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CBSE Class 10 Maths Question Paper Solution 2020
Set 430/2/1

QUESTION PAPER CODE 430/2/1
EXPECTED ANSWER/VALUE POINTS

SECTION A

Question numbers 1 to 10 are multiple choice questions of 1 mark each. Select the correct option.

1. **HCF of two numbers is 27 and their LCM is 162. If one of the number is 54, then the other number is**

(a) 36 **(b) 35** **(c) 9** **(d) 81**

Sol. (d) 81 1

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2. **The cumulative frequency table is useful in determining**

(a) Mean **(b) Median** **(c) Mode** **(d) All of these**

Sol. (b) Median 1

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3. **In Fig. 1, O is the centre of circle. PQ is a chord and PT is tangent at P which makes an angle of 50° with PQ. ∠POQ is**

(a) 130° **(b) 90°** **(c) 100°** **(d) 75°**

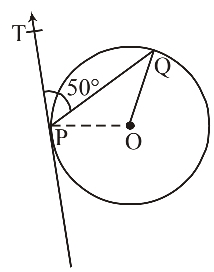

Fig. 1

Sol. (c) 100° 1

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4. **$2\sqrt{3}$ is**

(a) an integer **(b) a rational number**

(c) an irrational number **(d) a whole number**

Sol. (c) an irrational no. 1

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5. **Two coins are tossed simultaneously. The probability of getting at most one head is**

(a) $\frac{1}{4}$ **(b) $\frac{1}{2}$** **(c) $\frac{2}{3}$** **(d) $\frac{3}{4}$**

Sol. (d) $\frac{3}{4}$ 1

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6. **If one zero of the polynomial ($3x^2 + 8x + k$) is the reciprocal of the other, then value of k is**

(a) 3 **(b) –3** **(c) $\frac{1}{3}$** **(d) $-\frac{1}{3}$**

Sol. (a) 3 1

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7. **The decimal expansion of $\frac{23}{2^5 \times 5^2}$ will terminate after how many places of decimal?**

(a) 2 **(b) 4** **(c) 5** **(d) 1**

Sol. (c) 5 1

---

8. **The maximum number of zeroes a cubic polynomial can have, is**

(a) 1 **(b) 4** **(c) 2** **(d) 3**

Sol. (d) 3 1

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9. **The distance of the point (–12, 5) from the origin is**

(a) 12 **(b) 5** **(c) 13** **(d) 169**

Sol. (c) 13 1

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10. **If the centre of a circle is (3, 5) and end points of a diameter are (4, 7) and (2, y), then the value of y is**

(a) 3 **(b) –3** **(c) 7** **(d) 4**

Sol. (a) 3 1

Question numbers 11 to 15, fill in the blanks:

11. **The area of triangle formed with the origin and the points (4, 0) and (0, 6) is ________.**

Sol. 12 sq units 1

OR

The co-ordinate of the point dividing the line segment joining the points A(1, 3) and B(4, 6) in the ratio 2 : 1 is ________.

Sol. (3, 5) 1

---

12. **Value of the roots of the quadratic equation, $x^2 - x - 6 = 0$ are ________.**

Sol. 3 and –2 1

---

13. **If $\sin \theta = \frac{5}{13}$, then the value of $\tan \theta$ is ________.**

Sol. $\tan \theta = \frac{5}{12}$ 1

---

14. **The value of ($\tan^2 60° + \sin^2 45°$) is ________.**

Sol. $\frac{7}{2}$ or 3.5 — 1

---

15. **The corresponding sides of two similar triangles are in the ratio 3 : 4, then the ratios of the area of triangles is _________.**

Sol. 9 : 16 — 1

Question numbers 16 to 20, answer the following :

16. **Find the value of (cos 48° – sin 42°).**

Sol. cos 48° – cos (90 – 42°) — $\frac{1}{2}$

cos 48° – cos 48° = 0 — $\frac{1}{2}$

OR

Evaluate: (tan 23°) × (tan 67°)

Sol. tan (90 – 67°) × tan 67° — $\frac{1}{2}$

cot 67° × tan 67° — $\frac{1}{2}$

= 1

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17. **In figure-2 $\overset{\frown}{PQ}$ and $\overset{\frown}{AB}$ are two arcs of concentric circles of radii 7 cm and 3.5 cm resp., with centre O. If ∠POQ = 30°, then find the area of shaded region.**

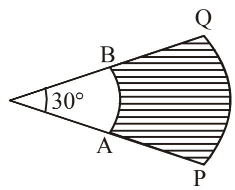

Fig.-2

Sol. Area of shaded region $= \frac{22}{7} \times \frac{30°}{360°}(7^2 - (3.5)^2)$ — $\frac{1}{2}$

$= 9.625 \text{ cm}^2$ — $\frac{1}{2}$

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18. **A card is drawn at random from a well shuffled deck of 52 playing cards. What is the probability of getting a black king?**

Sol. P(Black king) = $\frac{2}{52}$ or $\frac{1}{26}$ — 1

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19. **A ladder 25 m long just reaches the top of a building 24 m high from the ground. What is the distance of the foot of ladder from the base of the building?**

Sol. Distance $= \sqrt{(25)^2 - (24)^2} = 7 \text{ m}$ — $\frac{1}{2} + \frac{1}{2}$

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20. If 3k – 2, 4k – 6 and k + 2 are three consecutive terms of A.P., then find the value of k.

