	44	49	31	60	79	62	39	51	67	65	43	54	51	42
52	43	46	40	60	63	72	46	34	55	76	55	30	67	44	57
62	50	65	58	25	35	54	59	43	46	58	58	56	59	59	45
42	44														

In this data, if you want to locate the five highest marks, is it going to be easy? You have to search for them; in case you want the third rank among them, it is further complicated. If you need how many scored less than, say 56, the task will be quite time consuming.

Hence **arrangement of an array of marks** will make the job simpler.

With some difficulty you may note in the list that 79 is the highest mark and 25 is the least. Using these you can subdivide the data into convenient classes and place each mark into the appropriate class. Observe how one can do it.

Class Interval	Marks
25-30	30, 25
31-35	31, 34, 35
36-40	39, 40
41-45	44, 43, 42, 43, 44, 43, 45, 42, 44
46-50	49, 46, 46, 50, 46
51-55	51, 54, 51, 52, 55, 55, 54
56-60	60, 60, 60, 57, 58, 59, 58, 58, 56, 59, 59
61-65	61, 62, 65, 63, 62, 65
66-70	67, 67
71-75	72
76-80	79, 76

From this table can you answer the questions raised above? To answer the question, "how many scored below 56", you do not need the actual marks. You just want "how many" were there. To answer such cases, which often occur in a study, we can modify the table slightly and just note down how many items are there in each class. We then may have a slightly simpler and more useful arrangement, as given in the table.

Class Interval	Number of items
25-30	2
31-35	3
36-40	2
41-45	9
46-50	5
51-55	7
56-60	11
61-65	6
66-70	2
71-75	1
76-80	2

This table gives us the number of items in each class; each such number tells you how many times the required item occurs in the class and is called the frequency in that class.

The table itself is called a frequency table.

We use what are known as tally marks to compute the frequencies. (Under the column 'number of items', we do not write the actual marks but just tally marks). For example, against the class 31-35, instead of writing the actual marks 31, 34, 35 we simply put |||. You may wonder if for the class 56-60 in the example one has to write |||||||||||, making it difficult to count. To avoid confusion, every fifth tally mark is put across the four preceding it, like this 𝍸 . For example, 11 can be written as 𝍸 𝍸 | . The frequency table for the above illustration will be seen as follows:

Class Interval	Tally Marks	Frequency
25-30	||	2
31-35	|||	3
36-40	||	2
41-45	~~||||~~ ||||	9
46-50	~~||||~~	5
51-55	~~||||~~ ||	7
56-60	~~||||~~ ~~||||~~ |	11
61-65	~~||||~~ |	6
66-70	||	2
71-75	|	1
76-80	||	2
	TOTAL	**50**

Note

Consider any class, say 56- 60; then 56 is called the lower limit and 60 is called the upper limit of the class.

Progress Check

Form a frequency table for the following data:

23	44	12	11	45	55	79	20
52	37	77	97	82	56	28	71
62	58	69	24	12	99	55	78
21	39	80	65	54	44	59	65
17	28	65	35	55	68	84	97
80	46	30	49	50	61	59	33
11	57						

5.3 Measures of Central Tendency

It often becomes necessary in everyday life to express a quantity that is typical for a given data. Suppose a researcher says that on an average, people watch TV serials for 3 hours per day, it does not mean that everybody does so; some may watch more and some less. The average is an acceptable indicator of the data regarding programmes watched on TV.

Averages summarise a large amount of data into a single value and indicate that there is some variability around this single value within the original data.

A mathematician's view of an average is slightly different from that of the commoner. There are three different definitions of average known as the Mean, Median and Mode. Each of them is found using different methods and when they applied to the same set of original data they often result in different average values. It is important to figure out what each of these measures of average tells you about the original data and consider which one is the most appropriate to calculate.

5.3.1 Arithmetic Mean-Raw Data

The Arithmetic Mean of a data is the most commonly used of all averages and is found by adding together all the values and dividing by the number of items.

For example, a cricketer, played eight (T20) matches and scored the following scores 25, 32, 36, 38, 45, 41, 35, 36.

Then the mean of his scores (that is the arithmetic average of the scores) is obtained by

$$\overline{X} = \frac{\sum x}{n} = \frac{25+32+36+38+45+41+35+36}{8} = \frac{288}{8} = 36$$

In general, if we have n number of observations $x_1, x_2, x_3, \ldots, x_n$ then their arithmetic mean denoted by $\overline{X}$ (read as X bar) is given by

$$\overline{X} = \frac{\text{sum of all the observations}}{\text{number of observations}} \quad \overline{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

We express this as a formula: $\overline{X} = \dfrac{\sum x}{n}$

Assumed Mean method: Sometimes we can make calculations easy by working from an entry that we guess to be the right answer. This guessed number is called the assumed mean.

In the example above on cricket scores, let us assume that 38 is the assumed mean. We now list the differences between the assumed mean and each score entered:

Note

It does not matter which number is chosen as the assumed mean; we need a number that would make our calculations simpler. Perhaps a choice of number that is closer to most of the entries would help; it need not even be in the list given.

25–38 = –13, 32–38 = –6, 36–38 = –2, 38–38 = 0,

45–38 = 7, 41–38 = 3, 35–38 = –3, 36–38 = –2

The average of these differences is $\dfrac{-13-6-2+0+7+3-3-2}{8} = \dfrac{-16}{8} = -2$

We add this 'mean difference' to the assumed mean to get the correct mean.

Thus the correct mean = Assumed Mean +Mean difference = 38 – 2 = 36 .

This method will be very helpful when large numbers are involved.

5.3.2 Arithmetic Mean-Ungrouped Frequency Distribution

Consider the following list of heights (in cm) of 12 students who are going to take part in an event in the school sports.

140, 142, 150, 150, 140, 148, 140, 147, 145, 140, 147, 145.

How will you find the Mean height?

There are several options.

(i) You can add all the items and divide by the number of items.

$$\frac{140+142+150+150+140+148+140+147+145+140+147+145}{12}=\frac{1734}{12}=144.5$$

(ii) You can use Assumed mean method. Assume, 141 as the assumed mean.

Then the mean will be given by

$$=141+\frac{(-1)+(1)+(9)+(9)+(-1)+(7)+(-1)+(6)+(4)+(-1)+(6)+(4)}{12}$$

$$=141+\frac{-4+46}{12}=141+\frac{42}{12}=141+3.5=144.5$$

(iii) A third method is to deal with an ungrouped frequency distribution. You find that 140 has occurred 4 times, (implying 4 is the frequency of 140), 142 has occurred only once (indicating that 1 is the frequency of 142) and so on. This enables us to get the following frequency distribution.

Height(cm)	140	142	150	148	145	147
No. of students	4	1	2	1	2	2

You find that there are four 140s; their total will be 140 × 4 = 560

There is only one 142; so the total in this case is 142 × 1 = 142

There are two 150s; their total will be 150 × 2 = 300 etc.

These details can be neatly tabulated as follows:

Height (x)	Frequency (f)	fx
140	4	560
142	1	142
150	2	300
148	1	148
145	2	290
147	2	294
	12	1734

$$\text{Mean}=\frac{\text{Sum of all } fx}{\text{No.of items}}$$

$$=\frac{1734}{12}=144.5 \text{ cm}$$

Looking at the procedure in general terms, you can obtain a formula for ready use. If $x_1, x_2, x_3, \ldots x_n$ are n observations whose corresponding frequencies are $f_1, f_2, f_3, \ldots f_n$ then the mean is given by

$$\overline{X}=\frac{f_1x_1+f_2x_2+\ldots+f_nx_n}{f_1+f_2+\ldots f_n}=\frac{\sum_{i=1}^{n} f_ix_i}{\sum_{i=1}^{n} f_i}=\frac{\sum fx}{\sum f}$$

Note

Study each step and understand the meaning of each symbol.

