WBBSE Class 10th Maths Question Paper With Solutions 2017

Question 1: Choose the correct option in each case from the following questions: [1 x 6 = 6]

(i) If the ratio of principal and yearly amount be in the ratio 25:28, then the yearly rate of interest is

(a) 3% (b) 12% (c) 10 (5 / 7)% (d) 8%

Answer: (b)

R = SI * 100 / PT
= 28x - 25x * 100 / 25x * 1
= 3x / 25x
= 0.12 * 100
= 12%

(ii) Under what condition one root of the quadratic equation $ax^2 + bx + c = 0$ is zero?

(a) a = 0 (b) b = 0 (c) c = 0 (d) None of these

Answer: (c)

(iii) The number of common tangents of two circles when they do not touch or intersect each other is

(a) 2 (b) 1 (c) 3 (d) 4

Answer: (d)

(iv) If sin θ = cos θ, then the value of 2θ will be

(a) 30° (b) 60° (c) 45° (d) 90°

Answer: (d)

$\sin\theta = \cos\theta$

$\sin\theta = \cos(90 - \theta)$

$\theta = 90^o - \theta$

$\theta = 45^o$

$2\theta = 90^o$

(v) If each of radius of the base and height of a cone is doubled, then the volume of it will be

(a) 3 times (b) 4 times (c) 6 times (d) 8 times of the previous one

Answer: (d)

Let the height of the cone = h cm

The radius of the cone = r cm

Then the volume of the cone = $(1 / 3)\pi r^2 h$

If the height and radius are doubled then,

Height = 2h

Radius = 2r

Then new volume = $(1 / 3)\pi * (2r)^2 * 2h$

$\Rightarrow (1 / 3)\pi * 4r^2 * 2h = 8 * (1 / 3) * \pi r^2 h$

Hence it is 8 times of the old volume.

(vi) The median of the numbers 2, 8, 2, 3, 8, 5, 9, 5, 6 is

(a) 8 (b) 6.5 (c) 5.5 (d) 5

Answer: (d)

Question 2: Fill in the blanks (any five): **[1 x 5 = 5]**

(i) At the same rate per cent per annum, the simple interest and compound interest of the same principal are the same in _____ year. **[1]**
(ii) If in a quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ $b^2 = 4ac$, then the roots of the equation will be real and _____. **[equal]**
(iii) If the length of the sides of two triangles are in proportion, then two triangles are ______. **[similar]**
(iv) If $\cos^2 \theta - \sin^2 \theta = 1 / x$ $(x > 1)$, then $\cos^4 \theta - \sin^4 \theta =$ ______ . **[1 / x]**
(v) The numbers of the plane surface of a solid hemisphere are ______. **[1]**
(vi) If the mean of $x_1, x_2, x_3, x_4 \ldots\ldots x_n$ be the mean, then the mean of $Kx_1, Kx_2, Kx_3 \ldots\ldots Kx_n$ is _____ $(K \neq 0)$. **[k * (mean)]**

Question 3: Write True or False (any five): [1 x 5 = 5]

(i) A starts a business with Rs. 10,000 and B give Rs. 20,000 after 6 months. At the end of the year, their profit will be equal. **[True]**
(ii) If $x + 2\sqrt{3}$, the value of $x + 1 / x$ is $2\sqrt{3}$. **[False]**
(iii) If two circles of radii 7 cm and 3 cm touch each other. externally, then the distance between their centres will be 4 cm. **[False]**
(iv) If $0° < \theta < 90°$, then $\sin (\sin \theta) > \sin^2\theta$. **[True]**
(v) If the total surface area of a hemisphere is 36π sq. cm., then its radius will be 3 cm. **[False]**
(vi) If the perpendicular drawn on the x-axis from the point of intersection of both ogive, the abscissa of the point of intersection of this perpendicular with the x-axis will be the median. **[True]**

Question 4: Answer the following questions (any ten): [2 x 10 = 20]

(i) A sum of money is doubled in 8 years at r% rate of compound interest per annum. At the same rate in how many years will it be four times the sum?

Solution:
$A = P [1 + (r / 100)]^t$
$2P = P [1 + (r / 100)]^8$

$[1 + (r / 100)]^8 = 2$

$1 + (r / 100) = 2^{1/8}$

$4P = P [1 + (r / 100)]^t$

$[1 + (r / 100)]^t = 4$

$(2^{1/8)})^t = 4$

$2^{1/8)t} = 2^2$

$2^{t/8} = 2^2$

$t / 8 = 2$

t = 16 years

(ii) A invests 1½ times more than B invests in a business. At the end of the year, B receives Rs. 1,500 as profit. How much profit A will get at the end of that year?

Solution:

A = 1½ * B

= 3 / 2 * B

= 3 / 2 * 1500

= Rs. 2250

(iii) Without solving, find the values of 'p' for which the equation $x^2 + (p - 3)x + p = 0$ has real and equal roots.

Solution:

$x^2 + (p - 3) x + p = 0$

Here, a = 1, b = (p - 3), c = p

Since, the roots are real and equal, D = 0

$b^2 - 4ac = 0$

$(p - 3)^2 - 4(1) (p) = 0$

$p^2 + 9 - 6p - 4p = 0$

$p^2 - 10p + 9 = 0$

$(p - 1)(p - 9) = 0$

p = 1 or p = 9

(iv) If $x \propto yz$ and $y \propto zx$ show that z is a non−zero constant.

Solution:

$x \propto yz$

$x = k_1yz$

$y \propto zx$

$y = k_2zx$

$y = k_2z(k_1yz)$

$y = k_2k_1yz^2$

$k_2k_1z^2 = 1$

$z^2 = 1 / k_2k_1$

$z = \pm \sqrt{1} / k_2k_1$

So, z is a non zero constant.

(v) The perimeter of two similar triangles is 20 cm and 16 cm respectively. If the length of one side of the first triangle is 9 cm, then find the length of the corresponding side of the second triangle.

Solution:

Let the side of the second triangle be x.

In similar triangles,

Ratios of perimeters = ratios of lengths of sides - (1)

The ratio of perimeter = 20:16 = 5:4 - (2)

The ratio of lengths of sides = 9:x - (3)

From (1), (2) and (3),

5:4 = 9:x

5 / 4 = 9 / x

5x = 36

x = 36 / 5

x = 7.2 cm

(vi) In ABC, $\angle ABC = 90^o$, AB = 5 cm, BC = 12 cm. Find the length of the circumradius of ABC.