Sol. $(4k - 6) - (3k - 2) = (k + 2) - (4k - 6)$ $\frac{1}{2}$

$\Rightarrow k = 3$ $\frac{1}{2}$

SECTION B

Question numbers 21 to 26 carry 2 marks each.

21. In a lottery, there are 10 prizes and 25 blanks. What is the probability of getting a prize?

Sol. Total = 10 + 25 = 35, $P(\text{getting prize}) = \frac{10}{35}$ or $\frac{2}{7}$ 1+1

22. In a family of three children, find the probability of having at least two boys.

Sol. Total outcomes = 8 {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG} 1

$P(\text{atleast 2 boys}) = \frac{4}{8}$ or $\frac{1}{2}$ 1

OR

Two dice are tossed simultaneously. Find the probability of getting

(i) an even number on both dice.

(ii) the sum of two numbers more than 9.

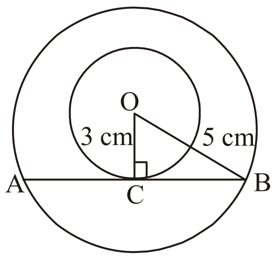
Total outcomes = 36 1

$P(\text{even no. on both side}) = \frac{9}{36}$ or $\frac{1}{4}$ $\frac{1}{2}$

$P(\text{sum} > 9) = \frac{6}{36}$ or $\frac{1}{6}$ $\frac{1}{2}$

23. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of larger circle which touches the smaller circle.

Sol.



In ΔOCB Fig. $\frac{1}{2}$

$BC = \sqrt{5^2 - 3^2} = 4$ cm 1

$AB = 2 \times BC = 8$ cm $\frac{1}{2}$

24. Prove that: $\dfrac{1}{1+\sin\theta}+\dfrac{1}{1-\sin\theta} = 2\sec^2\theta$

Sol. $\text{L.H.S} = \dfrac{1}{1+\sin\theta}+\dfrac{1}{1-\sin\theta} = \dfrac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$ 1

$$= \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta}$$ $\frac{1}{2}$

$$= 2\sec^2\theta$$ $\frac{1}{2}$

OR

Prove that: $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta - \sin^2\theta$

Sol. $\text{L.H.S} = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{1+\dfrac{\sin^2\theta}{\cos^2\theta}} = \dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}$ 1

$$= \cos^2\theta - \sin^2\theta$$ 1

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25. The wheel of a motorcycle is of radius 35 cm. How many revolutions are required to travel a distance of 11 m?

Sol. Distance in 1 revolution $= 2\times\dfrac{22}{7}\times 35 = 220\text{ cm}$ 1

No. of revolution $= \dfrac{1100}{220} = 5$ 1

---

26. Divide $(2x^2 - x + 3)$ by $(2 - x)$ and write the quotient and the remainder.

Sol.

$$\begin{array}{r|l} & -2x-3 \\ \hline -x+2 & 2x^2 - x + 3 \\ & 2x^2 - 4x \\ & - \quad + \\ \hline & 3x + 3 \\ & 3x - 6 \\ & - \quad + \\ \hline & 9 \end{array}$$ 1

Quotient $= -2x - 3$

$R = 9$ 1

SECTION C

Question numbers 27 to 34 carry 3 marks each.

27. If α and β are the zeroes of the polynomial $f(x) = 5x^2 - 7x + 1$, then find the value of $\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$.

Sol. $\alpha + \beta = \frac{7}{5}$ and $\alpha\beta = \frac{1}{5}$ $\quad \frac{1}{2} + \frac{1}{2}$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$ 1

$$= \frac{\left(\frac{7}{5}\right)^2 - 2 \times \frac{1}{5}}{\frac{1}{5}}$$ $\frac{1}{2}$

$$= \frac{39}{5} \text{ or } 7.8$$ $\frac{1}{2}$

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28. Draw a line segment of length 7 cm and divide it in the ratio 2 : 3.

Sol. Correct construction 3

OR

Draw a circle of radius 4 cm and construct the pair of tangents to the circle from an external point, which is at a distance of 7 cm from its centre.

Sol. Correct construction 3

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29. The minute hand of a clock is 21 cm long. Calculate the area swept by it and the distance travelled by its tip in 20 minutes.

Sol. Angle in 20 min = 120° $\quad \frac{1}{2}$

$$\text{Area} = \frac{22}{7} \times \frac{120°}{360°} \times (21)^2 = 462 \text{ cm}^2$$ $1 + \frac{1}{2}$

$$\text{Distance} = \frac{120°}{360°} \times 2\pi r = 44 \text{ cm}$$ 1

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30. If $x = 3\sin\theta + 4\cos\theta$ and $y = 3\cos\theta - 4\sin\theta$ then prove that $x^2 + y^2 = 25$.