Can you adopt the above method combining with the assumed mean method? Here is an attempt in that direction:

(iv) Let the assumed mean be 145. Then we can prepare the following table:

Height(x)	**d = deviation from the assumed mean**	**Frequency (f)**	**fd**
140	140 – 145 = – 5	4	–20
142	142– 145 = – 3	1	– 3
150	150 – 145 = + 5	2	+10
148	148 – 145 = + 3	1	+ 3
145 (Assumed)	145 – 145 = 0	2	0
147	147 – 145 = + 2	2	+ 4
Total		$\sum f = 12$	$\sum fd = -23 + 17 = -6$

Arithmetic mean = Assumed mean + Average of the sum of deviations

$$= A + \frac{\sum fd}{\sum f} = 145 + \left(\frac{-6}{12}\right) = 145.0 - 0.5 = 144.5$$

When large numbers are involved this, method could be useful.

Progress Check

You are given the following ungrouped data of the number of units of electricity consumed by 68 householders of a locality:

75, 75, 75, 75, 95, 95, 95, 95, 95, 115, 115, 115, 115, 115, 115, 115, 115, 115, 115, 115, 115, 115, 135, 135, 135, 135, 135, 135, 135, 135, 135, 135, 135, 135, 135, 135, 135, 135, 135, 135, 135,155, 155, 155, 155, 155, 155, 155, 155, 155, 155, 155, 155, 155, 155,175, 175, 175, 175, 175, 175, 175, 175, 195, 195, 195, 195.

Try to find the Mean of this data by the following methods:

(i) Adding all the items and dividing the total by the total number of items (i).

(ii) Using Assumed mean (ii) in the above method

(iii) Forming an ungrouped frequency distribution and then using the formula

$$\bar{X} = \frac{\sum fx}{\sum f}$$

(iv) Using Assumed mean (iv) in the above method

Which procedure do you find simpler? Why?

5.3.3 Arithmetic Mean-Grouped Frequency Distribution

When data are grouped in class intervals and presented in the form of a frequency table, we get a frequency distribution like this one:

Age (in years)	10-20	20-30	30 – 40	40 – 50	50 - 60
Number of customers	80	120	50	22	8

The above table shows the number of customers in the various age groups. For example, there are 120 customers in the age group 20 – 30, but does not say anything about the age of any individual. (When we form a grouped frequency table the identity of the individual observations is lost). Hence we need a value that represents the particular class interval. Such a value is called mid value (mid-point or class mark) The mid-point or class mark can be found using the formula given below.

Mid Value = $\frac{UCL + LCL}{2}$, *UCL – Upper Class Limit, LCL – Lower Class Limit*

5.4 Arithmetic Mean

In grouped frequency distribution, arithmetic mean may be computed by applying any one of the following methods.

(i) Direct Method (ii) Assumed Mean Method (iii) Step Deviation Method

5.4.1 Direct Method

When direct method is used, the formula for finding the arithmetic mean is

$$\overline{X} = \frac{\sum fx}{\sum f}$$

Where x is the midpoint of the class interval and f is the frequency

Steps

(i) Obtain the midpoint of each class and denote it by x

(ii) Multiply those midpoints by the respective frequency of each class and obtain the sum of fx

(iii) Divide $\sum fx$ by $\sum f$ to obtain mean

Example 5.1 The following data gives the number of residents in an area based on their age. Find the average age of the residents.

Age	0-10	10-20	20-30	30-40	40-50	50-60
Number of Residents	2	6	9	7	4	2

Solution

Age	Number of Residents(f)	Midvalue(x)	fx
0-10	2	5	10
10-20	6	15	90
20-30	9	25	225
30-40	7	35	245
40-50	4	45	180
50-60	2	55	110
	$\sum f = 30$		$\sum fx = 860$

Mean= $\overline{X} = \dfrac{\sum fx}{\sum f} = \dfrac{860}{30} = 28.67$

Hence the average age $= 28.67$.

5.4.2 Assumed Mean Method

We have seen how we can find the arithmetic mean of a grouped data quickly using the direct method formula. However, if the observations are large, finding the products of the observations and their frequencies, and then adding them is not only difficult and time consuming but also has chances of errors. In such cases, we can use the Assumed Mean Method to find the arithmetic mean of grouped data.

Steps

1. Assume any value of the observations as the Mean (A) . Consider the value in the middle preferably.
2. Calculate the deviation $d = x - A$ for each class
3. Multiply each of the corresponding frequency f with d and obtain Σfd
3. Apply the formula $\overline{X} = A + \dfrac{\sum fd}{\sum f}$

Example 5.2 Find the mean for the following frequency table:

Class Interval	100-120	120-140	140-160	160-180	180-200	200-220	220-240
Frequency	10	8	4	4	3	1	2

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Solution

Let Assumed mean $A = 170$

Class Interval	Frequency f	Mid value x	$d = x - A$ $d = x - 170$	fd
100-120	10	110	–60	–600
120-140	8	130	–40	–320
140-160	4	150	–20	–80
160-180	4	170	0	0
180-200	3	190	20	60
200-220	1	210	40	40
220-240	2	230	60	120
	$\sum f = 32$			$\sum fd = -780$

Mean $\overline{X} = A + \frac{\sum fd}{\sum f}$

$= 170 + \left(\frac{-780}{32}\right)$

Therefore , $\overline{X} = 170 - 24.375$

$= 145.625$

5.4.3 Step Deviation Method

In order to simplify the calculation, we divide the deviation by the width of class intervals (i.e. calculate $\frac{x - A}{c}$) and then multiply by c in the formula for getting the mean of the data. The formula to calculate the Arithmetic Mean is

$$\overline{X} = A + \left[\frac{\sum fd}{\sum f} \times c\right], \text{ where } d = \frac{x - A}{c}$$

Example 5.3 Find the mean of the following distribution using Step Deviation Method.

Class Interval	0-8	8-16	16-24	24-32	32-40	40-48
Frequency (f)	10	20	14	16	18	22

Solution

Let Assumed mean $A = 28$, class width c =8

Class Interval	Mid Value x	Frequency f	$d = \frac{x - A}{c}$	fd
0-8	4	10	–3	–30
8-16	12	20	–2	–40
16-24	20	14	–1	–14
24-32	28	16	0	0
32-40	36	18	1	18
40-48	44	22	2	44
		$\sum f = 100$		$\sum fd = -22$

Mean

$$\overline{X} = A + \frac{\sum fd}{\sum f} \times c$$

$$= 28 + \left(\frac{-22}{100}\right) \times 8$$

$$= 28 - 1.76 = 26.24$$

Note

- When x_i and f_i are small, then Direct Method is the appropriate choice.
- When x_i and f_i are numerically large numbers, then Assumed Mean Method or Sep Deviation Method can be used.
- When class sizes are unequal and d numerically large,we can still use Step Deviation Method.

5.4.4 A special property of the Arithmetic Mean

Let a, b and c be three numbers. Their Mean is $\frac{a+b+c}{3}$. How far is each number from this average? (We call it deviation of the number from the Mean). And what is the sum of all these deviations?

Number	Deviation from the Mean
a	$a - \frac{a+b+c}{3} = \frac{2a-b-c}{3}$
b	$b - \frac{a+b+c}{3} = \frac{2b-c-a}{3}$
c	$c - \frac{a+b+c}{3} = \frac{2c-a-b}{3}$
	Total = $\frac{2a-b-c}{3} + \frac{2b-c-a}{3} + \frac{2c-a-b}{3} = 0$

This can be generalised as follows:

The sum of the deviations of the entries from the arithmetic mean is always zero.