Solution:

ABC is a triangle, in which $\angle B = 90°$, AB = 12 cm , BC = 5 cm

BC is the base of the ΔABC, AB is the perpendicular of ΔABC and AC is the hypotenuse of ΔABC.

By Pythagoras theorem,

$H^2 = P^2 + B^2$

$AC^2 = 12^2 + 5^2$

$AC^2 = 144 + 25$

$AC^2 = 169$

$AC = \sqrt{169}$

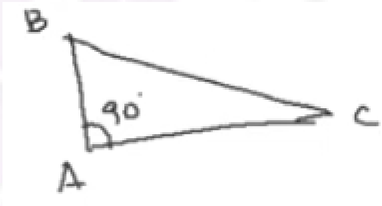
AC = 13

So, the hypotenuse is 13 cm

The hypotenuse is the circumradius of the triangle.

(vii) In ABC, If AB = (2a – 1) cm, AC = $2\sqrt{2}a$ cm and BC = (2a + 1) cm, then write down the value of $\angle ABC$.

Solution:



$AB^2 + AC^2 = BC^2$

$AB^2 = (2a - 1)^2$

$= 4a^2 - 4a + 1$

$AC^2 = (2\sqrt{2a})^2 = 8a$

$AB^2 + AC^2 = 4a^2 - 4a + 1 + 8a = 4a^2 + 4a + 1$

$BC^2 = (2a + 1)^2$

$= 4a^2 + 4a + 1$

Hence, $AB^2 + AC^2 = BC^2$.

So, $\angle ABC = 90^o$

(viii) If $x = a \sec \theta$ and $y = b \tan \theta$, then find the relation between x and y free from θ.

Solution:

$x = a \sec \theta$, $y = b \tan \theta$

$\sec \theta = x / a$

$\tan \theta = y / b$

$\sec^2 \theta - \tan^2 \theta = x^2 / a^2 - y^2 / b^2$

$1 = x^2b^2 - y^2a^2 / a^2b^2$

$a^2b^2 = x^2b^2 - y^2a^2$

(ix) If $\tan (\theta + 15°) = \sqrt{3}$, find the value of $\sin \theta + \cos \theta$.

Solution:

$\tan (\theta + 15°) = \sqrt{3}$

$\tan (\theta + 15°) = \tan 60^o$

$(\theta + 15°) = 60^o$

$\theta = 60^o - 15^o$

$\theta = 45^o$

$\sin 45^o + \cos 45^o$

$= 1 / \sqrt{2} + 1 / \sqrt{2}$

$= 2 / \sqrt{2}$

$= (2 / \sqrt{2}) * (\sqrt{2} / \sqrt{2})$

$= \sqrt{2}$

(x) The diameter of one sphere is double the diameter of another sphere. If the numerical value of the total surface area of the larger sphere is equal to the volume of the smaller sphere, then find the radius of the smaller sphere.

Solution:

$R = 2r$

$4\pi R^2 = [4 / 3]\pi r^3$

$4\pi(2r)^2 = [4 / 3]\pi r^3$

$4r^2 = [1 / 3]r^3$

$4 = (1 / 3)r$
$r = 12$

(xi) If the number of surfaces of a cuboid is x, the number of edges is y, the number of vertices is z and the number of diagonals is P, then find the value of x - y + z + P.

Solution:
$x = 6$
$y = 12$
$z = 8$
$P = 4$
$x - y + z + P$
$= 6 - 12 + 8 + 4$
$= 6 - 12 + 12$
$= 6$

(xii) If 11, 12, 14, x - 2, x + 4, x + 9, 32, 38, 47 are arranged in ascending order and their median is 24, find x.

Solution:
Median = 24
Number of observations = 9
Median = $[(n + 1) / 2]^{th}$ observation
= $[(9 + 1) / 2]^{th}$ observation
= $[10 / 2]^{th}$ observation
= 5^{th} observation
$= x + 4$
Median = $x + 4$
$24 = x + 4$
$x = 24 - 4$
$x = 20$

Question 5: Answer any one question: [5 x 1 = 5]

(i) The difference between simple interest and compound interest for 2 years of a sum of money becomes Rs. 80 at 4% interest per annum. Calculate the sum of money.
(ii) A, B, C start a business jointly investing Rs. 1,80,000 together. A gives Rs. 20,000 more than that of B and B gives Rs 20,000 more than that of C. Distribute the profit of Rs 10, 800 among them.

Solution:
[i] r = 4%
t = 2 years
I = PTR / 100
= P * 2 * 4 / 100
= 8P / 100
= 2P / 25
A = P [1 + (r / 100)]^t
= P [1 + (4 / 100)]^2
= P * (104 / 100)^2
Sum of money = P * (104 / 100)^2 - P
= P [(104 / 100)^2 - 1^2]
= P [(104 / 100) - 1] [(104 / 100) + 1]
= P [(104 + 100 / 100)] [(104 - 100 / 100]
= 204P / 2500
[204P / 2500] - [2P / 25] = 80
[204P - 200P] / 2500 = 80
4P / 2500 = 80
P = 80 * 2500 / 4
= Rs. 50000

[ii] Let the Money invested by C be Rs. x.
Money invested by B is Rs. x + 20000
Money invested by A is Rs. x + 20000 + 20000 = x + 40000
According to the question,

x + (x + 20000) + (x + 40000) = 180000
3x + 60000 = 180000

3x = 180000 - 60000
3x = 120000
x = 120000 ÷ 3
x = 40000
Money invested by A = 40000 + 40000 = 80000
Money invested by B = 40000 + 20000 = 60000
Money invested by C = 40000
So the ratio of money invested by A, B, C is 80000:60000:40000
Profit earned by A is 80000 / 180000 * 10,800 = 4800
Profit earned by B is 60000 / 180000 * 10,800 = 3600
Profit earned by C is 40000 / 180000 * 10800 = 2400

Question 6: Solve any one: **[3 x 1 = 3]**

(i) 1 / [a + b + x] = [1 / a + 1 / b + 1 / x], [x ≠ 0, (a + b)]
(ii) If 5 times a positive whole number is less by 3 than twice of its square, then find the number.