Sol. $x^2 = 9\sin^2\theta + 16\cos^2\theta + 24\sin\theta\cos\theta$ 1

$y^2 = 9\cos^2\theta + 16\sin^2\theta - 24\sin\theta\cos\theta$ 1

$x^2 + y^2 = 25$ 1

OR

If $\sin\theta + \sin^2\theta = 1$; then prove that $\cos^2\theta + \cos^4\theta = 1$.

Sol. $\sin\theta = 1 - \sin^2\theta = \cos^2\theta$ 1

L.H.S $= \cos^2\theta + (\cos^2\theta)^2 = \cos^2\theta + \sin^2\theta$ 1+1

$= 1 =$ R.H.S

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31. Prove that $\sqrt{3}$ is an irrational number.

Sol. Let $\sqrt{3}$ be a rational number

$\sqrt{3} = \frac{p}{q}$ p, q are coprime, $q \neq 0$ $\frac{1}{2}$

$3q^2 = p^2 \Rightarrow 3 \mid p^2 \Rightarrow 3 \mid p$ Let $p = 3m$ 1

$3q^2 = 9m^2 \Rightarrow q^2 = 3m^2 \Rightarrow 3 \mid q^2 \Rightarrow 3 \mid q$ $\frac{1}{2}$

$\therefore$ 3 is common factor of p and q

Contraction to our assumption 1

Hence $\sqrt{3}$ is irrational No.

OR

Using Euclid's algorithm, find the HCF of 272 and 1032.

Sol. $1032 = 272 \times 3 + 216$

$272 = 216 \times 1 + 56$ $\frac{1}{2}+\frac{1}{2}$

$216 = 56 \times 3 + 48$

$56 = 48 \times 1 + 8$ $\frac{1}{2}+\frac{1}{2}$

$48 = 8 \times 6 + 0$ HCF(1032, 272) = 8 $\frac{1}{2}+\frac{1}{2}$

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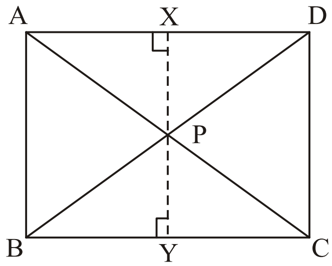
32. In a rectangle ABCD, P is any interior point. Then prove that $PA^2 + PC^2 = PB^2 + PD^2$.

Sol.



Correct figure & Construction $\frac{1}{2}+\frac{1}{2}$

In rt ΔAPX $AP^2 = AX^2 + PX^2$

In rt ΔPCY $PC^2 = PY^2 + YC^2$ $\frac{1}{2}$

In rt ΔPBY $PB^2 = PY^2 + BY^2$

In rt ΔPXD $PD^2 = DX^2 + PX^2$ $\frac{1}{2}$

$PA^2 + PC^2 = AX^2 + PX^2 + PY^2 + YC^2$

$= BY^2 + PY^2 + PX^2 + XD^2$ 1

$= PB^2 + PD^2$

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33. In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. 3. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.

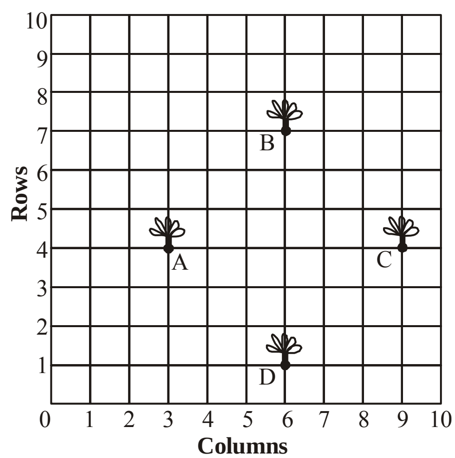

Fig. 3

Sol. A = (3, 4), B = (6, 7), C = (9, 4), D = (6, 1) 1

$AB = 3\sqrt{2}$, $BC = 3\sqrt{2}$, $CD = 3\sqrt{2}$, $DA = 3\sqrt{2}$ 1

AC = 6 unit BD = 6 unit $\frac{1}{2}$

AB = BC = CD = DA and AC = BD

ABCD is a square

$\therefore$ Champa is correct $\frac{1}{2}$

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34. Solve graphically:

$2x - 3y + 13 = 0$; $3x - 2y + 12 = 0$

Sol. Correct graph of $2x - 3y + 13 = 0$, $3x - 2y + 12 = 0$ 1+1

Solution $x = -2$, $y = 3$ 1

SECTION D

Question numbers 35 to 40 carry 4 marks each.