If $x_1, x_2, x_3, \ldots, x_n$ are n observations taken from the arithmetic mean $\overline{X}$

then $\left(x_1 - \overline{X}\right) + \left(x_2 - \overline{X}\right) + \left(x_3 - \overline{X}\right) + \ldots + \left(x_n - \overline{X}\right) = 0$ .Hence $\sum_{i=1}^{n}(x_i - \overline{X}) = 0$

1. *If each observation is increased or decreased by k then the arithmetic mean is also increased or decreased by k respectively.*

2. *If each observation is multiplied or divided by k, $k \neq 0$, then the arithmetic mean is also multiplied or divided by the same quantity k respectively.*

Example 5.4 Find the sum of the deviations from the arithmetic mean for the following observations:

21, 30, 22, 16, 24, 28, 18, 17

Solution

$$\overline{X} = \frac{\sum x}{n} = \frac{21+30+22+16+24+28+18+17}{8} = \frac{176}{8} = 22$$

Deviation of an entry x from the arithmetic mean $\overline{X}$ is $x - \overline{X}$

Sum of the deviations

$= (21\text{-}22)+(30\text{-}22)+(22\text{-}22)+(16\text{-}22)+(24\text{-}22)+(28\text{-}22)+(18\text{-}22)+(17\text{-}22)$

$= 16 - 16 = 0.$ or equivalently, $\sum (x - \overline{X}) = 0$

Hence, we conclude that sum of the deviations from the Arithmetic Mean is zero.

Example 5.5 The arithmetic mean of 6 values is 45 and if each value is increased by 4, then find the arithmetic mean of new set of values.

Solution

Let $x_1, x_2, x_3, x_4, x_5, x_6$ be the given set of values then $\dfrac{\sum_{i=1}^{6} x_i}{6} = 45.$

If each value is increased by 4, then the mean of new set of values is

$$\text{New A.M. } \overline{X} = \frac{\sum_{i=1}^{6}(x_i + 4)}{6}$$

$$= \frac{(x_1+4)+(x_2+4)+(x_3+4)+(x_4+4)+(x_5+4)+(x_6+4)}{6}$$

$$= \frac{\sum_{i=1}^{6} x_i + 24}{6} = \frac{\sum_{i=1}^{6} x_i}{6} + 4$$

$$\overline{X} = 45 + 4 = 49.$$

Progress Check

Mean of 10 observations is 48 and 7 is subtracted to each observation, then mean of new observation is __________

Example 5.6 If the arithmetic mean of 7 values is 30 and if each value is divided by 3, then find the arithmetic mean of new set of values

Solution

Let X represent the set of seven values $x_1, x_2, x_3, x_4, x_5, x_6, x_7$.

Then $$\overline{X} = \frac{\sum_{i=1}^{7} x_i}{7} = 30 \text{ or } \sum_{i=1}^{7} x_i = 210$$

If each value is divided by 3, then the mean of new set of values is

$$\frac{\sum_{i=1}^{7} \frac{x_i}{3}}{7} = \frac{\left(\frac{x_1}{3}+\frac{x_2}{3}+\frac{x_3}{3}+\frac{x_4}{3}+\frac{x_5}{3}+\frac{x_6}{3}+\frac{x_7}{3}\right)}{7}$$

$$= \frac{\sum_{i=1}^{7} x_i}{21} = \frac{210}{21} = 10$$

Progress Check

1. The Mean of 12 numbers is 20. If each number is multiplied by 6, then the new mean is __
2. The Mean of 30 numbers is 16. If each number is divided by 4, then the new mean is___

Aliter

If Y is the set of values obtained by dividing each value of X by 3.

Then, $\bar{Y} = \frac{\bar{X}}{3} = \frac{30}{3} = 10.$

Example 5.7 The average mark of 25 students was found to be 78.4. Later on, it was found that score of 96 was misread as 69. Find the correct mean of the marks.

Solution

Given that the total number of students $n = 25, \bar{X} = 78.4$

So, Incorrect $\sum x = \bar{X} \times n \;\; = 78.4 \times 25 = 1960$

Correct Σx $= incorrect \sum x - wrong\ entry + correct\ entry$

$= 1960 - 69 + 96 = 1987$

Correct $\bar{X} = \frac{correct \sum x}{n} = \frac{1987}{25} = 79.48$

Progress Check

There are four numbers. If we leave out any one number, the average of the remaining three numbers will be 45, 60, 65 or 70. What is the average of all four numbers?

Exercise 5.1

1. In a week, temperature of a certain place is measured during winter are as follows 26°C, 24°C, 28°C, 31°C, 30°C, 26°C, 24°C. Find the mean temperature of the week.

2. The mean weight of 4 members of a family is 60kg.Three of them have the weight 56kg, 68kg and 72kg respectively. Find the weight of the fourth member.

3. In a class test in mathematics, 10 students scored 75 marks, 12 students scored 60 marks, 8 students scored 40 marks and 3 students scored 30 marks. Find the mean of their score.

4. In a research laboratory scientists treated 6 mice with lung cancer using natural medicine. Ten days later, they measured the volume of the tumor in each mouse and given the results in the table.

Mouse marking	1	2	3	4	5	6
Tumor Volume(mm^3)	145	148	142	141	139	140

Find the mean.

5. If the mean of the following data is 20.2, then find the value of p

Marks	10	15	20	25	30
No. of students	6	8	p	10	6

6. In the class, weight of students is measured for the class records. Calculate mean weight of the class students using Direct method.

Weight in kg	15-25	25-35	35-45	45-55	55-65	65-75
No. of students	4	11	19	14	0	2

7. Calculate the mean of the following distribution using Assumed Mean Method:

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	5	7	15	28	8

8. Find the Arithmetic Mean of the following data using Step Deviation Method:

Age	15-19	20-24	25-29	30-34	35-39	40-44
No. of persons	4	20	38	24	10	9

5.5 Median

The arithmetic mean is typical of the data because it 'balances' the numbers; it is the number in the 'middle', pulled up by large values and pulled down by smaller values.

Suppose four people of an office have incomes of ₹5000, ₹6000, ₹7000 and ₹8000. Their mean income can be calculated as $\frac{5000+6000+7000+8000}{4}$ which gives ₹6500. If a fifth person with an income of ₹ 29000 is added to this group, then the arithmetic mean of all the five would be $\frac{5000+6000+7000+8000+29000}{5} = \frac{55000}{5}$ = ₹11000. Can one say that the average income of ₹11000 truly represents the income status of the individuals in the office? Is it not misleading? The problem here is that an extreme score affects the Mean and can move the mean away from what would generally be considered the central area.

In such situations, we need a different type of average to provide reasonable answers.

> Median is the value which occupies the middle position when all the observations are arranged in an ascending or descending order. It is a positional average.

For example, the height of nine students in a class are 122 cm, 124 cm, 125 cm, 135 cm, 138 cm, 140cm, 141cm, 147 cm, and 161 cm

(i) Usual calculation gives Arithmetic Mean to be 137 cm.

(ii) If the heights are neatly arranged in, say, ascending order, as follows 122 cm, 124 cm, 125 cm, 135 cm, 138 cm, 140cm, 141cm, 147 cm, 161 cm, one can observe the value 138 cm is such that equal number of items lie on either side of it. Such a value is called the Median of given readings.

(iii) Suppose a data set has 11 items arranged in order. Then the median is the 6th item because it will be the middlemost one. If it has 101 items, then 51st item will be the Median.

If we have an odd number of items, one can find the middle one easily. In general, if a data set has n items and n is odd, then the median will be the $\left(\frac{n+1}{2}\right)^{th}$ item.

(iv) If there are 6 observations in the data, how will you find the Median? It will be the average of the middle two terms. (Shall we denote it as 3.5th term?) If there are 100 terms in the data, the Median will be 50.5th term!

In general, if a data set has n items and n is even, then the Median will be the average of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ items.

Example 5.8 The following are scores obtained by 11 players in a cricket match 7, 21, 45, 12, 56, 35, 25, 0, 58, 66, 29. Find the median score.

Solution

Let us arrange the values in ascending order.

0,7,12,21,25,29,35,45,56,58,66

The number of values = 11 which is odd

$$\text{Median} = \left(\frac{11+1}{2}\right)^{\text{th}} \text{ value}$$

$$= \left(\frac{12}{2}\right)^{\text{th}} \text{ value} = 6^{\text{th}} \text{ value} = 29$$

Example 5.9 For the following ungrouped data 10, 17, 16, 21, 13, 18, 12, 10, 19, 22. Find the median.

Solution

Arrange the values in ascending order.

10, 10, 12, 13, 16, 17, 18, 19, 21, 22.