Solution:
(1 / [a + b + x]) = (1 / a) + (1 / b) + (1 / x)
(1 / [a + b + x]) - (1 / x) = (1 / a) + (1 / b)
=> {x - [a + b + x]} / ([a + b + x] * x) = {a + b} / ab
=> - {a + b} / ([a + b + x] * x) = {a + b} / ab
=> -1 / ([a + b + x] * x) = 1 / ab
Cross Multiply : -ab = [a + b + x] * x
On Simplification : x^2 + (a + b) x + ab = 0
Applying the Splitting the middle term method :
=> x^2 + (a + b) x + ab = 0
=> [x^2 + ax]+ [bx + ab] = 0
So : x (x + a) + b (x + a) = 0
=> (x + a) * (x + b) = 0
Therefore : (x + a) = 0 or (x + b) = 0
Now : x = -a or x = -b
The value of x is: -a or -b.

[ii] Let the number be x,
5 times of x = 5x
3 less than twice of the square of x = $2x^2 - 3$
According to the Question,
$5x = 2x^2 - 3$
$2x^2 - 3 - 5x = 0$
$2x^2 - 5x - 3 = 0$
$2x^2 - (6 - 1)x - 3 = 0$
$2x^2 - 6x + x - 3 = 0$
$2x(x - 3) + 1(x - 3) = 0$
$(x - 3)(2x + 1) = 0$
By Zero Product Rule
$x - 3 = 0$ OR $2x + 1 = 0$
$x = 3$ OR $x = -1 / 2$
X can't be negative.
So, the number is 3.

Question 7: [i] Simplify: [1 / √2 + √3] – [√3 + 1 / 2 + √3] + [√2 + 1 / 3 + 2√2].
[ii] The total expenses of a hostel are partly constant and partly vary directly as the number of boarders. When the number of boarders is 120 and 100 the total expenses are Rs. 2,000 and Rs. 1,700, respectively. What will be the number of boarders when the total expenses are Rs. 1,880?

Solution:
[i] $[1 / \sqrt{2} + \sqrt{3}]$
$= 1 / \sqrt{2} + \sqrt{3} * [\sqrt{2} - \sqrt{3} / \sqrt{2} - \sqrt{3}]$
$= \sqrt{2} - \sqrt{3} / (\sqrt{2})^2 - (\sqrt{3})^2$
$= \sqrt{2} - \sqrt{3} / -1$
$= -(\sqrt{2} - \sqrt{3})$
$= -\sqrt{2} + \sqrt{3}$
$[\sqrt{3} + 1 / 2 + \sqrt{3}]$
$= \sqrt{3} + 1 / 2 + \sqrt{3} * [\sqrt{2} - \sqrt{3} / \sqrt{2} - \sqrt{3}]$

= $2\sqrt{3} - 3 + 2 - \sqrt{3} / 2^2 - (\sqrt{3})^2$

= $\sqrt{3} - 1 / 4 - 3$

= $\sqrt{3} - 1$

[$\sqrt{2} + 1 / 3 + 2\sqrt{2}$]

= $\sqrt{2} + 1 / 3 + 2\sqrt{2}$ * [$3 - 2\sqrt{2} / 3 - 2\sqrt{2}$]

= $3\sqrt{2} - 4 + 3 - 2\sqrt{2} / 3^2 - (2\sqrt{2})^2$

= $\sqrt{2} - 1 / 9 - 8$

= $\sqrt{2} - 1$

[$1 / \sqrt{2} + \sqrt{3}$] − [$\sqrt{3} + 1 / 2 + \sqrt{3}$] + [$\sqrt{2} + 1 / 3 + 2\sqrt{2}$]

= $-\sqrt{2} + \sqrt{3} + \sqrt{3} - 1 + \sqrt{2} - 1$

= $-\sqrt{2} + \sqrt{3} - \sqrt{3} + 1$

[ii] $A = k_1 + y$

$y \propto n$

$A = k_1 + k_2 n$ ---- (1)

n = 120

A = 2000

$2000 = k_1 + k_2 * 120$ ---- (2)

n = 100

A = 1700

$1700 = k_1 + k_2 * 100$ ---- (3)

(2) - (3),

$k_1 + 120k_2 = 2000$

$k_1 + 100k_2 = 1700$

____

$20k_2 = 300$

$k_2 = 300 / 20$

$k_2 = 15$

Put $k_2 = 15$ in (3),

$1700 = k_1 + 15 * 100$

$1700 - 1500 = k_1$

$k_1 = 200$

A = 200 + 15n

$1880 = 200 + 15n$
$15n = 1680$
$n = 1680 / 15$
$n = 112$

Question 8: Answer any one question: [3 x 1 = 3]

[i] If a / (b + c) = b / (c + a) = c / (a + b) then prove that each ratio is equal to 1 / 2 or -1.
[ii] If (b + c - a)x = (c + a - b)y = (a + b - c)z = 2, then show that [(1 / x) + (1 / y)] [(1 / y) + (1 / z)] [(1 / z) + (1 / x)] = abc.

Solution:
[i] $a / (b + c) = b / (c + a) = c / (a + b) = k$
$[a + b + c] / (b + c) + (c + a) + (a + b) = k$
$[a + b + c] / b + c + c + a + a + b = k$
$[a + b + c] / 2a + 2b + 2c = k$
$[a + b + c] / 2 [a + b + c] = k$
$k = 1 / 2$
$[a + b + c] = 0$
$a + b = -c$
$c / a + b = k$
$c / -c = k$
$k = -1$
$a / b + c = b / c + a = c / a + b = -1$ or $1 / 2$.