35. The product of two consecutive positive integers is 306. Find the integers.

Sol. Let two consecutive integers x, x + 1

$x(x + 1) = 306 \Rightarrow x^2 + x - 306 = 0$ 1

$\Rightarrow$ $(x + 18)(x - 17) = 0$ 1

$\Rightarrow$ $x = -18$, (Rejected), 17 1

$\therefore$ Two consecutive integers 17, 18 1

36. The 17th term of an A.P. is 5 more than twice its 8th term. If 11th term of A.P. is 43; then find its nth term.

Sol. $a_{17} = 2a_8 + 5 \Rightarrow a + 16d = 2(a + 7d) + 5$ 1

$\Rightarrow$ $2d - a = 15$ ...(1)

$a_{11} = 43 \Rightarrow a + 10d = 43$ ...(2) 1

Solving (1) & (2) $a = 3$ $d = 4$ 1

$a_n = 4n - 1$ 1

OR

How many terms of A.P. 3, 5, 7, 9, ... must be taken to get the sum 120?

Sol. $a = 3$, $d = 3$, $Sn = 120$ 1

$\frac{n}{2}[2 \times 3 + (n - 1)2] = 120 \Rightarrow n^2 + 2n - 120 = 0$ 1

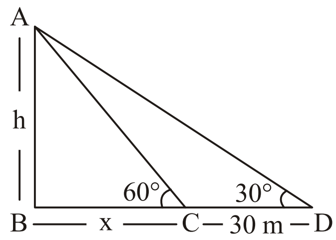
$(n + 12)(n - 10) = 0$ 1

$n = -12$, $n = 10$ 1

Reject $n = -12$, $n = 10$

37. **A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on opposite bank is 60°. When he moves 30 m away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and width of the river. [Take $\sqrt{3}$ = 1.732]**

Sol.



Correct figure 1

In right ΔABC

$\tan 60° = \frac{h}{x}$ $\frac{1}{2}$

$\sqrt{3}x = h$ ...(1) $\frac{1}{2}$

In rt ΔABD $\tan 30° = \frac{h}{30+x} \Rightarrow \frac{30+x}{\sqrt{3}} = h$ ...(2) $\frac{1}{2}+\frac{1}{2}$

Solving (1) & (2) $x = 15\text{m}$, $h = 15\sqrt{3}\text{ m} = 25.98\text{ m}$ $\frac{1}{2}+\frac{1}{2}$

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38. **Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.**

Sol. Correct Fig., given, to prove, construction $4\times\frac{1}{2}=2$

Correct proof given, to prove, construction, 2

OR

Prove that the length of tangents drawn from an external point to a circle are equal.

Correct Fig., given, to prove, construction $4\times\frac{1}{2}=2$

Correct proof given, to prove, construction, 2

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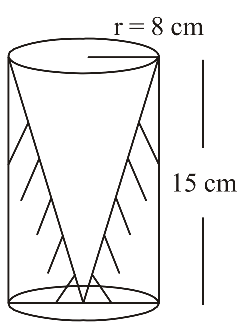
39. **From a solid cylinder whose height is 15 cm and the diameter is 16 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of remaining solid. (Give your answer in terms of π).**

Sol. Correct figure $\frac{1}{2}$



$l = 17$ 1

$r = 8$ cm $\frac{1}{2}$

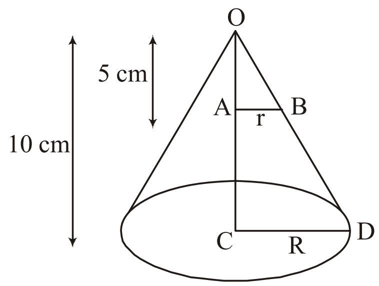
Total S.A. of remaining solid= C.S.A of cylinder + C.S.A of cone + Area of base

$= 2\pi rh + \pi rl + \pi r^2 = \pi r(2h + l + r)$ 1

$= \pi \times 8(2 \times 15 + 17 + 18) = 8\pi(55) = 440\pi\ \text{cm}^2$ 1

OR

The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the volumes of the two parts.



For correct fig 1

$\Delta OAB \sim \Delta OCD$

$$\frac{OA}{OC} = \frac{AB}{CD} \Rightarrow \frac{5}{10} = \frac{r}{R}$$

$\Rightarrow R = 2r$ 1

$$\frac{\text{V of cone}}{\text{V of frustum}} = \frac{\frac{1}{3}\pi r^2 5}{\frac{1}{3}\pi(r^2 + R^2 + rR)5} = \frac{r^2}{7r^2} = \frac{1}{7}$$ 1+1

or 7 : 1

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40. The mode of the following frequency distribution is 36. Find the missing frequency (f).

Classes	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**	**50 – 60**	**60 – 70**
Frequency	**8**	**10**	**f**	**16**	**12**	**6**	**7**

Sol. Modal class 30 – 40

$l = 30 \quad f_0 = f \quad f_1 = 16 \quad f_2 = 12 \quad h = 10$ 1

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$36 = 30 + \frac{16 - f}{32 - f - 12} \times 10$$ 2

$f = 10$ 1

QUESTION PAPER CODE 430/2/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

Question numbers 1 to 10 are multiple choice questions of 1 mark each. Select the correct option.