The number of values = 10

$$\text{Median} = \text{Average of } \left(\frac{10}{2}\right)^{\text{th}} \text{ and } \left(\frac{10}{2}+1\right)^{th} \text{ values}$$

$$= \text{Average of } 5^{\text{th}} \text{ and } 6^{\text{th}} \text{ values}$$

$$= \frac{16+17}{2} = \frac{33}{2} = 16.5$$

Example 5.10 The following table represents the marks obtained by a group of 12 students in a class test in Mathematics and Science.

Marks (Mathematics)	52	55	32	30	60	44	28	25	50	75	33	62
Marks (Science)	54	42	48	49	27	25	24	19	28	58	42	69

Indicate in which subject, the level of achievement is higher?

Solution

Let us arrange the marks in the two subjects in ascending order.

Marks (Mathematics)	25	28	30	32	33	44	50	52	55	60	62	75
Marks (Science)	19	24	25	27	28	42	42	48	49	54	58	69

Since the number of students is 12, the marks of the middle-most student would be the mean mark of 6^{th} and 7^{th} students.

Therefore , Median mark in Mathematics $= \dfrac{44+50}{2} = 47$

Median mark in Science $= \dfrac{42+42}{2} = 42$

Here the median mark in Mathematics is greater than the median mark in Science. Therefore, the level of achievement of the students is higher in Mathematics than Science.

5.5.1 Median-Ungrouped Frequency Distribution

(i) Arrange the data in ascending (or) decending order of magnitude.

(ii) Construct the cumulative frequency distribution. Let N be the total frequency.

(iii) If N is odd, median $= \left(\dfrac{N+1}{2}\right)^{th}$ observation.

(iv) If N is even, median $= \left(\dfrac{\left(\frac{N}{2}\right)^{\text{th}} \text{observation} + \left(\frac{N}{2}+1\right)^{\text{th}} \text{observation}}{2}\right)$

Example 5.11 Calculate the median for the following data:

Height (cm)	160	150	152	161	156	154	155
No. of Students	12	8	4	4	3	3	7

Solution

Let us arrange the marks in ascending order and prepare the following data:

Height (cm)	Number of students (f)	Cumulative frequency (cf)
150	8	8
152	4	12
154	3	15
155	7	22
156	3	25
160	12	37
161	4	41

Here N = 41

Median = size of $\left(\frac{N+1}{2}\right)^{th}$ value = size of $\left(\frac{41+1}{2}\right)^{th}$ value = size of 21^{st} value.

If the 41 students were arranged in order (of height), the 21^{st} student would be the middle most one, since there are 20 students on either side of him/her. We therefore need to find the height against the 21^{st} student. 15 students (see cumulative frequency) have height less than or equal to 154 cm. 22 students have height less than or equal to 155 cm. This means that the 21^{st} student has a height 155 cm.

Therefore, Median = 155 cm

5.5.2 Median - Grouped Frequency Distribution

In a grouped frequency distribution, computation of median involves the following

Steps

(i) Construct the cumulative frequency distribution.

(ii) Find $\left(\frac{N}{2}\right)^{th}$ term.

(iii) The class that contains the cumulative frequency $\frac{N}{2}$ is called the median class.

(iv) Find the median by using the formula:

$$\text{Median} = l + \frac{\left(\frac{N}{2} - m\right)}{f} \times c$$

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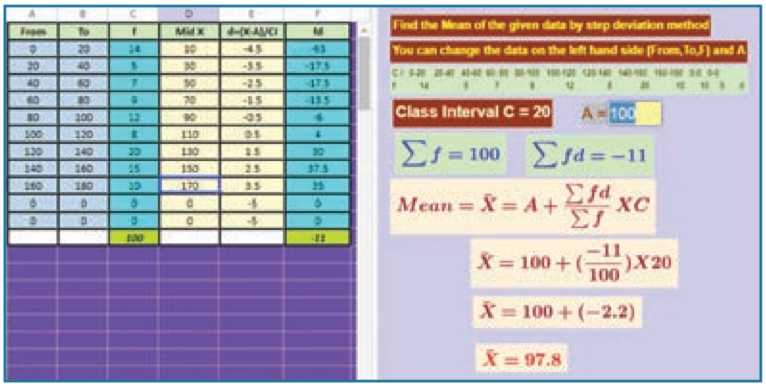
Expected Result is shown in this picture

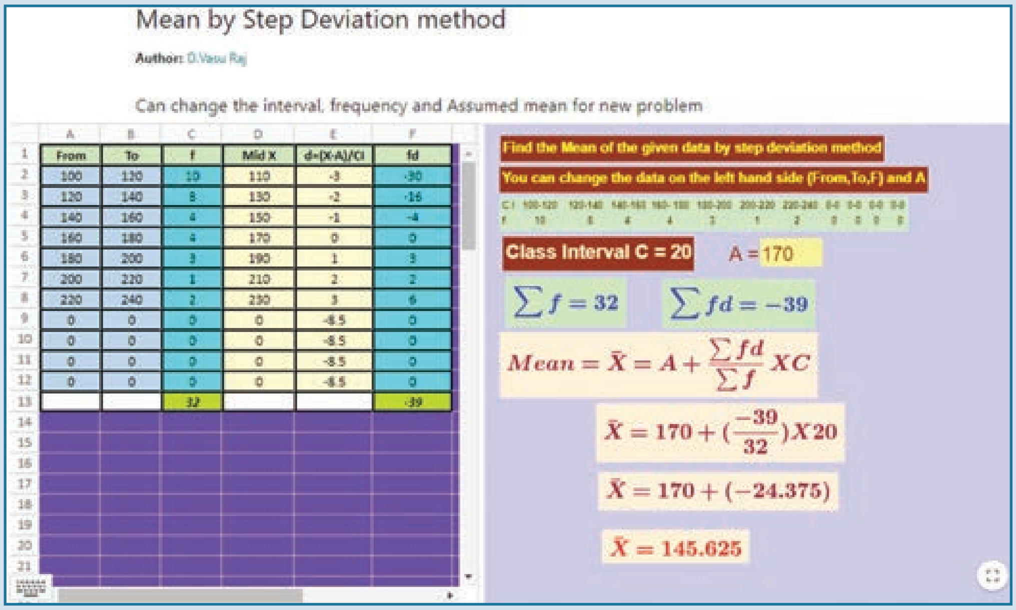


Step

Open the browser, type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Mean by step deviation method" will open.

In the work sheet Example 5.5 is given. Observe the steps. You can change the question by typing new data "From", "To" and "Frequency f" in the spread sheet on Left hand side. After that change, the Assumed mean on the right-hand side and check the calculation.

Step 1



Browse in the link

Mean by step deviation method:
https://ggbm.at/NWcKTRtA or Scan the QR Code

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Where l = *Lower limit of the median class,* f = *Frequency of the median class*

c = *Width of the median class,* N = *The total frequency* $\left(\sum f\right)$

m = *cumulative frequency of the class preceeding the median class*

Example 5.12 The following table gives the weekly expenditure of 200 families. Find the median of the weekly expenditure.

Weekly expenditure (₹)	0-1000	1000-2000	2000-3000	3000-4000	4000-5000
Number of families	28	46	54	42	30

Solution

Weekly Expenditure	Number of families (f)	Cumulative frequency (cf)
0-1000	28	28
1000-2000	46	74
2000-3000	54	128
3000-4000	42	170
4000-5000	30	200
	N=200	

Median class $= \left(\frac{N}{2}\right)^{\text{th}}$ value $= \left(\frac{200}{2}\right)^{\text{th}}$ value

$= 100^{\text{th}}$ value

Median class $= 2000 - 3000$

$\frac{N}{2} = 100 \quad l = 2000$

$m = 74, \quad c = 1000, \quad f = 54$

$$\text{Median} = l + \frac{\left(\frac{N}{2} - m\right)}{f} \times c$$

$$= 2000 + \left(\frac{100 - 74}{54}\right) \times 1000$$

$$= 2000 + \left(\frac{26}{54}\right) \times 1000 = 2000 + 481.5$$

$$= 2481.5$$

Progress Check

1. The median of the first four whole numbers _____.

2. If 4 is also included to the collection, then the difference of the medians in the two cases ____________.

Example 5.13 The Median of the following data is 24. Find the value of x.