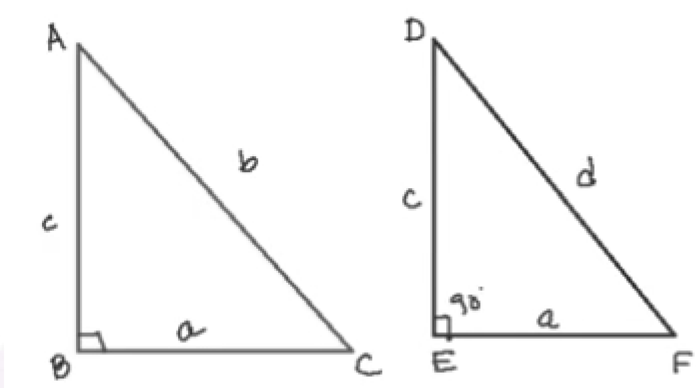
[ii] $(b + c - a)x = 2$
$1 / x = (b + c - a) / 2 = b / 2 + c / 2 - a / 2$
$1 / y = c / 2 + a / 2 - b / 2$
$1 / z = a / 2 + b / 2 - c / 2$
$(1 / x) + (1 / y) = (b / 2 + c / 2 - a / 2) + (c / 2 + a / 2 - b / 2)$
$= c / 2 + c / 2$
$= 2c / 2$
$= c$
Similarly $(1 / y) + (1 / z) = a$ and $(1 / z) + (1 / x) = b$

LHS = [(1 / x) + (1 / y)] [(1 / y) + (1 / z)] [(1 / z) + (1 / x)]
= abc
= RHS

Question 9: Answer any one question: [5 x 1 = 5]

(i) If in a triangle, the area of the square drawn on one side is equal to the sum of the areas of squares drawn on the other two sides, then prove that the angle opposite to the first side will be a right angle.
(ii) If two tangents are drawn to a circle from a point outside it, then the line segments joining the point of contacts and the exterior point are equal.

Solution:
[i]



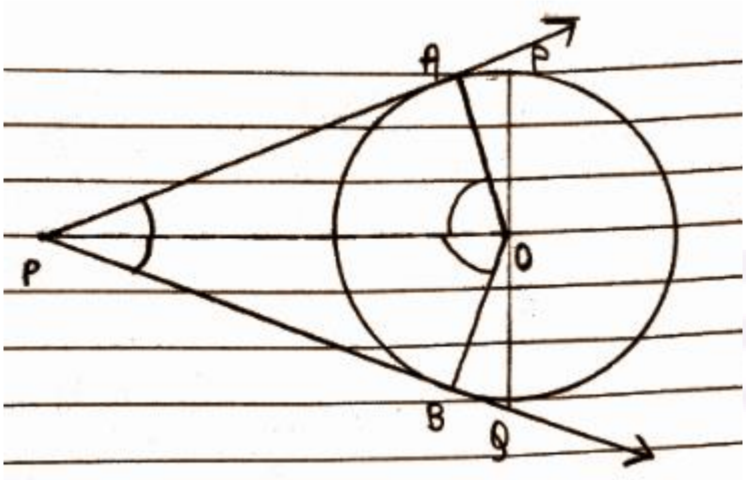
In $\triangle$ABC, AC = b, BC = a, AB = c,
$b^2 = c^2 + a^2$ --- (1)
$\angle ABC = 90^o$
In $\triangle$DEF, DE = AB = c, EF = BC = a, DF = d,
$d^2 = c^2 + a^2$ --- (2)
$b^2 = d^2$
b = d
In $\triangle$ABC and $\triangle$DEF,
DE = AB = c
EF = BC = a
DF = AC
By SSS congruence,

$\triangle ABC \doteqdot \triangle DEF$

$\angle DEF = \angle ABC = 90^o$

So, $\angle ABC = 90^o$

[ii]



In ΔAPO and ΔBPO

OA = OB (radii)

AP = BP (theorem)

OP = OP (common)

$\Delta APO \cong \Delta BPO$ (by SSS congruence)

$\angle AOP = \angle BOP$ (hence, they subtend equal angle at centre P)

$\angle APO = \angle BPO$ (hence, they are equally inclined)

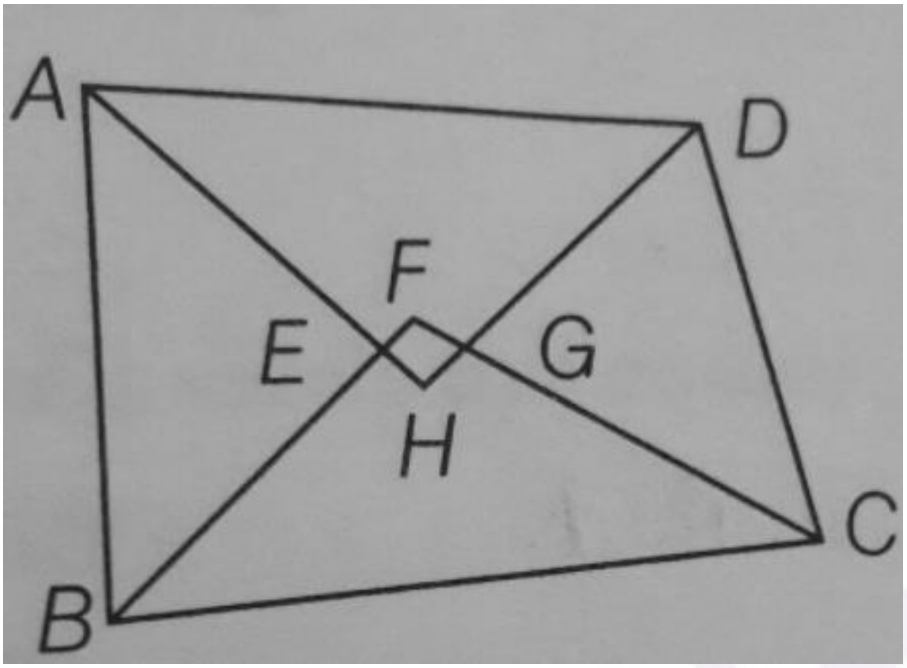
Question 10: Answer any one question: **[3 x 1 = 3]**

(i) Prove that the quadrilateral formed by the internal bisectors of the four angles of a quadrilateral is cyclic.

(ii) O is the circumcentre of $\triangle ABC$ and $OD \perp BC$; prove that $\angle BOD = \angle BAC$.

Solution:

[i]



Let ABCD be a quadrilateral in which the angle bisectors AH, BF, CF & DH of internal $\angle A$, $\angle B$, $\angle C$ & $\angle D$ respectively form a quadrilateral EFGH.

EFGH is a cyclic quadrilateral.

we have to prove that the sum of one pair of opposite angles of a quadrilateral is 180°.