1. If the centre of a circle is (3, 5) and end points of a diameter are (4, 7) and (2, y), then the value of y is

(a) 3 **(b) –3** **(c) 7** **(d) 4**

Sol. (a) 3 1

---

2. The decimal expansion of $\frac{23}{2^5 \times 5^2}$ will terminate after how many places of decimal?

(a) 2 **(b) 4** **(c) 5** **(d) 1**

Sol. (c) 5 1

---

3. Two coins are tossed simultaneously. The probability of getting at most one head is

(a) $\frac{1}{4}$ **(b) $\frac{1}{2}$** **(c) $\frac{2}{3}$** **(d) $\frac{3}{4}$**

Sol. (d) $\frac{3}{4}$ 1

---

4. The cumulative frequency table is useful in determining

(a) Mean **(b) Median** **(c) Mode** **(d) All of these**

Sol. (b) Median 1

---

5. HCF of two numbers is 27 and their LCM is 162. If one of the number is 54, then the other number is

(a) 36 **(b) 35** **(c) 9** **(d) 81**

Sol. (d) 81 1

---

6. $2\sqrt{3}$ is

(a) an integer **(b) a rational number**

(c) an irrational number **(d) a whole number**

Sol. (c) an irrational no. 1

---

7. **The maximum number of zeroes a cubic polynomial can have, is**

(a) 1 **(b) 4** **(c) 2** **(d) 3**

Sol. (d) 3 1

8. **If α and β are the zeroes of the polynomial $2x^2 - 13x + 6$, then $\alpha + \beta$ is equal to**

(a) –3 **(b) 3** **(c) $\frac{13}{2}$** **(d) $-\frac{13}{2}$**

Sol. (c) $\frac{13}{2}$ 1

9. **The mid-point of the line-segment AB is P(0, 4). If the coordinates of B are (–2, 3) then the co-ordinates of A are**

(a) (2, 5) **(b) (–2, –5)** **(c) (2, 9)** **(d) (–2, 11)**

Sol. (a) (2, 5) 1

10. **In Fig.-1 AP, AQ and BC are tangents to the circle with centre O. If AB = 5 cm, AC = 6 cm and BC = 4 cm, then the length of AP (in cm) is**

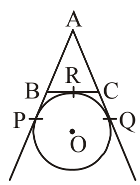

Fig. 1

(a) 15 **(b) 10** **(c) 9** **(d) 7.5**

Sol. (d) 7.5 1

Question numbers 11 to 15, fill in the blanks:

11. **The corresponding sides of two similar triangles are in the ratio 3 : 4, then the ratios of the area of triangles is _________.**

Sol. 9 : 16 1

12. **The area of triangle formed with the origin and the points (4, 0) and (0, 6) is ________.**

Sol. 12 sq units 1

OR

The co-ordinate of the point dividing the line segment joining the points A(1, 3) and B(4, 6) in the ratio 2 : 1 is ________.

Sol. (3, 5) 1

13. The value of $(\tan^2 60° + \sin^2 45°)$ is ________.

Sol. $\frac{7}{2}$ or 3.5 — 1

14. Value of the roots of the quadratic equation, $x^2 - x - 6 = 0$ are ________.

Sol. 3 and –2 — 1

15. The value of $(\sin 43° \cdot \cos 47° + \sin 47° \cos 43°)$ is __________.

Sol. 1 — 1

Question numbers 16 to 20, answer the following :

16. In figure-2 $\overset{\frown}{PQ}$ and $\overset{\frown}{AB}$ are two arcs of concentric circles of radii 7 cm and 3.5 cm resp., with centre O. If $\angle POQ = 30°$, then find the area of shaded region.

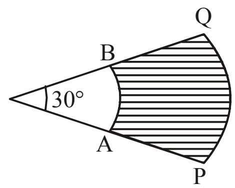

Fig.-2

Sol. Area of shaded region $= \frac{22}{7} \times \frac{30°}{360°}(7^2 - (3.5)^2)$ — $\frac{1}{2}$

$= 9.625 \text{ cm}^2$ — $\frac{1}{2}$

17. If 3k – 2, 4k – 6 and k + 2 are three consecutive terms of A.P., then find the value of k.

Sol. $(4k - 6) - (3k - 2) = (k + 2) - (4k - 6)$ — $\frac{1}{2}$

$\Rightarrow \; k = 3$ — $\frac{1}{2}$

18. Find the value of $(\cos 48° - \sin 42°)$.

Sol. $\cos 48° - \cos (90° - 42°)$ — $\frac{1}{2}$

$\cos 48° - \cos 48° = 0$ — $\frac{1}{2}$

OR

Evaluate: $(\tan 23°) \times (\tan 67°)$

Sol. $\cos (90° - 67°) \times \tan 67°$ — $\frac{1}{2}$

$= \cot 67° \times \tan 67° = 1$ — $\frac{1}{2}$

19. In a ΔPQR, S and T are points on the sides PQ and PR respectively, such that ST || QR. If PT = 2 cm and TR = 4 cm, find the ratio of the areas of ΔPST and ΔPQR.