Class Interval (*CI*)	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency (*f*)	6	24	x	16	9

Solution

Class Interval (*CI*)	Frequency (*f*)	Cumulative frequency (*cf*)
0-10	6	6
10-20	24	30
20-30	x	$30 + x$
30-40	16	$46 + x$
40-50	9	$55 + x$
	$N = 55 + x$	

Since the median is 24 and median class is 20 – 30

$l = 20 \quad N = 55 + x, \quad m = 30, \quad c = 10, \quad f = x$

$$\text{Median} = l + \frac{\left(\frac{N}{2} - m\right)}{f} \times c$$

$$24 = 20 + \frac{\left(\frac{55 + x}{2} - 30\right)}{x} \times 10$$

$$4 = \frac{5x - 25}{x} \qquad \text{(after simplification)}$$

$$4x = 5x - 25$$

$$5x - 4x = 25$$

$$x = 25$$

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Exercise 5.2

1. Find the median of the given values : 47, 53, 62, 71, 83, 21, 43, 47, 41.
2. Find the Median of the given data: 36, 44, 86, 31, 37, 44, 86, 35, 60, 51
3. The median of observation 11, 12, 14, 18, x+2, x+4, 30, 32, 35, 41 arranged in ascending order is 24. Find the values of x.
4. A researcher studying the behavior of mice has recorded the time (in seconds) taken by each mouse to locate its food by considering 13 different mice as 31, 33, 63, 33, 28, 29, 33, 27, 27, 34, 35, 28, 32. Find the median time that mice spent in searching its food.

5. The following are the marks scored by the students in the Summative Assessment exam

Class	0-10	10-20	20-30	30-40	40-50	50-60
No. of Students	2	7	15	10	11	5

Calculate the median.

6. The mean of five positive integers is twice their median. If four of the integers are 3, 4, 6, 9 and median is 6, then find the fifth integer.

5.6 Mode

(i) The votes obtained by three candidates in an election are as follows:

Name of the Candidate	Votes Polled
Mr. X	4, 12, 006
Mr. Y	9, 87, 991
Mr. Z	7, 11, 973
Total	21, 11, 970

Who will be declared as the winner? Mr. Y will be the winner, because the number of votes secured by him is the highest among the three candidates. Of course, the votes of Mr. Y do not represent the majority population (because there are more votes against him). However, he is declared winner because the mode of selection here depends on the highest among the candidates.

(ii) An Organisation wants to donate sports shoes of same size to maximum number of students of class IX in a School. The distribution of students with different shoe sizes is given below.

Shoe Size	5	6	7	8	9	10
No. of Students	10	12	27	31	19	1

If it places order, shoes of only one size with the manufacturer, which size of the shoes will the organization prefer?

In the above two cases, we observe that mean or median does not fit into the situation. We need another type of average, namely the Mode.

The mode is the number that occurs most frequently in the data.

When you search for some good video about Averages on You Tube, you look to watch the one with maximum views. Here you use the idea of a mode.

Note

Fashion is generally an answer to the question, "What is the mode?"

e.g. "What shirt style would most people want to wear?"

Where there is a grouped frequency table, the group with the greatest frequency is called the modal group.

5.6.1 Mode - Raw Data

For an individual data mode is the value of the variable which occurs most frequently.

Example 5.14 In a rice mill, seven labours are receiving the daily wages of ₹500, ₹600, ₹600, ₹800, ₹800, ₹800 and ₹1000, find the modal wage.

Solution

In the given data ₹800 occurs thrice.Hence the mode is ₹ 800.

Example 5.15 Find the mode for the set of values 17, 18, 20, 20, 21, 21, 22, 22.

Solution

In this example, three values 20, 21, 22 occur two times each. There are three modes for the given data!

Note

- A distribution having only one mode is called unimodal.
- A distribution having two modes is called bimodal.
- A distribution having Three modes is called trimodal.
- A distribution having more than three modes is called multimodal.

5.6.2 Mode for Ungrouped Frequency Distribution

In a ungrouped frequency distribution, the value of the item having maximum frequency is taken as the mode.

Example 5.16 A set of numbers consists of five 4's, four 5's, nine 6's,and six 9's. What is the mode.

Solution

Size of item	4	5	6	9
Frequency	5	4	9	6

6 has the maximum frequency 9. Therefore 6 is the mode.

5.6.3 Mode – Grouped Frequency Distribution:

In case of a grouped frequency distribution, the exact values of the variables are not known and as such it is very difficult to locate mode accurately. In such cases, if the class intervals are of equal width, an appropriate value of the mode may be determined by

$$\text{Mode} = l + \left(\frac{f - f_1}{2f - f_1 - f_2}\right) \times c$$

The class interval with maximum frequency is called the modal class.

Where l - lower limit of the modal class; f - frequency of the modal class

f_1 - frequency of the class just preceding the modal class

f_2 - frequency of the class succeeding the modal class

c - width of the class interval

Example 5.17 Find the mode for the following data.

Marks	1-5	6-10	11-15	16-20	21-25
No. of students	7	10	16	32	24

Solution

Marks	f
0.5-5.5	7
5.5-10.5	10
10.5-15.5	16
15.5-20.5	32
20.5-25.5	24

Note

Convert discrete class interval into contiunous class interval for doing this substract 0.5 to the lower limit and add 0.5 to the upper limit of each class.

Modal class is 16 -20 since it has the maximum frequency.

$l = 15.5$, $f = 32$, $f_1 = 16$, $f_2 = 24$, $c = 20.5 - 15.5 = 5$

$$\text{Mode} = l + \left(\frac{f - f_1}{2f - f_1 - f_2}\right) \times c$$

$$= 15.5 + \left(\frac{32 - 16}{64 - 16 - 24}\right) \times 5$$

$$= 15.5 + \left(\frac{16}{24}\right) \times 5 = 15.5 + 3.33 = 18.83.$$

5.6.4 An Empirical Relationship between Mean, Medan and Mode

We have seen that there is an approximate relation that holds among the three averages we have seen, when the frequencies are nearly symmetrically distributed.

$$\text{Mode} \approx 3\ \text{Median} - 2\ \text{Mean}$$

Example 5.18 In a distribution, the mean and mode are 66 and 60 respectively. Calculate the median.

Solution

Given, Mean = 66 and Mode = 60.

Using, Mode $\approx$ 3Median – 2Mean

$$60 \approx 3\text{Median} - 2(66)$$

$$3\text{ Median} \approx 60 + 132$$

$$\text{Therefore, Median} \approx \frac{192}{3} \approx 64$$

Exercise 5.3

1. The monthly salary of 10 employees in a factory are given below :
₹5000, ₹7000, ₹5000, ₹7000, ₹8000, ₹7000, ₹7000, ₹8000, ₹7000, ₹5000
Find the mean, median and mode.

2. Find the mode of the given data : 3.1, 3.2, 3.3, 2.1, 1.3, 3.3, 3.1

3. For the data 11, 15, 17, x+1, 19, x–2, 3 if the mean is 14 , find the value of x. Also find the mode of the data.

4. The demand of track suit of different sizes as obtained by a survey is given below:

Size	38	39	40	41	42	43	44	45
No. of Persons	36	15	37	13	26	8	6	2

Which size is demanded more?

5. Find the mode of the following data:

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	22	38	46	34	20

6. Find the mean , median and mode of the following distribution:

Weight(in kgs)	25-34	35-44	45-54	55-64	65-74	75-84
Number of students	4	8	10	14	8	6

Project

1. Prepare a frequency table of the top speeds of 20 different land animals. Find mean, median and mode. Justify your answer.

2. From the record of students particulars of the class,
(i) Find the mean age of the class (using class interval)
(ii) Calculate the mean height of the class(using class intervals)

Exercise 5.4

Multiple choice questions

BF4WHW

1. Data available in an unorganized form is called ------------- data

   (1) Grouped data (2) class interval. (3) mode (4) raw data.