$\angle E + \angle G = 180^\circ$ OR

$\angle F + \angle H = 180^\circ$

In Δ AEB,

$\angle ABE + \angle BAE + \angle AEB = 180^\circ$

$\angle AEB = 180^\circ - \angle ABE - \angle BAE$

$\angle AEB = 180^\circ - (1/2 \angle B + 1/2 \angle A)$

$\angle AEB = 180^\circ - 1/2 (\angle B + \angle A)$ ....(1)

AH & BF are bisectors of $\angle A$ & $\angle B$.

Lines AH and BF intersect.

$\angle FEH = \angle AEB$ (vertically opposite angles)

$\angle FEH = 180^\circ - 1/2 (\angle B + \angle A)$ ……....(2)

Similarly, $\angle FGH = \angle GCD$

$\angle FGH = 180^\circ - (1/2)(\angle C + \angle D)$............... (3)

On adding equations (2) and (3)

$\angle FEH + \angle FGH = 180° - 1/2\ (\angle A + \angle B) + 180° - (1/2)\ (\angle C + \angle D)$

$\angle FEH + \angle FGH = 180° + 180° - (1/2)\ (\angle A + \angle B + \angle C + \angle D)$

$\angle FEH + \angle FGH = 360° - (1/2)\ (\angle A + \angle B + \angle C + \angle D)$

$\angle FEH + \angle FGH = 360° - (1/2) \times 360°$

[$\angle A + \angle B + \angle C + \angle D = 360°$, Sum of angles of Quadrilateral is 360°]

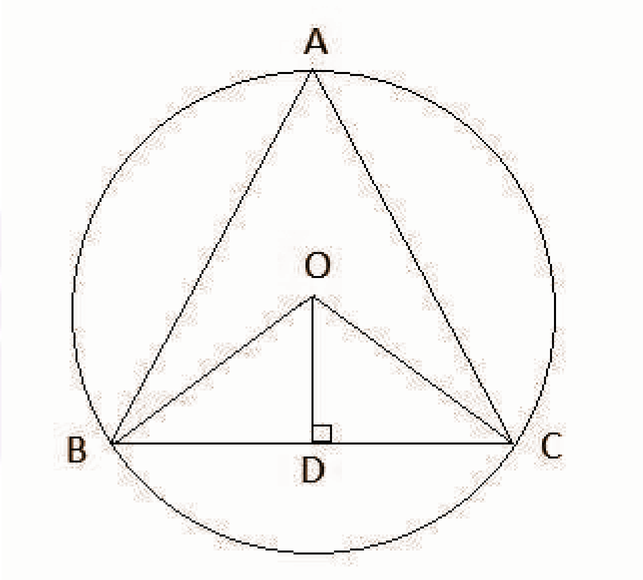
$\angle FEH + \angle FGH = 360° - 180°$

$\angle FEH + \angle FGH = 180°$

Hence, EFGH is a cyclic quadrilateral in which the sum of one pair of opposite angles is 180°.

$\angle FEH + \angle FGH = 180°$

[ii]



O is the circumcenter of ΔABC and OD ⊥ BC

∴ O is the point of intersection of perpendicular bisectors of the sides of ΔABC.

⇒ D is the midpoint of BC

⇒ BD = DC

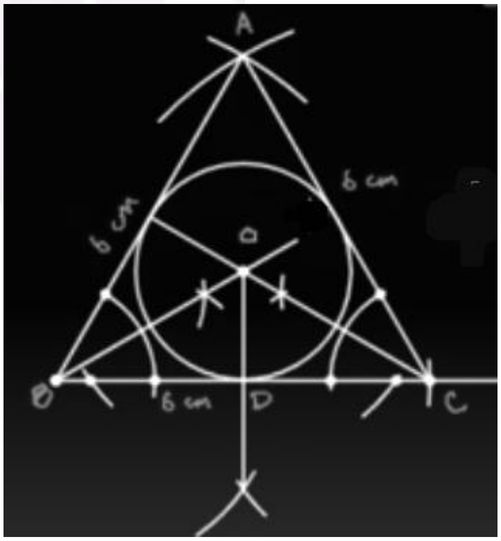
In ΔOBD and ΔOCD,

OB = OC (Radius of circle)
OD = OD (Common)
BD = DC (Proved)
$\Delta OBD \cong \Delta OCD$ (SSS congruence criterion)
$\Rightarrow \angle BOD = \angle COD$ (By CPCT)
$\angle BOD = \angle COD = (1/2) \angle BOC$ ---- (1)
The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
$\Rightarrow \angle BOC = 2\angle BAC$
$\Rightarrow 2\angle BOD = 2\angle BAC$ (From equation (1))
$\Rightarrow \angle BOD = \angle BAC$

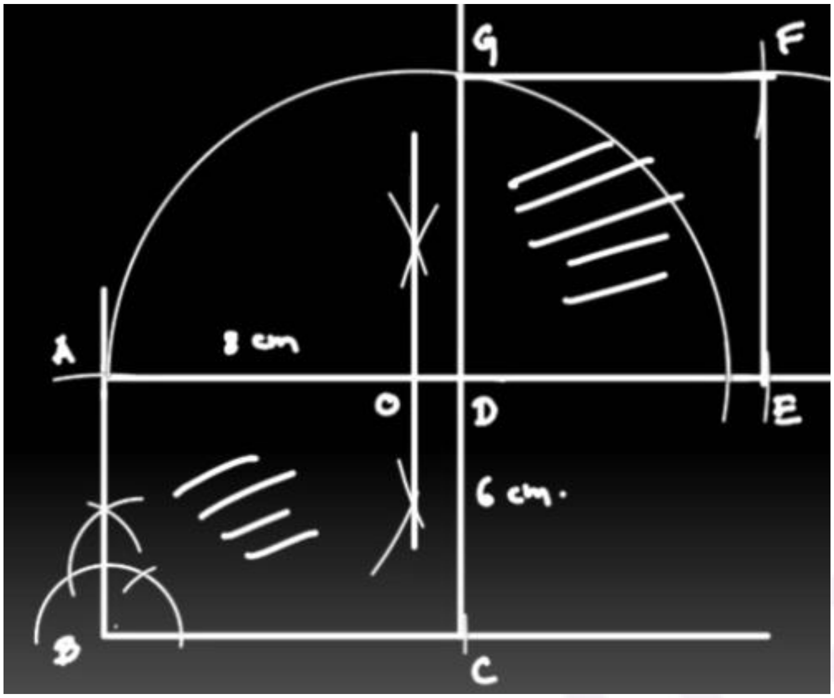
Question 11: Answer any one question: [5 x 1 = 5]

(i) Draw an equilateral triangle of side 6 cm and draw the incircle of the triangle (only traces of construction are required.)
(ii) Construct a rectangle with sides 8 cm and 6 cm and construct a square equal in area to that of the rectangle (only traces of construction are required.)