Sol. $\frac{ar\,(\Delta PST)}{ar\,(\Delta PQR)} = \left(\frac{PT}{PR}\right)^2$ $\frac{1}{2}$

$= \left(\frac{2}{2+4}\right)^2 = \frac{1}{9}$

$\therefore$ ratio is 1 : 9 $\frac{1}{2}$

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20. Two different coins are tossed simultaneously. What is the probability of getting at least one head?

Sol. Total outcomes = 4 {HH, HT, TH, TT} $\frac{1}{2}$

P(atleast one head) = $\frac{3}{4}$ $\frac{1}{2}$

SECTION B

Question numbers 21 to 26 carry 2 marks each.

21. Prove that: $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

Sol. $L.H.S = \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = \frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$ 1

$= \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta}$ $\frac{1}{2}$

$= 2\sec^2\theta$ $\frac{1}{2}$

OR

Prove that: $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta - \sin^2\theta$

Sol. $L.H.S = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}} = \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}$ 1

$= \cos^2\theta - \sin^2\theta$ 1

---

22. Divide $(2x^2 - x + 3)$ by $(2 - x)$ and write the quotient and the remainder.

Sol.

$$\begin{array}{r|l} & \quad -2x - 3 \\ -x + 2 & \cancel{2x^2} - x + 3 \\ & \cancel{2x^2} - 4x \\ & - \quad + \\ \hline & \quad \cancel{3x} + 3 \\ & \quad \cancel{3x} - 6 \\ & \quad - \quad + \\ \hline & \qquad 9 \end{array}$$ 1

$$\text{Quotient} = -2x - 3$$
$$R = 9$$ 1

---

23. In a family of three children, find the probability of having at least two boys.

Sol. Total outcomes = 8 {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG} 1

P(atleast 2 boys) = $\frac{4}{8}$ or $\frac{1}{2}$ 1

OR

Two dice are tossed simultaneously. Find the probability of getting

(i) an even number on both dice.

(ii) the sum of two numbers more than 9.

Sol. Total outcomes = 36 cases 1

P(even no. on both side) = $\frac{9}{36}$ or $\frac{1}{4}$ $\frac{1}{2}$

P(sum > 9) = $\frac{6}{36}$ or $\frac{1}{6}$ $\frac{1}{2}$

---

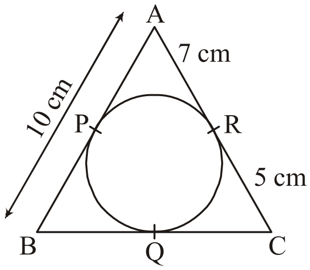
24. In a lottery, there are 10 prizes and 25 blanks. What is the probability of getting a prize?

Sol. Total = 10 + 25 = 35, P(getting prize) = $\frac{10}{35}$ or $\frac{2}{7}$ 1+1

---

25. A circle is inscribed in a ΔABC touching AB, BC and AC at P, Q and R respectively. If AB = 10 cm, AR = 7 cm and CR = 5 cm, then find the length of BC.

Sol.



AP = AR = 7 cm

PB = 10 – 7 = 3 cm $\frac{1}{2}$

BQ = BP = 3 cm $\frac{1}{2}$

QC = RC = 5 cm $\frac{1}{2}$

BC = 5 + 3 = 8 cm $\frac{1}{2}$

---

26. The length of the minute hand of clock is 14 cm. Find the area swept by the minute hand in 15 minutes.

Sol. Angle swept in 15 minutes = 90° $\frac{1}{2}$

$$\text{Area} = \frac{22}{7}\times 14\times 14\times \frac{90°}{360°}$$ 1

$= 154 \text{ cm}^2$ $\frac{1}{2}$

SECTION C

Question numbers 27 to 34 carry 3 marks each.

27. Solve graphically:

$2x - 3y + 13 = 0;\ 3x - 2y + 12 = 0$

Sol. Correct graph of $2x - 3y + 13 = 0$, $3x - 2y + 12 = 0$ 1+1

Solution $x = -2$, $y = 3$ 1

---

28. Prove that $\sqrt{3}$ is an irrational number.

Sol. Let $\sqrt{3}$ be a rational number

$\sqrt{3} = \frac{p}{q}$ p, q are coprime, $q \neq 0$ $\frac{1}{2}$

$3q^2 = p^2 \Rightarrow 3 \mid p^2 \Rightarrow 3 \mid p$ Let $p = 3m$ 1

$3q^2 = 9m^2 \Rightarrow q^2 = 3m^2 \Rightarrow 3 \mid q^2 \Rightarrow 3 \mid q$ $\frac{1}{2}$

$\therefore$ 3 is common factor of p and q

Contraction to our assumption 1

Hence $\sqrt{3}$ is irrational No.

OR

Using Euclid's algorithm, find the HCF of 272 and 1032.