2. Let m be the mid point and b be the upper limit of a class in a continuous frequency distribution. The lower limit of the class is

   (1) $2m - b$ (2) $2m+b$ (3) $m-b$ (4) $m-2b$.

3. Which one of the following is not a measure of central tendency?

   (1) Mean (2) Range (3) Median (4) Mode.

4. The mean of a set of seven numbers is 81. If one of the numbers is discarded , the mean of the remaining numbers is 78 . The value of discarded number is

   (1) 101 (2) 100 (3) 99 (4) 98.

5. A particular observation which occurs maximum number of times in a given data is called its

   (1) Frequency (2) range (3) mode (4) Median.

6. For which set of numbers do the mean, median and mode all have the same values?

   (1) 2,2,2,4 (2) 1,3,3,3,5 (3) 1,1,2,5,6 (4) 1,1,2,1,5.

7. The algebraic sum of the deviations of a set of n values from their mean is

   (1) 0 (2) n-1 (3) n (4) n+1.

8. The mean of a,b,c,d and e is 28 . If the mean of a, c and e is 24 , then mean of b and d is_

   (1) 24 (2) 36 (3) 26 (4) 34

9. If the mean of five observations x, x+2 , x+4 , x+6 , x+8 , is 11 , then the mean of first three observations is

   (1) 9 (2) 11 (3) 13 (4) 15.

10. The mean of 5, 9, x, 17, and 21 is 13, then find the value of x

    (1) 9 (2) 13 (3) 17 (4) 21

11. The mean of the square of first 11 natural numbers is

    (1) 26 (2) 46 (3) 48 (4) 52.

12. The mean of the first 10 prime numbers is

(1) 12.6 (2) 12.7 (3) 12.8 (4) 12.9.

13. The median of the first 10 whole numbers is

(1) 4 (2) 4.5 (3) 5 (4) 5.5.

14. The mean of a set of numbers is $\bar{X}$. If each number is multiplied by z, the mean is

(1) $\bar{X}+z$ (2) $\bar{X}-z$ (3) $z\bar{X}$ (4) $\bar{X}$

15. Find the mean of the prime factors of 165.

(1) 5 (2) 11 (3) 13 (4) 55

Points to Remember

- The information collected for a definite purpose is called data.
- The data collected by the investigator are known as primary data. When the information is gathered from an external source,the data are called secondary data.
- Initial data obtained through unorganized form are called Raw data.
- When data are arranged into classes or group we call the data as Grouped data.
- The number of times the observations are repeated is called its Frequency.
- Mid Value $=\dfrac{UCL+LCL}{2}$ (where UCL–Upper Class Limit, LCL–Lower Class Limit).
- Size of the class interval = $UCL - LCL$.
- The mean for grouped data:

Direct Method	Assumed Mean Method	Step-Deviation Method
$\bar{X}=\dfrac{\sum fx}{\sum f}$	$\bar{X}=A+\dfrac{\sum fd}{\sum f}$	$\bar{X}=A+\left[\dfrac{\sum fd}{\sum f}\times c\right]$

- The cumulative frequency of a class is the frequency obtained by adding the frequency of all up to the classes preceeding the given class.
- Formula to find the median for grouped data: Median $=l+\dfrac{\left(\dfrac{N}{2}-m\right)}{f}\times c$.
- Formula to find the mode for grouped data: Mode $=l+\left(\dfrac{f-f_1}{2f-f_1-f_2}\right)\times c$.

ANSWERS

1 Set Language

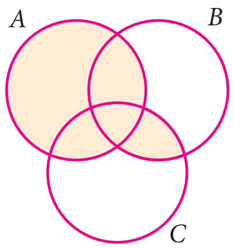
Exercise 1.1

1. (i) {1,2,3,4,5,7,9,11} (ii) {2,5} (iii) {3,5 }

Exercise 1.2

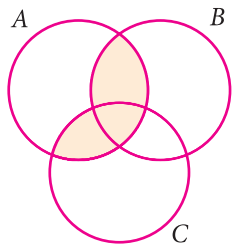
1. (i) $\{a,b,c,d,e,f\}$ (ii) $\{a,b,d\}$ (iii) { a,b,c,d,e,f } (iv) {a, b, d }

2. (i)



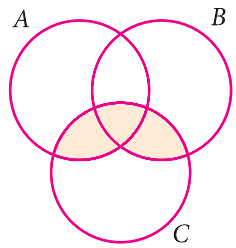
$A\cup(B\cap C)$

(ii)



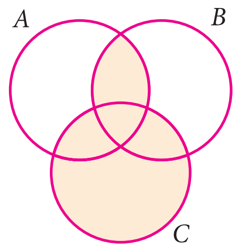
$A\cap(B\cup C)$

(iii)



$(A\cup B)\cap C$

(iv)

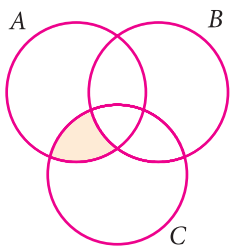


$(A\cap B)\cup C$

Exercise 1.3

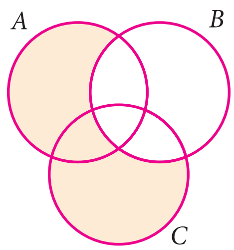
1. (i) $\{3,4,6\}$ (ii) $\{-1,5,7\}$ (iii) $\{-3,0,1,2,3,4,5,6,7,8\}$
(iv) $\{-3,0,1,2\}$ (v) $\{1,2,4,6\}$ (vi) $\{4,6\}$ (vii) $\{-1,3,4,6\}$

2.(i)



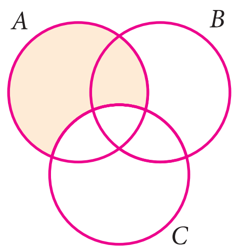
$(A-B)\cap C$

(ii)



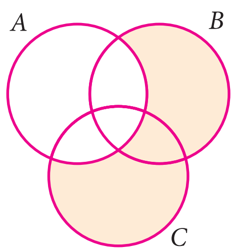
$(A\cup C)-B$

(iii)



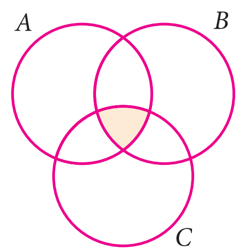
$A-(A\cap C)$

(iv)



$(B\cup C)-A$

(v)



$A\cap B\cap C$

Exercise 1.4

2. (i) 185 (ii) 141 (iii) 326 3.(i) 125
(ii) 695 (iii) 105 4. 70

5. $x = 20$, $y = 40$, $z = 30$ 6. (i) 5   (ii) 7   (iii) 8   7. 5

Exercise 1.5

1.(1) 1   2. (3) $(P - Q) \cup (P - R)$

3.(4) $(A \cap B)' = A' \cup B'$ 4.(2) 10   5.(1) 10

6.(4) ϕ 7. (3) Set of isosceles right triangles.   8.(1) A

9.(3) $Z - (X \cap Y)$ 10.(1) 5

2 Real Numbers

Exercise 2.1

1.(i) 5^4 (ii) 5^{-1} (iii) $5^{\frac{1}{2}}$ (iv) $5^{\frac{3}{2}}$

2.(i) 4^2 (ii) $4^{\frac{3}{2}}$ (iii) $4^{\frac{5}{2}}$ 3.(i) 7

(ii) 9   (iii) 32   (iv) $\frac{1}{27}$ (v) 9

(vi) $\frac{25}{16}$ 4.(i) $5^{\frac{1}{2}}$ (ii) $7^{\frac{1}{2}}$ (iii) $7^{\frac{10}{3}}$