Solution:
[i]



[ii]



Question 12: Answer any two questions: **[3 x 2 = 6]**

[i] If the measures of three angles of a quadrilateral are π / 3, 5π / 6 and 90º, then determine the fourth angle in sexagesimal and circular measure.
[ii] If sin θ / x = cos θ / y then prove that sin θ – cos θ = [x – y] / √x² + y².
[iii] If tan 9º = a / b, that prove that sec² 81º / 1 + cot² 81º = b² / a².

Solution:
[i] **π** / 3 = 180 / 3 = 60º
5**π** / 6 = 5 * 180 / 6 = 150º
Sum of the angles = 60 + 150 + 90 = 300º
Sum of the angles of a quadrilateral = 360º
Fourth angle = 360º - 300º = 60º
180 = **π**
60 = **π** / 180 * 60
= **π** / 3

[ii] sin θ / x = cos θ / y
sin θ / cos θ = x / y

tan θ = x / y

cos θ = ± y / √x² + y²

sin θ / cos θ = tan θ

sin θ = tan θ * cos θ

LHS = sin θ - cos θ

= tan θ * cos θ - cos θ

= cos θ (tan θ - 1)

= ± y / √x² + y² * (x / y - 1)

= [x − y] / √x² + y²

[iii] tan 9° = a / b

$\sec^2 81^o / 1 + \cot^2 81^o$

$= \sec^2 81^o / \text{cosec}^2 81^o$

$= (1 / \cos^2 81^o) / (1 / \sin^2 81^o)$

$= \tan^2 81^o$

$= \cot (90^o - 81^o)^2$

$= (\cot 9^o)^2$

$= (1 / \tan 9^o)^2$

$= 1 / (a / b)^2$

$= b^2 / a^2$

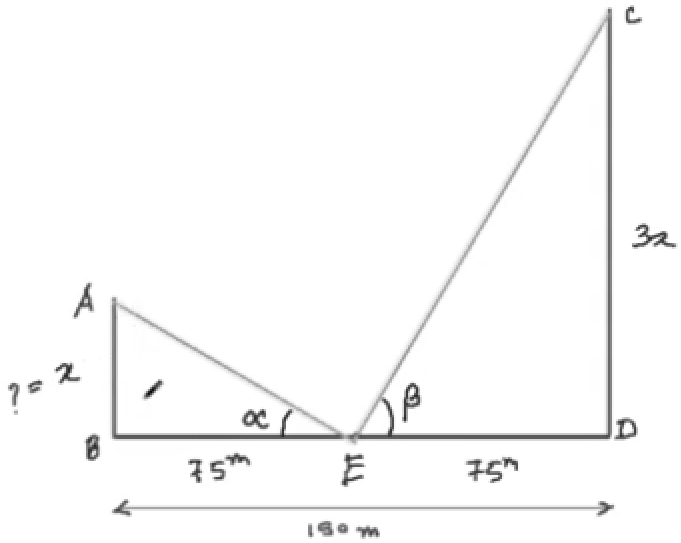
Question 13: Answer any one question: [5 x 1 = 5]

(i) The distance between the two pillars is 150m. Height of one is thrice the other. From the midpoint of the line segment joining the foot of the pillars, the angle of elevation of the top of the pillars is complementary to each other. Find the height of the shorter pillar.

(ii) If the angle of depression from a lighthouse of two ships situated in the same line with the lighthouse is 60° and 30° and if the nearer ship is 150 m away from the lighthouse, then find the distance of the other ship from the lighthouse.

Solution:

[i]



$\alpha + \beta = 90^o$

AB = x m

CD = 3x m

BE = DE = BD / 2 = 150 / 2 = 75 m

In $\triangle$ABE,

$\tan \alpha = AB / BE$

$\tan \alpha = x / 75$

$x = 75 \tan \alpha$

In $\triangle$CDE,

$\tan \beta = CD / DE$

$= 3x / 75$

$= 3 * (x / 75)$

$= 3 * \tan \alpha$

$\tan (90^o - \alpha) = 3 \tan \alpha$

$\cot \alpha = 3 \tan \alpha$

$1 / \tan \alpha = 3 \tan \alpha$

$3 \tan^2 \alpha = 1$

$\tan^2 \alpha = 1 / 3$

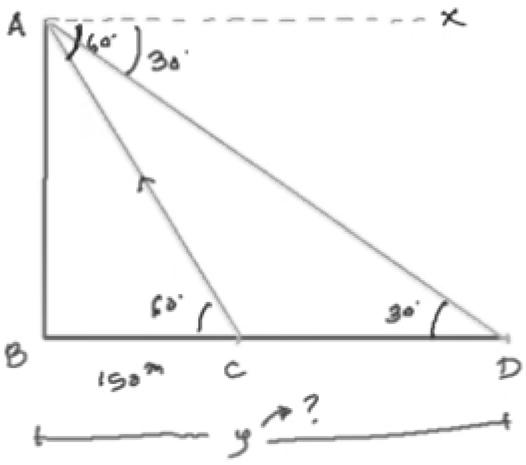
$\tan \alpha = 1 / \sqrt{3}$

$x = 75 \tan \alpha$

$x = 75 * (1 / \sqrt{3})$

x = 25√3 m

[ii]



AB = x m
BC = 150m
BD = y m
AX || BD
∠XAD = ∠BDA = 30°
∠XAC = ∠BCA = 60°
In △ABC,
tan 60° = x / 150
√3 = x / 150
x = 150√3m
In △ABD,
tan 30° = AB / BD
1 / √3 = x / y
y = √3x
y = √3 * (150√3)
y = 450m

Question 14: Answer any two questions: **[4 x 2 = 8]**

(i) Determine the volume of a solid right circular cone which can be made from a solid wooden cube of 4.2 dcm edge length by wasting a minimum quantity of wood.
(ii) A hemispherical bowl with a radius of 9 cm is completely filled with water. How many cylindrical bottles of diameter 3 cm and height 4 cm can be filled, up with the water in the bowl?
(iii) Area of the base of a closed cylindrical water tank is 616 square meter and the height is 21 meter. Find the total surface area of the tank.