Sol. $1032 = 272 \times 3 + 216$

$272 = 216 \times 1 + 56$ $\frac{1}{2}+\frac{1}{2}$

$216 = 56 \times 3 + 48$

$56 = 48 \times 1 + 8$ $\frac{1}{2}+\frac{1}{2}$

$48 = 8 \times 6 + 0$ HCF(1032, 272) = 8 $\frac{1}{2}+\frac{1}{2}$

---

29. If $x = 3 \sin \theta + 4 \cos \theta$ and $y = 3 \cos \theta - 4 \sin \theta$ then prove that $x^2 + y^2 = 25$.

Sol. $x^2 = 9 \sin^2 \theta + 16 \cos^2 \theta + 24 \sin \theta \cos \theta$ 1

$y^2 = 9 \cos^2 \theta + 16 \sin^2 \theta - 24 \sin \theta \cos \theta$ 1

$x^2 + y^2 = 25$ 1

OR

If $\sin \theta + \sin^2\theta = 1$; then prove that $\cos^2\theta + \cos^4\theta = 1$.

Sol. $\sin \theta = 1 - \sin^2 \theta = \cos^2 \theta$ 1

$\text{L.H.S} = \cos^2 \theta + (\cos^2 \theta)^2 = \cos^2 \theta + \sin^2 \theta$ 1+1

$= 1 = \text{R.H.S}$

---

30. In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. 3. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.

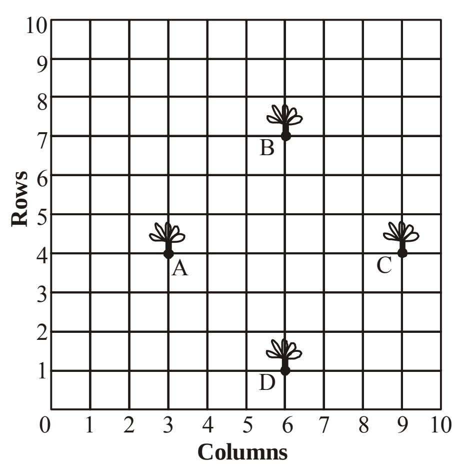


Fig. 3

Sol. A = (3, 4), B = (6, 7), C = (9, 4), D = (6, 1) 1

$AB = 3\sqrt{2}, \quad BC = 3\sqrt{2}, \quad CD = 3\sqrt{2}, \quad DA = 3\sqrt{2}$ 1

AC = 6 unit BD = 6 unit $\frac{1}{2}$

AB = BC = CD = DA and AC = BD

ABCD is a square

∴ Champa is correct $\frac{1}{2}$

---

31. Draw a line segment of length 7 cm and divide it in the ratio 2 : 3.

Sol. Correct construction 3

OR

Draw a circle of radius 4 cm and construct the pair of tangents to the circle from an external point, which is at a distance of 7 cm from its centre.

Sol. Correct construction 3

---

32. If α and β are the zeroes of the polynomial f(x) = $5x^2 - 7x + 1$, then find the value of $\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$.

Sol. $\alpha + \beta = \frac{7}{5}$ and $\alpha\beta = \frac{1}{5}$ $\frac{1}{2} + \frac{1}{2}$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$ 1

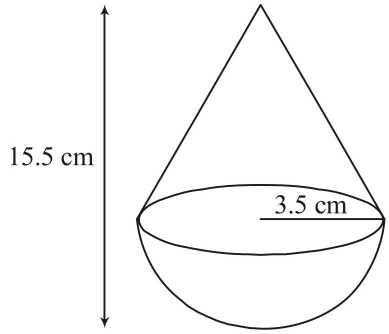
$$= \frac{\left(\frac{7}{5}\right)^2 - 2 \times \frac{1}{5}}{\frac{1}{5}}$$ $\frac{1}{2}$

$$= \frac{39}{5} \text{ or } 7.8$$ $\frac{1}{2}$

---

33. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. If the total height of the toy is 15.5 cm, find the total surface area of the toy.

Sol.



Height of cone = 15.5 – 3.5 = 12 cm $\frac{1}{2}$

Slant height, $l = \sqrt{(12)^2 + (3.5)^2} = 12.5\text{ cm}$ 1

TSA of toy = $\pi(3.5) \times 12 + 2\pi(3.5)^2$ 1

= 66.5π or 209 cm^2 $\frac{1}{2}$

34. In the Fig.-4, two circles touch each other at a point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.

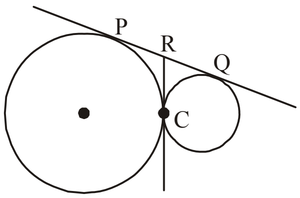


Fig. 4

Sol. $PR = RC$ ...(1) [Tangents from external point] 1

$PQ = RC$ ...(2) 1

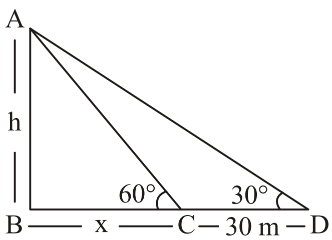
From (1) and (2), PR = PQ 1

SECTION D

Question numbers 35 to 40 carry 4 marks each.

35. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on opposite bank is 60°. When he moves 30 m away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and width of the river. [Take $\sqrt{3}$ = 1.732]

Sol.



