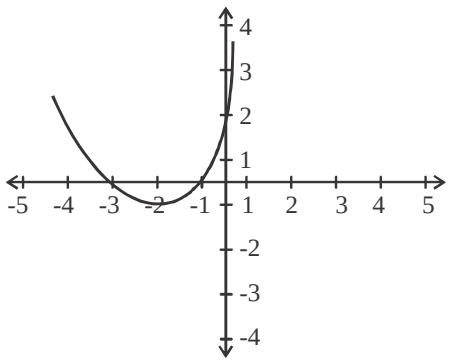
QUESTION PAPER CODE 30/3/1

EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question numbers 1 to 10 are multiple choice questions of 1 mark each.

You have to select the correct choice :

Q.No.		Marks
1.	The HCF of 135 and 225 is **(a)** 15 **(b)** 75 **(c)** 45 **(d)** 5 **Ans:** (c) 45	**1**
2.	The exponent of 2 in the prime factorization of 144, is **(a)** 2 **(b)** 4 **(c)** 1 **(d)** 6 **Ans:** (b) 4	**1**
3.	The common difference of an AP, whose n^{th} term is $a_n = (3n + 7)$, is **(a)** 3 **(b)** 7 **(c)** 10 **(d)** 6 **Ans:** (a) 3	**1**
4.	The value of λ for which $(x^2 + 4x + \lambda)$ is a perfect square, is **(a)** 16 **(b)** 9 **(c)** 1 **(d)** 4 **Ans:** (d) 4	**1**
5.	The value of k, for which the pair of linear equations $kx + y = k^2$ and $x + ky = 1$ have infinitely many solutions is **(a)** ± 1 **(b)** 1 **(c)** -1 **(d)** 2 **Ans:** (b) 1	**1**
6.	The value of p for which $(2p + 1)$, 10 and $(5p + 5)$ are three consecutive terms of an AP is **(a)** -1 **(b)** -2 **(c)** 1 **(d)** 2 **Ans:** (d) 2 **OR** The number of terms of an AP 5, 9, 13, … 185 is **(a)** 31 **(b)** 51 **(c)** 41 **(d)** 40 **Ans:** 1 mark should be given to each candidate.	**1** **1**
7.	In Fig. 1, the graph of the polynomial p(x) is given. The number of zeroes of the polynomial is  **Fig. 1**	

(a) 1 **(b)** 2 **(c)** 3 **(d)** 0

Ans: (b) 2 **1**

8. If (a, b) is the mid-point of the line segment joining the points A(10, −6) and B(k, 4) and a − 2b = 18, the value of k is

(a) 30 **(b)** 22 **(c)** 4 **(d)** 40

Ans: (b) 22 **1**

9. The value of k for which the points A (0, 1), B (2, k) and C(4, −5) are collinear is

(a) 2 **(b)** −2 **(c)** 0 **(d)** 4

Ans: (b) −2 **1**

10. If $\Delta ABC \sim \Delta DEF$ such that AB = 1.2 cm and DE = 1.4 cm, the ratio of the areas of ΔABC and ΔDEF is

(a) 49 : 36 **(b)** 6 : 7 **(c)** 7 : 6 **(d)** 36 : 49

Ans: (d) 36 : 49 **1**

In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 mark :

11. $\sqrt{2}$ times the distance between (0, 5) and (−5, 0) is _____.

Ans: 10 **1**

12. The distance between two parallel tangents of a circle of radius 4 cm is _____.

Ans: 8 cm **1**

13. In Fig. 2, PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$, then the measure of $\angle OAB$ is _____.

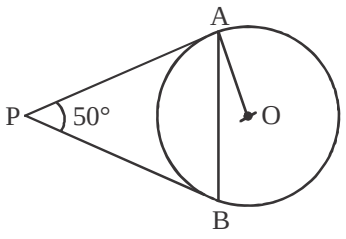


Fig. 2

Ans: 25° **1**

OR

In Fig. 3, PQ is a chord of a circle and PT is tangent at P such that $\angle QPT = 60°$, then the measure of $\angle PRQ$ is _____.

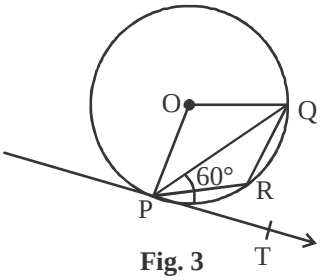


Fig. 3

Ans: 120° **1**

14. $\frac{3\cot 40°}{\tan 50°} - \frac{1}{2}\left(\frac{\cos 35°}{\sin 55°}\right) =$ _____.