(iv) $10^{\frac{-14}{3}}$ 5.(i) 2   (ii) 3   (iii) 10   (iv) $\frac{4}{5}$

Exercise 2.2

1.(i) $21\sqrt{3}$ (ii) $3\sqrt[3]{5}$ (iii) $26\sqrt{3}$ (iv) $8\sqrt[3]{5}$

2. (i) $\sqrt{30}$ (ii) $\sqrt{5}$ (iii) 30   (iv) $49a - 25b$

(v) $\frac{5}{16}$ 3.(i) 1.852   (ii) 23.978

4. (i) $\sqrt[3]{5} > \sqrt[6]{3} > \sqrt[9]{4}$ (ii) $\sqrt{\sqrt{3}} > \sqrt[2]{\sqrt[3]{5}} > \sqrt[3]{\sqrt[4]{7}}$

5. (i) yes   (ii) yes   (iii) yes   (iv) yes

6. (i) yes   (ii) yes   (iii) yes   (iv) yes

Exercise 2.3

1.(i) $\frac{\sqrt{2}}{10}$ (ii) $\frac{\sqrt{5}}{3}$ (iii) $\frac{5\sqrt{6}}{6}$ (iv) $\frac{\sqrt{30}}{2}$

2. (i) $\frac{4}{3}(5 + 2\sqrt{6})$ (ii) $13 - 4\sqrt{6}$ (iii) $\frac{9 + 4\sqrt{30}}{21}$ (iv) $-2\sqrt{5}$

3. $a = \frac{-4}{3}, b = \frac{11}{3}$ 4. $x^2 + \frac{1}{x^2} = 18$ 5. 5.414

Exercise 2.4

1. (i) 5.6943×10^{11} (ii) 2.00057×10^3 (iii) 6.0×10^{-7} (iv) 9.000002×10^{-4}

2. (i) 3459000 (ii) 56780 (iii) 0.0000100005 (iv) 0.0000002530009

3. (i) 1.44×10^{28} (ii) 8.0×10^{-60} (iii) 2.5×10^{-36} 4.(i) 7.0×10^{9}

(ii) 9.4605284×10^{15} km (iii) $9.1093822 \times 10^{-31}$ kg

5. (i) 1.505×10^{8} (ii) 1.5522×10^{17} (iii) 1.224×10^{7} (iv) 1.9558×10^{-1}

Exercise 2.5

1.(4) $\sqrt{25} = \pm 5$ 2.(4) $\sqrt{13}$ 3.(1) $8\sqrt{10}$ 4.(3) $5\sqrt{3}$

5.(2) 4 6.(2) $8\sqrt{21}$ 7. (3) $\frac{\sqrt{6}}{3}$ 8.(2) $22 - 4\sqrt{10}$

9.(2) $\sqrt[3]{9}$ 10.(4) $\frac{10^6}{3^6}$ 11.(2) $\frac{4}{3}$ 12.(3) 5.367×10^{-3}

13.(2) 0.00592 14.(3) 2×10^{10} m^2

3 Algebra

Exercise 3.1

1. (i) $(x-1)$ is a factor (ii) $(x-1)$ is not a factor

2. $x+2$ is not a factor 3. $(x-5)$ is a factor of $p(x)$

4. $m = 10$ 6. Yes 7. $k = 3$ 8. Yes

Exercise 3.2

1.(i) $4x^2 + 9y^2 + 16z^2 + 12xy + 24yz + 16xz$ (ii) $4a^2 + 9b^2 + 16c^2 - 12ab - 24bc + 16ac$

(iii) $p^2 + 4q^2 + 9r^2 - 4pq + 12qr - 6pr$ (iv) $\frac{a^2}{16} + \frac{b^2}{9} + \frac{c^2}{4} + \frac{ab}{6} + \frac{bc}{3} + \frac{ac}{4}$

2.(i) $x^3 + 15x^2 + 74x + 120$ (ii) $8p^3 - 24p^2 - 14p + 60$

(iii) $27a^3 + 27a^2 - 18a - 8$ (iv) $64m^3 + 64m^2 - 100m - 100$

3.(i) 18,107,210 (ii) −32, −6, +90

4.(i) 14 (ii) $\frac{59}{70}$ (iii) 78 (iv) $\frac{78}{70}$

5.(i) $8a^3 + 27b^3 + 36a^2b + 54ab^2$ (ii) $27x^3 - 64y^3 - 108x^2y + 144xy^2$

(iii) $x^3 + \frac{1}{y^3} + \frac{3x^2}{y} + \frac{3x}{y^2}$ (iv) $a^3 + \frac{1}{a^3} + 3a + \frac{3}{a}$

6.(i) 941192 (ii) 1092727 (iii) 970299 (iv) 1003003001

7. 29 8. 280 9. 335 10. 198 11. ± 5, ± 110

12. 36 13.(i) $8a^3 + 27b^3 + 64c^3 - 72abc$ (ii) $x^3 - 8y^3 + 27z^3 + 18xyz$

14.(i) −630 (ii) 486 (iii) $\frac{-5}{12}$ (iv) $\frac{-9}{4}$

15. $(x+y)(y+z)(x+z)$ 16. $\frac{1}{15}$ 18. 72xyz

Exercise 3.3

1. (i) p^5 (ii) 1 (iii) $3a^2b^2c^3$ (iv) $16x^6$
(v) abc (vi) $7xyz^2$ (vii) 25ab (viii) 1

2. (i) 1 (ii) a^{m+1} (iii) $(2a+1)$ (iv) 1
(v) $(x+1)(x-1)$ (vi) $(a-3x)$

Exercise 3.4

1.(i) $2a^2(1+2b+4c)$ (ii) $(a-m)(b-c)$
(iii) $(p+q)(p+r)$ (iv) $(y+1)(y-1)(2y+1)$ 2.(i) $(x+2)^2$ (ii) $3(a-4b)^2$
(iii) $x(x+2)(x-2)(x^2+4)$ (iv) $\left(m+\frac{1}{m}+5\right)\left(m+\frac{1}{m}-5\right)$
(v) $6(1+6x)(1-6x)$ (vi) $\left(a-\frac{1}{a}+4\right)\left(a-\frac{1}{a}-4\right)$
(vii) $(m^2+3m+1)(m^2-3m+1)$ (viii) $(x^n+1)^2$
(ix) $\left(\frac{a}{\sqrt{3}}-\sqrt{3}\right)^2$ (x) $(a^2+b^2+ab)(a^2+b^2-ab)$ (xi) $(x^2+2y^2+2xy)(x^2+2y^2-2xy)$

3. (i) $(2x+3y+5z)^2$ (ii) $(1+x-3y)^2$ (or) $(-1-x+3y)^2$
(iii) $(-5x+2y+3z)^2$ (or) $(5x-2y-3z)^2$ (iv) $\left(\frac{1}{x}+\frac{2}{y}+\frac{3}{z}\right)^2$

4. (i) $(2x+5y)(4x^2-10xy+25y^2)$ (ii) $(a-9)(a^2+9a+81)$
(iii) $(3x-2y)(9x^2+6xy+4y^2)$ (iv) $(m+8)(m^2-8m+64)$
(v) $(a+2b)(a^2+b^2+ab)$ (vi) $(a+2)(a-2)(a^2+4-2a)(a^2+4+2a)$

5. (i) $(x+2y+3z)(x^2+4y^2+9z^2-2xy-6yz-3xz)$
(ii) $(a+b+1)(a^2+b^2+1-ab-b-a)$
(iii) $(x+2y-1)(x^2+4y^2+1-2xy+2y+x)$
(iv) $(l-2m-3n)(l^2+4m^2+9n^2+2lm-6mn+3ln)$

Exercise 3.5

1.(i) $(x+6)(x+4)$ (ii) $(x-11)(x+9)$
(iii) $(z+6)(z-2)$ (iv) $(x+15)(x-1)$
(v) $(p-8)(p+2)$ (vi) $(t-9)(t-8)$
(vii) $(x-5)(x-3)$ (viii) $(y-20)(y+4)$
(ix) $(a+30)(a-20)$

2. (i) $(2a+5)(a+2)$ (ii) $(11-6m)(m+1)$ (iii) $(2x-5)^2$ (iv) $(-12x-8)(5x-4)$

(v) $(x-7y)(5x+6y)$ (vi) $(2x-3)(4x-3)$ (vii) $2(3x+2y)(x+2y)$ (viii) $-3(4x+3)(x-1)$