Correct figure 1

In right ΔABC

$\tan 60° = \frac{h}{x}$ $\frac{1}{2}$

$\sqrt{3}x = h$ ...(1) $\frac{1}{2}$

In rt ΔABD $\tan 30° = \frac{h}{30 + x} \Rightarrow \frac{30 + x}{\sqrt{3}} = h$ ...(2) $\frac{1}{2}+\frac{1}{2}$

Solving (1) & (2) x = 15m, $h = 15\sqrt{3}\text{ m} = 25.98\text{ m}$ $\frac{1}{2}+\frac{1}{2}$

36. **Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.**

Sol. Correct Fig., given, to prove, construction $4\times\frac{1}{2}=2$

Correct proof given, to prove, construction, 2

OR

Prove that the length of tangents drawn from an external point to a circle are equal.

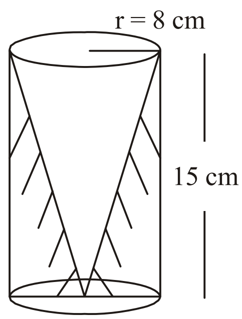
Sol. Correct Fig., given, to prove, construction $4\times\frac{1}{2}=2$

Correct proof given, to prove, construction, 2

---

37. **From a solid cylinder whose height is 15 cm and the diameter is 16 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of remaining solid. (Give your answer in terms of π).**

Sol. Correct figure $\frac{1}{2}$



$l = 17$ 1

$r = 8$ cm $\frac{1}{2}$

Total S.A. of remaining solid= C.S.A of cylinder + C.S.A of cone + Area of base

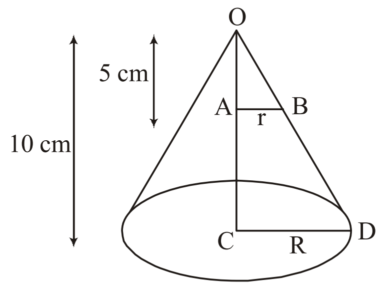
$= 2\pi rh + \pi rl + \pi r^2 = \pi r(2h + l + r)$ 1

$= \pi \times 8(2 \times 15 + 17 + 8) = 8\pi(55) = 440\pi \text{ cm}^2$ 1

OR

The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the volumes of the two parts.

Sol. For correct fig 1



$\Delta OAB \sim \Delta OCD$

$\frac{OA}{OC} = \frac{AB}{CD} \Rightarrow \frac{5}{10} = \frac{r}{R}$

$\Rightarrow R = 2r$ 1

$\frac{\text{V of cone}}{\text{V of frustum}} = \frac{\frac{1}{3}\pi r^2 5}{\frac{1}{3}\pi(r^2 + R^2 + rR)} = \frac{r^2}{7r^2} = \frac{1}{7}$ 1+1

or 7 : 1

---

38. The 17th term of an A.P. is 5 more than twice its 8th term. If 11th term of A.P. is 43; then find its nth term.

Sol. $a_{17} = 2a_8 + 5 \Rightarrow a + 16d = 2(a + 7d) + 5$ 1

$\Rightarrow 2d - a = 15$ ...(1)

$a_{11} = 43 \Rightarrow a + 10d = 43$ ...(2) 1

Solving (1) & (2) $a = 3 \quad d = 4$ 1

$a_n = 4n - 1$ 1

OR

How many terms of A.P. 3, 5, 7, 9, ... must be taken to get the sum 120?

Sol. $a = 3, \quad d = 3, \quad Sn = 120$ 1

$\frac{n}{2}[2 \times 3 + (n-1)2] = 120 \Rightarrow n^2 + 2n - 120 = 0$ 1

$(n + 12)(n - 10) = 0$ 1

$n = -12, n = 10$ 1

Reject $n = -12$, $n = 10$

---

39. Find the median for the given frequency distribution:

Classes	**40 – 45**	**45 – 50**	**50 – 55**	**55 – 60**	**60 – 65**	**65 – 70**	**70 – 75**
Frequency	**2**	**3**	**8**	**6**	**6**	**3**	**2**

Sol.

Class	**Frequency**	**cf**
40-50	2	2
45-50	3	5
55-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30

Correct table 1

Median class = 55 – 60 $\frac{1}{2}$

$$\text{Median} = 55 + \frac{\left(\frac{30}{2} - 13\right)}{6} \times 5$$ 2

$$= 56\frac{2}{3} \text{ or } 56.67$$ $\frac{1}{2}$

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40. If the price of a book is reduced by ₹ 5, a person can buy 4 more books for ₹ 600. Find the original price of the book.

Sol. Let original price of the book be ₹x

A.T.Q.

$$\frac{600}{x-5} - \frac{600}{x} = 4$$ $1\frac{1}{2}$

$x^2 - 5x - 750 = 0$ 1

$(x - 30)(x + 25) = 0$ 1

$x = 30$ or -25

Price is always positive, so original price of book is is ₹30 $\frac{1}{2}$