Ans: $\frac{5}{2}$ **1**

15. If $\cot\theta = \frac{7}{8}$, then the value of $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} =$ _____.

Ans: $\frac{49}{64}$ **1**

Q. Nos. 16 to 20 are short answer type questions of 1 mark each.

16. What is the value of $\left(\frac{1}{1+\cot^2\theta} + \frac{1}{1+\tan^2\theta}\right)$?

Ans: Given expression = $\sin^2\theta + \cos^2\theta$ **1/2**

$= 1$ **1/2**

17. Two right circular cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1, what is the ratio of their volumes?

Ans: $V_1 : V_2 = \frac{1}{3}\pi(3r)^2 h : \frac{1}{3}\pi r^2(3h)$ **1/2**

$= 3 : 1$ **1/2**

18. Using the empirical formula, find the mode of a distribution whose mean is 8.32 and the median is 8.05.

Ans: Mode = $3 \times 8.05 - 2 \times 8.32$ **1/2**

$= 7.51$ **1/2**

19. The probability that it will rain tomorrow is 0.85. What is the probability that it will not rain tomorrow ?

Ans: Prob (no rain tomorrow) = $1 - 0.85$ **1/2**

$= 0.15$ **1/2**

20. What is the arithmetic mean of first n natural numbers?

Ans: Sum of first n natural numbers = $\frac{n(n+1)}{2}$ **1/2**

$\therefore$ Mean = $\frac{n+1}{2}$ **1/2**

SECTION – B

Q. Nos. 21 to 26 carry 2 marks each.

21. Find the 11^{th} term from the last term (towards the first term) of the AP 12, 8, 4, …, –84.

Ans: $l = -84$ **1/2**

$d = -4$ **1/2**

t_{11} (from the end) = –84 + 40 = –44 **1**

OR

Solve the equation : 1 + 5 + 9 + 13 + … + x = 1326

Ans: $\frac{n}{2}(1 + x) = 1326$ … (i) **1/2**

$x = 1 + (n - 1) \times 4$ … (ii) **1/2**

Solving (i) and (ii) x = 101 **1**

22. In Fig. 4 AB is a chord of circle with centre O, AOC is diameter and AT is tangent at A. Prove that ∠BAT = ∠ACB.

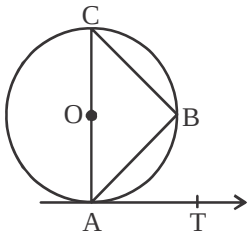


Fig. 4

Ans: ∠BAC = 90° – ∠BAT … (i) **1/2**

In ΔBAC, ∠B = 90° **1/2**

∴ ∠BCA = 90° – ∠BAC

or ∠ACB = ∠BAT (Using (i)) **1**

23. If $\tan\theta = \frac{3}{4}$, find the value of $\left(\frac{1-\cos^2\theta}{1+\cos^2\theta}\right)$

Ans: $\sec^2\theta = 1 + \frac{9}{16} = \frac{25}{16}$

$\therefore \cos^2\theta = \frac{16}{25}$ **1**

Hence $\frac{1-\cos^2\theta}{1+\cos^2\theta} = \frac{1-\frac{16}{25}}{1+\frac{16}{25}} = \frac{9}{41}$ **1**

OR

If $\tan\theta = \sqrt{3}$, find the value of $\left(\frac{2\sec\theta}{1+\tan^2\theta}\right)$

Ans: $\sec^2\theta = 1 + 3 = 4$

$\therefore \sec\theta = 2$ **1**

Hence $\frac{2\sec\theta}{1+\tan^2\theta} = \frac{2\times 2}{4} = 1$ **1**

24. Read the following passage and answer the questions given at the end :

Students of Class XII presented a gift to their school in the form of an electric lamp in the shape of a glass hemispherical base surmounted by a metallic cylindrical top of same radius 21 cm and height 3.5 cm. The top was silver coated and the glass surface was painted red.

(i) What is the cost of silver coating the top at the rate of ₹ 5 per 100 cm^2 ?

(ii) What is the surface area of glass to be painted red ?