(ix) $-1(3a+10)(a-1)$ (x) $3x^2(3y+2)^2$ (xi) $(a+b+6)(a+b+3)$

3. (i) $(p-q-8)(p-q+2)$ (ii) $(18x-9y-13)(2x-y+1)$

(iii) $(m+6n)(m-4n)$ (iv) $\left(a+\sqrt{5}\right)\left(\sqrt{5}a-3\right)$ (v) $(a+1)(a-1)(a^2-2)$

(vi) $m(4m+5n)(2m-3n)$ (vii) $\left(\sqrt{3}x+2\right)\left(4x-\sqrt{3}\right)$

(viii) $(a^2+3a+1)(a^2-3a+1)$ (ix) $\left(a-\frac{1}{a}+4\right)\left(a-\frac{1}{a}-4\right)$

(x) $\left(\frac{1}{x}+\frac{1}{y}\right)^2$ (xi) $\left(\frac{1}{x}+\frac{2}{y}\right)\left(\frac{3}{x}+\frac{2}{y}\right)$

Exercise 3.6

1.(i) $x^2+4x+5,\ 12$ (ii) $(x^2-1),\ -2$

(iii) $x^2+8x+48,\ 253$ (iv) $3x^2-11x+40,\ -125$

(v) $x^2-\frac{2x}{3}-\frac{34}{9},\ \frac{4}{9}$ (vi) $2x^3-\frac{x^2}{2}-\frac{3x}{8}+\frac{51}{32},\ \frac{109}{32}$

2. $4x^3-2x^2+3,\ p=-2,\ q=0,$ remainder=-10

3. $a=20,\ b=94$ & remainder=388

Exercise 3.7

1.(i) $(x-2)(x+3)(x-4)$ (ii) $(x+1)(x-2)(2x-1)$

(iii) $(x-1)(4x^2-x+6)$ (iv) $(x-1)(2x-1)(2x+3)$

(v) $(x+2)(x+3)(x-4)$ (vi) $(x-1)(x-2)(x+3)$

(vii) $(x-1)(x-10)(x+1)$ (viii) $(x-1)(x^2+x-4)$

Exercise 3.8

1.(4) factor 2.(3) $\frac{2}{3}$ 3.(2) $(3x-3)$ 4.(3) $p(3)$

5.(3) $\left(x^3+y^3\right)$ 6.(3) $(x+2)$ 7.(2) $\left(-a-b+c\right)^2$ 8.(2) b,ac

9.(3) 1,2, −15 10.(3) 3 11.(4) 0 12.(3) 1

13.(2) 31 14.(1) a^k 15.(2) $\left(x^2-y^2\right)$ 16.(3) 7

4 Geometry

Exercise 4.1

1. (i) diameter (ii) centre (iii) radius (iv) arc (v) three

2. (i) false (ii) true (iii) true (iv) true (v) false

Exercise 4.2

1. 15cm 2. 24cm 3. 17cm 4. 8cm, 45°, 45°
5. 18cm 6. 14 cm 7. 6 cm

Exercise 4.3

1. (i) 45° (ii) 10° (iii) 55° (iv) 120° (v) 60°

2. $\angle BDC = 25^\circ, \angle DBA = 65^\circ, \angle COB = 50^\circ$

Exercise 4.4

1. 30° 2.(i) $\angle ACD = 55^\circ$ (ii) $\angle ACB = 50^\circ$ (iii) $\angle DAE = 25^\circ$
3. $\angle A = 64^\circ$; $\angle B = 80^\circ$; $\angle C = 116^\circ$; $\angle D = 100^\circ$ 4. 17cm
5.(i) $\angle CAD = 40^\circ$ (ii) $\angle BCD = 80^\circ$ 6. Radius=5cm 7. 3.25m
8. $\angle OAC = 30^\circ$ 9. 5.6m 10. $\angle RPO = 60^\circ$

Exercise 4.7

1.(1) 55° 2.(3) 40cm 3.(1) 80° 4.(1) 30°
5.(4) 10cm 6.(1) 80° 7.(2) 105° 8.(3) 120°
9.(2) 9cm 10.(4) 9cm

5 Statistics

Exercise 5.1

1. $27^\circ C$ 2. 44kg 3. 56.96 (or) 57 (approximately)
4. 142.5 mm3 5. $p = 20$ 6. 40.2 7. 29.29 8. 29.05

Exercise 5.2

1. 47 2. 44 3. 21 4. 32
5. 31 6. 38

Exercise 5.3

1. 6600, 7000, 7000 2. 3.1 and 3.3 (bimodal) 3. 15
4. 40 5. 24 6. 55.9, 56.64, 58.5

Exercise 5.4

1.(4) raw data 2.(1) 2m−b 3.(2) Range 4.(3) 99
5.(3) mode 6.(2) 1, 3, 3, 3, 5 7.(1) 0 8.(4) 34
9.(1) 9 10.(2) 13 11.(2) 46 12.(4) 12.9
13.(2) 4.5 14.(3) $z\overline{X}$ 15.(4) 55

MATHEMATICAL TERMS

Angle	கோணம்
Arithmetic mean	கூட்டுச் சராசரி
Associative property	சேர்ப்புப் பண்பு
Assumed mean	ஊகச் சராசரி
Binomial surds	ஈருறுப்பு முறுடுகள்
Centre	மையம்
Centroid	நடுக்கோட்டுமையம்
Chord	நாண்
Circumference	பரிதி
Commutative property	பரிமாற்றுப் பண்பு
Compound surds	கூட்டு முறுடுகள்
Concentric circle	பொதுமைய வட்டங்கள்
Congruent circle	சர்வசம வட்டங்கள்
Conjugate	இணை
Cyclic Quadrilateral	வட்ட நாற்கரம்
Diameter	விட்டம்
Disjoint sets	வெட்டாக் கணங்கள்
Distributive property	பங்கீட்டுப் பண்பு
Division algorithm	வகுத்தல் படிமுறை
Factor theorem	காரணித் தேற்றம்
Factorisation	காரணிப்படுத்துதல்
Finite set	முடிவுறு கணம்
Frequency table	நிகழ்வெண் பட்டியல்
Greatest Common Divisor	மீப்பெரு பொது வகுத்தி
Grouped data	தொகுக்கப்பட்ட தரவுகள்
Identities	முற்றொருமைகள்
Incentre	உள்வட்ட மையம்
Incircle	உள்வட்டம்
Indices	அடுக்குகள்
Inradius	உள்வட்ட ஆரம்
Irrational number	விகிதமுறா எண்
Major sector	பெரிய வட்டக்கோணப்பகுதி
Measures of central tendency	மையப்போக்கு அளவைகள்
Median	இடைநிலை அளவு
Minor sector	சிறிய வட்டக் கோணப்பகுதி
Mixed surds	கலப்பு முறுடுகள்
Mode	முகடு
Overlapping sets	வெட்டும் கணங்கள்
Primary data	முதல்நிலைத் தரவுகள்
Pure surds	முழுமையான முறுடுகள்
Quadrilateral	நாற்கரம்
Radical	மூலக்குறியீடு
Radicand	மூல அடிமானம்
Rationalisation	விகிதப்படுத்துதல்
Raw data	செப்பனிடப்படாத தரவுகள்
Scientific notation	அறிவியல் குறியீடு
Secondary data	இரண்டாம்நிலைத் தரவுகள்
Sector	வட்டக்கோணப்பகுதி
Segment	வட்டத்துண்டு
Semi-circle	அரைவட்டம்
Set complementation	கண நிரப்பி
Set difference	கண வித்தியாசம்
Set operations	கணச் செயல்கள்
Step-deviation method	படி விலக்க முறை
Surds	முறுடுகள்
Synthetic division	தொகுமுறை வகுத்தல்
Ungrouped data	தொகுக்கப்படாத தரவுகள்
Universal set	அனைத்துக் கணம்
Venn diagram	வென்படம்
Zero / Factor	பூச்சியம் / காரணி

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This book has been printed on 80 GSM
Elegant Maplitho paper
Printed by offset at :