Solution:
[i] The volume of the cone should be maximum
∴ The radius of the base of the cone: = edge of cube / 2
= 4.2 / 2
= 2.1 dm
Height of cone = edge of cube = 4.2 dm.
∴ The volume of the cone = [1 / 3] $\pi r^2 h$
= (1 / 3 × 22 / 7 × 2.1 × 2.1 × 4.2) cu.dm.
= 58.1 cu.dm

[ii] Volume of hemispherical bowl = Volume of one cylindrical shaped ball
R = 9cm
r = 3 / 2 cm
h = 4cm
$(2 / 3) \pi R^3 = x * (\pi r^2 h)$
$(2 / 3) * 9^3 = x * (3 / 2)^2 * 4$
486 = x * 9
x = 486 / 9
x = 54 bottles

[iii] Area of base = 616 sq. m
Let the radius be x metre
Area of circle = 616 sq. m
$\pi r^2 = 616$
22 / 7 * r * r = 616

r * r = 616 * 7 / 22
= 28 * 7
=196
r = √196
r = 14 m
Height of cylinder = 21 m
Radius of cylinder = 14 m
Total surface area of cylinder = 2πr (r + h)
= 2 * 22 / 7 * 14 (14 + 21)
= 2 * 22 * 2 * 35
= 88 * 35
= 3080 sq. m

Question 15: Answer any two questions: [2 x 4 = 8]

[i] If the median of the following data is 32, find the values of x and y when the sum of the frequencies is 100.

Class Interval	Frequency
0 - 10	10
10 - 20	x
20 - 30	25
30 - 40	30
40 - 50	y
50 - 60	10

[ii] Find the mode from the following distribution table:

Class Interval	Frequency
0 - 5	5
5 - 10	12

10 - 15	**18**
15 - 20	**28**
20 - 25	**17**
25 - 30	**12**
30 - 35	**8**

[iii] By preparing a cumulative frequency (greater than type) table from the following data, draw an ogive in a graph paper.

Class Interval	**Frequency**
0 - 5	**4**
5 - 10	**10**
10 - 15	**15**
15 - 20	**8**
20 - 25	**3**
25 - 30	**5**

Solution:

[i]

Class Interval	**Frequency**	**CF**
0 - 10	10	10
10 - 20	x	10 + x
20 - 30	25	35 + x
30 - 40	30	65 + x
40 - 50	y	65 + x + y
50 - 60	10	75 + x + y

h / 2 = 50

75 + x + y = 100

x + y = 25 ---- (1)

f = 30

c = 10

CF = 35 + x

L = 30

Median = L + [(N / 2 – CF) / f]* c

32 = 30 + [(50 - 35 - x) / 30] * 10

32 - 30 = 50 - 35 - x / 3

6 - 15 = -x

x = 9

Substitute the value of x in (1),

9 + y = 25

y = 16

[ii] Mode = Z = $L_1 + (F_1 - F_0) / (2F_1 - F_0 - F_2) * i$

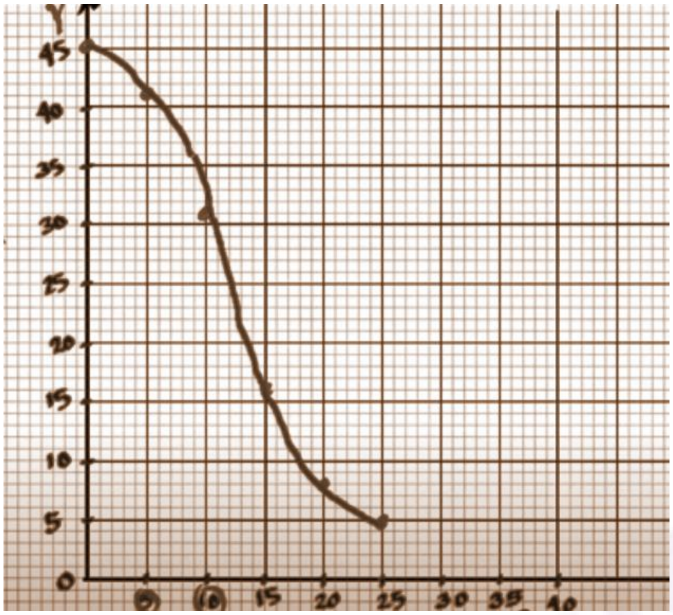
= 15 + [28 - 18] / [2 * 28 - 18 - 17] * 5

= 15 + 10 / 21 * 5

= 15 + 2.38

= 17.38

[iii]



[Alternative Question for Sightless Candidates]

Question 11: Answer any one question: [5 x 1 = 5]

(i) Describe the procedure of drawing the incircle of an equilateral triangle whose side is given.
(ii) Describe the procedure of construction of a square of the equal area of a rectangle whose sides are given.

Solution:
An example of drawing the incircle of an equilateral triangle whose side is 8 cm is explained below.
[i] An equilateral triangle ABC in which AB = BC = CA = 8cm.
Construction of $\triangle$ABC
(1) Draw a line segment BC = 8cm
(2) Draw an arc of radius = 8cm taking B as the centre.
(3) Draw another arc of radius = 8cm taking B as centre
(4) Draw another arc of radius = 8cm taking C as the centre with intersects the previous arc at a point A.
(5) Join AB and AC.
Thus the required ABC is constructed.