Ans: (i) Surface Area of the top = $2 \times \frac{22}{7} \times 21 \times 3.5 = 462\ cm^2$ **1/2**

Cost of silver coating = $462 \times \frac{5}{100}$ = Rs. 23.10 **1/2**

(ii) Surface Area of glass = $2 \times \frac{22}{7} \times 21 \times 21$ **1/2**

$= 2772\ cm^2$ **1/2**

25. Find the probability that a leap year selected at random will contain 53 Sundays and 53 Mondays.

Ans: 366 days = 52 weeks + 2 days **1/2**

Total possible outcomes are 7 (SM, MT, TW, WTh, ThF, FS, SS) **1**

Prob (having 53 Sundays & 53 Mondays) = $\frac{1}{7}$ **1/2**

26. Find the value of p, if the mean of the following distribution is 7.5.

Classes	2-4	4-6	6-8	8-10	10-12	12-14
Frequency (fi)	6	8	15	p	8	4

Ans:

Class	**Frequency (f)**	**x**	**fx**
2-4	6	3	18
4-6	8	5	40
6-8	15	7	105
8-10	p	9	9p
10-12	8	11	88
12-14	4	13	52
	41 + p		303 + 9p

Correct table = 1

Mean = $7.5 = \frac{303+9p}{41+p} \Rightarrow p = 3$ **1**

SECTION – C

Q. Nos. 27 to 34 carry 3 marks each.

27. Find a, b and c if it is given that the numbers a, 7, b, 23, c are in AP.

Ans: a, 7, b, 23, c are in A.P

Let d be the common difference of AP.

$\therefore$ $a + d = 7$ … (i) **1/2**

$a + 3d = 23$ … (ii) **1/2**

Solving (i) & (ii) , $d = 8$ **1/2**

$\therefore$ $a = -1, b = 15, c = 31$ **1/2+1/2+1/2**

OR

If m times the m^{th} term of an AP is equal to n times its nth term, show that the $(m + n)^{th}$ term of the AP is zero.

Ans: Given $m[a + (m - 1)d] = n[a + (n - 1)d]$ **1**

$\Rightarrow$ $a(m - n) + d(m^2 - m - n^2 + n) = 0$

$\Rightarrow$ $(m - n)[a + (m + n - 1)d = 0$ **1**

$\because$ $m \neq n \Rightarrow a + (m + n - 1)d = 0$ **1/2**

$\Rightarrow a_{m+n} = 0$ **1/2**

28. Find the values of k, for which the quadratic equation $(k + 4)x^2 + (k + 1)x + 1 = 0$ has equal roots.

Ans: For equal roots $(k + 1)^2 - 4(k + 4) \times 1 = 0$ **1**

$\Rightarrow$ $k^2 - 2k - 15 = 0$ **1**

$\Rightarrow$ $(k + 3)(k - 5) = 0$ **1/2**

$\Rightarrow$ $k = -3, 5$ **1/2**

29. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$ respectively. Find g(x).

Ans: $x^3 - 3x^2 + x + 2 = (x - 2) \times g(x) + (-2x + 4)$ **1**

$\Rightarrow$ $(x - 2)\, g(x) = x^3 - 3x^2 + 3x - 2$ **1/2**

$\Rightarrow$ $g(x) = \dfrac{(x-2)(x^2 - x + 1)}{(x-2)}$ **1**

$= x^2 - x + 1$ **1/2**

OR

If the sum of the squares of zeros of the quadratic polynomial $f(x) = x^2 - 8x + k$ is 40, find the value of k.

Ans: Let the zeroes of polynomial f(x) be α and β.

$\therefore$ $\alpha + \beta = 8$ and $\alpha\beta = k$ **1/2+1/2**

$\because$ $\alpha^2 + \beta^2 = 40$

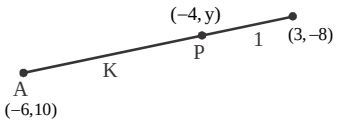
$\Rightarrow$ $(\alpha + \beta)^2 - 2\alpha\beta = 40$ **1**

$\Rightarrow$ $64 - 2k = 40$ **1/2**

$\Rightarrow$ $k = 12$ **1/2**

30. In what ratio does the point P(−4, y) divide the line segment joining the points A(−6, 10) and B(3, −8) if it lies on AB. Hence find the value of y.

Ans: Let AP : PB = k : 1



$\therefore \quad -4 = \dfrac{3k-6}{k+1}$ **1**

$\Rightarrow \quad k = \dfrac{2}{7}$ **1**

$\therefore \quad$ AP : PB = 2 : 7

Hence $y = \dfrac{-8k+10}{k+1} = \dfrac{-8\times\frac{2}{7}+10}{\frac{2}{7}+1} = 6$ **1**

31. Prove that, a tangent to a circle is perpendicular to the radius through the point of contact.

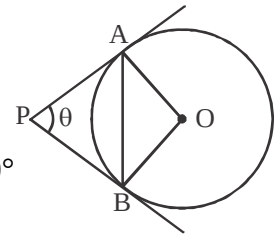
Ans: Given, To prove, figure **1/2 × 3 = $1\frac{1}{2}$**

Correct proof **$1\frac{1}{2}$**

OR

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Ans: ∠PAO = 90° (radius ⊥ tangent)
∠PBO = 90° } **cor. fig. 1/2**



Now

∠PAO + ∠AOB + ∠OBP + ∠BPA = 360°

$\Rightarrow$ 90° + ∠AOB + 90° + ∠BPA = 360° **1**

$\Rightarrow$ ∠AOB + ∠BPA = 180°

or ∠AOB and ∠BPA are supplementary. **1/2**

32. In a right triangle, prove that the square of the hypotenuse is equal to the sum of squares of the other two sides.

Ans: Correct given, To prove & figure **1/2 × 3 = $1\frac{1}{2}$**

Correct proof **$1\frac{1}{2}$**

33. If sin θ + cos θ = p and sec θ + cosec θ = q, show that q (p^2 – 1) = 2p.

Ans: LHS = $q(p^2 - 1) = (\sec\theta + \text{cosec}\,\theta)((\sin\theta + \cos\theta)^2 - 1)$

$= \dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} \times 2\sin\theta\cos\theta$ **1+1**

$= 2(\sin\theta + \cos\theta)$ **1/2**

$= 2p = \text{RHS}$ **1/2**

34. 500 persons are taking dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is 0.04 m^3 ?

Ans: Let the rise in the water level be h

$\therefore 500 \times .04 = 80 \times 50 \times h$ **2**

$\Rightarrow h = \dfrac{500 \times .04}{80 \times 50}$ **1/2**

$= .005$ m **1/2**

SECTION – D

Q. Nos. 35 to 40 carry 4 marks each.

35. Show that $(12)^n$ cannot end with digit 0 or 5 for any natural number n.

Ans: $12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$ **2**

Since there is no factor of the form 5^m therefore 12^n can not end with digit 0 or 5 for any natural number n. **2**

OR

Prove that $\left(\sqrt{2} + \sqrt{5}\right)$ is irrational.

Ans: Let us assume $\sqrt{2} + \sqrt{5}$ is rational number

Let $\sqrt{2} + \sqrt{5} = m$ where m is rational **1**

$\Rightarrow \left(\sqrt{2} + \sqrt{5}\right)^2 = m^2$ **1**

$\Rightarrow m^2 = 7 + 2\sqrt{10}$

$\Rightarrow \sqrt{10} = \dfrac{m^2 - 7}{2}$ **1**

$\because$ m is rational

$\therefore \dfrac{m^2 - 7}{2}$ is also rational

but $\sqrt{10}$ is irrational

$\Rightarrow$ LHS $\neq$ RHS

It means our assumption was wrong.

Hence $\sqrt{2} + \sqrt{5}$ is an irrational number. **1**

36. A train covered a certain distance at a uniform speed. If the train would have been 6 km/hr. faster, it would have taken 4 hours less than the scheduled time and if the train were slower by 6 km/hr., it would have taken 6 hrs. more than the scheduled time. Find the length of the journey.

Ans: Let usual speed of train be x km/hr and distance covered be d km.

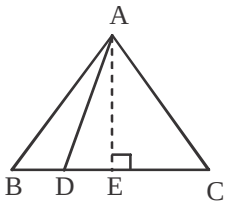
Therefore $\frac{d}{x} - \frac{d}{x+6} = 4$ …(i) **1**

$\frac{d}{x-6} - \frac{d}{x} = 6$ …(ii) **1**

Solving (i) and (ii) x = 30 and d = 720 **2**

∴ Length of journey = 720 km

37. In an equilateral triangle ABC, D is a point on the side BC such that BD = $\frac{1}{3}$BC. Prove that $9AD^2 = 7AB^2$. **cor. fig 1/2**



Ans: Draw AE ⊥ BC

∵ ΔABC is an equilateral Δ

∴ BE = $\frac{BC}{2}$ **1/2**

Now, $AD^2 = AE^2 + DE^2$ and $AB^2 = AE^2 + BE^2$ **1**

$\Rightarrow AB^2 = AD^2 - DE^2 + BE^2$

$= AD^2 + (BE + DE)(BE - DE)$ **1/2**

$= AD^2 + \frac{BC}{3} \times \left(\frac{BC}{2} + \frac{BC}{2} - \frac{BC}{3}\right)$