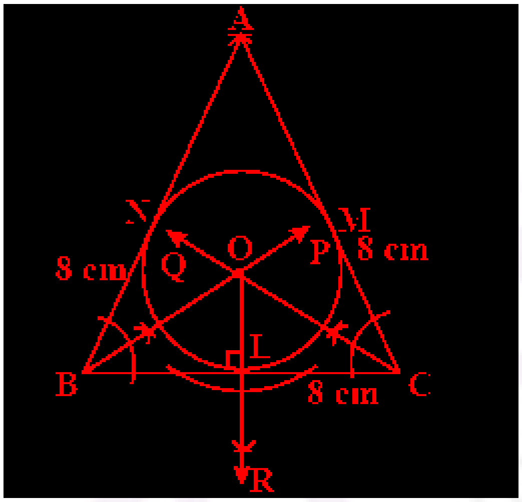
Construction of incircle of $\triangle ABC$

(1) Draw BP and CQ the bisectors of angles, $\angle B$ and $\angle C$ respectively and which intersect each other at point C.

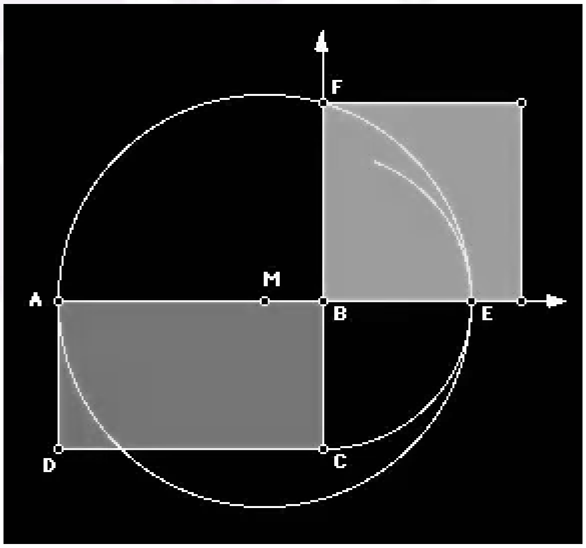
(2) Draw OR perpendicular to BC which intersects BC at L.

(3) Taking O as the centre and OL as radius draw a circle which touches the sides AB, BC and CA at points N, L and M, respectively.

(4) Thus the required incircle is constructed.



[ii]



- Start with rectangle ABCD.
- Extend side AB.

- Draw an arc with B as the centre and with radius BC. Let E be the intersection of the arc and ray AB.
- Construct the midpoint M of segment AE.
- Construct a circle with centre at M with radius MA.
- Extend side CB.
- Let F be the point where the circle and ray CB intersect.
- Then BF is the side of the desired square.
- Complete by constructing a square with side BF.

Question 16: (a) Answer any three questions: [3 x 2 = 6]

(i) If x ∝ y, y ∝ z and z ∝ x then find the relation between the constants of variations.

(ii) If in the case of compound interest, the rate of interest in 1st, 2nd and 3rd year are r_1%, r_2% and r_3%, respectively, then find the amount after 3 years for Rs P.

(iii) If √2 sin (2x + 5°) = tan 45°, find the value of sec 3x.

(iv) What is the ratio of the surface area of a solid sphere and a solid hemisphere of equal radius?

Solution:

[i] x ∝ y, y ∝ z and z ∝ x

It can be written as,

x = ky, y = mz and z = nx where k, m and n are constants of variations.

Put the value of y in x,

x = kmz

Now put the value of z in x,

⇒ x = kmnx

⇒ kmn = 1

[ii] Amount = P * [1 + (r_1 / 100)] * [1 + (r_2 / 100)] * [1 + (r_3 / 100)]

[iii] √2 sin (2x + 5°) = tan 45°

√2 sin (2x + 5°) = 1

$\sin(2x + 5°) = 1 / \sqrt{2}$
$(2x + 5^o) = \sin^{-1}(1 / \sqrt{2})$
2x + 5 = 45
2x = 40
x = 40 / 2
x = 20
sec (3 * 20) = sec 60^o = 2

[iv] The radius of both the hemisphere and sphere be 'r'.
The surface area of the sphere = $[4 / 3]\pi r^3$
The surface area of the hemisphere = $[2 / 3]\pi r^3$
The ratio of the total surface area of a sphere and a solid hemisphere of the same radius
= $[4 / 3]\pi r^3 / [2 / 3]\pi r^3$
= 2:1

(b) Answer any four questions:

(i) If (a + 2b) : (3a - 2b) = 9 : 13, then find the value of a : b.
(ii) What should be subtracted from $\sqrt{72}$ to get $\sqrt{32}$?
(iii) What is the length of the tangent drawn from a point at a distance of 13 cm from the centre of a circle whose radius is 5 cm?
(iv) Find the mean proportion of 3 and 12.
(v) Justify that the roots of the equation $4x^2 - 4x + 1 = 0$ are real and equal.

Solution:
[i] (a + 2b) : (3a - 2b) = 9 : 13
a + 2b / 3a - 2b = 9 / 13
13 * (a + 2b) = 9 * (3a - 2b)
13a + 26b = 27a - 18b
44b = 14a
a = 44 / 14 * b
44 = 14 (a / b)
44 / 14 = a / b

44:14 = a:B

[ii] Let the number be x

$\sqrt{72} - x = \sqrt{32}$

$6\sqrt{2} - x = 4\sqrt{2}$

$x = 6\sqrt{2} - 4\sqrt{2}$

$= \sqrt{2}\,[6 - 4]$

$= 2\sqrt{2}$

[iii] A tangent is drawn from P which touches the circle at T.
Length of the tangent is PT and the centre of the circle is O.
PO = 13cm , OT = radius = 5cm
The tangent is perpendicular to the radius of the circle.
ΔPOT is a right-angled triangle.

$PT^2 + OT^2 = OP^2$

$PT^2 + 5^2 = 13^2$

$PT^2 = 169 - 25 = 144$

PT = 12cm

The length of tangent = 12cm.

[iv] Let the mean proportion between 12 and 3 be a.

=> $12 / a = a / 3$

=> $12 \times 3 = a \times a$

=> $4 \times 3 \times 3 = a^2$

=> $6 \times 6 = a^2$

=> $6 = a$

Therefore, the mean proportion between 12 and 3 is 6.

[v] $4x^2 - 4x + 1 = 0$

On comparing the given equation to $ax^2 + bx + c = 0$,

a = 4, b = - 4, c = 1

The condition for the roots to be real and equal is $b^2 - 4ac = 0$.

$(-4)^2 - 4 * 4 * 1 = 16 - 16 = 0$

So, the roots are real and equal.