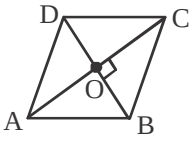
$= AD^2 + \frac{2}{9}BC^2 = AD^2 + \frac{2}{9}AB^2$ **1**

$\Rightarrow 7AB^2 = 9AD^2$ **1/2**

OR

Prove that the sum of squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Ans: $AB^2 + BC^2 + CD^2 + AD^2$ **cor. fig 1/2**



$= 4\,AB^2$ (∵ ABCD is a rhombus) **1**

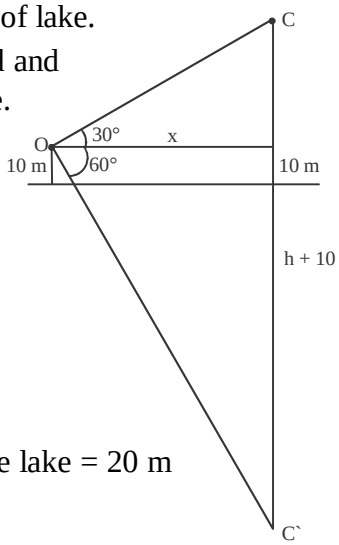
$= 4\,(OA^2 + OB^2)$ **1**

$= 4\left(\frac{AC^2}{4} + \frac{BD^2}{4}\right)$ **1**

$= AC^2 + BD^2$ **1/2**

38. If the angle of elevation of a cloud from a point 10 metres above a lake is 30° and the angle of depression of its reflection in the lake is 60°, find the height of the cloud from the surface of lake.

Ans: Let C represents the position of cloud and C` represents its reflection in the lake. **cor. fig 1**



$\tan 30° = \frac{1}{\sqrt{3}} = \frac{h}{x}$ **1**

$\Rightarrow x = h\sqrt{3}$ … (i)

$\tan 60° = \sqrt{3} = \frac{h+20}{x}$ … (ii) **1**

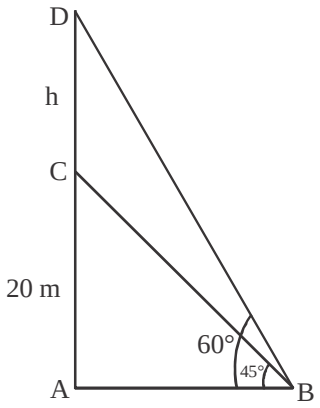
Solving (i) and (ii) h =10 **1/2**

$\therefore$ Height of cloud from surface of the lake = 20 m **1/2**

OR

A vertical tower of height 20 m stands on a horizontal plane and is surmounted by a vertical flag-staff of height h. At a point on the plane, the angle of elevation of the bottom and top of the flag staff are 45° and 60° respectively. Find the value of h.

Ans: Let AC be the tower and CD be the flag-staff. **cor. fig 1**



$\tan 45° = 1 = \frac{AC}{AB}$

$\Rightarrow$ AC = AB …(i) **1**

$\tan 60° = \sqrt{3} = \frac{AC + h}{AB}$

$\Rightarrow \sqrt{3}$ AB = AC + h …(ii) **1**

Using (i) and (ii)

$AC\left(\sqrt{3}-1\right) = h$

$\Rightarrow h = 20\left(\sqrt{3}-1\right)$ m **1**

39. A solid iron cuboidal block of dimensions 4.4 m × 2.6 m × 1 m is cast into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.

Ans: Internal radius of cylinder (r_2) = 30 cm = 0.30 m

Outer radius of cylinder (r_1) = 30 + 5 = 35 cm = 0.35 m **1**

Therefore $4.4 \times 2.6 \times 1 = \pi \times h \times ((0.35)^2 - (.30)^2)$ **2**

$= \pi \times h \times \frac{1}{100 \times 100} \times 65 \times 5$

$\Rightarrow h = \frac{352}{\pi}$ m or 112 m **1**

40.	For the following frequency distribution, draw a cumulative frequency curve of 'more than' type and hence obtain the median value.	

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	15	20	23	17	11	9

Ans: Plotting points (0, 100) (10, 95) (20, 80) (30, 60) (40, 37) (50, 20) (60, 9) **2**

and joining them. **$1\frac{1}{2}$**

Median = 34.3 (approx) **1/2**