QUESTION PAPER CODE 30/5/1

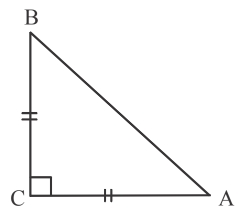
EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question numbers 1 to 20 are of 1 mark each.

Question numbers **1** to **10** are multiple choice questions.

You have to select the correct choice :

Q.No.		Marks
1.	On dividing a polynomial p(x) by $x^2 - 4$, quotient and remainder are found to be x and 3 respectively. The polynomial p(x) is	
	(a) $3x^2 + x - 12$ **(b)** $x^3 - 4x + 3$ **(c)** $x^2 + 3x - 4$ **(d)** $x^3 - 4x - 3$	
	Ans: (b) $x^3 - 4x + 3$	1
2.	In Figure 1, ABC is an isosceles triangle, right-angled at C. Therefore	
	(a) $AB^2 = 2AC^2$ **(b)** $BC^2 = 2AB^2$ **(c)** $AC^2 = 2AB^2$ **(d)** $AB^2 = 4AC^2$	
	 Figure 1	
	Ans: (a) $AB^2 = 2AC^2$	1
3.	The point on the x-axis which is equidistant from (–4, 0) and (10, 0) is	
	(a) (7, 0) **(b)** (5, 0) **(c)** (0, 0) **(d)** (3, 0)	
	Ans: (d) (3, 0)	1
	OR	
	The centre of a circle whose end points of a diameter are (–6,3) and (6, 4) is	
	(a) (8, –1) **(b)** (4, 7) **(c)** $\left(0, \frac{7}{2}\right)$ **(d)** $\left(4, \frac{7}{2}\right)$	
	Ans: (c) $\left(0, \frac{7}{2}\right)$	1
4.	The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is	
	(a) 4 **(b)** ±4 **(c)** –4 **(d)** 0	
	Ans: (b) ± 4	1
5.	Which of the following is *not* an A.P.?	
	(a) –1.2, 0.8, 2.8, … **(b)** $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$	
	(c) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \ldots$ **(d)** $\frac{-1}{5}, \frac{-2}{5}, \frac{-3}{5}, \ldots$	
	Ans: (c) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \ldots$	1

6. The pair of linear equations

$\frac{3x}{2}+\frac{5y}{3}=7$ and $9x + 10y = 14$ is

(a) consistent

(b) inconsistent

(c) consistent with one solution

(d) consistent with many solutions

Ans: (b) inconsistent **1**

7. In Figure 2, PQ is tangent to the circle with centre at O, at the point B. If $\angle AOB = 100°$, then $\angle ABP$ is equal to

(a) 50°

(b) 40°

(c) 60°

(d) 80°

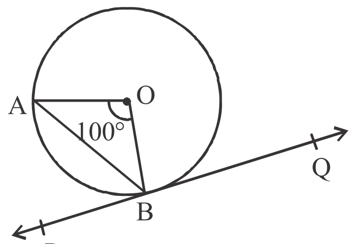


Figure 2

Ans: (a) 50° **1**

8. The radius of a sphere (in cm) whose volume is 12π cm^3, is

(a) 3

(b) $3\sqrt{3}$

(c) $3^{2/3}$

(d) $3^{1/3}$

Ans: (c) $3^{2/3}$ **1**

9. The distance between the points $(m, -n)$ and $(-m, n)$ is

(a) $\sqrt{m^2+n^2}$

(b) $m + n$

(c) $2\sqrt{m^2+n^2}$

(d) $\sqrt{2m^2+2n^2}$

Ans: (c) $2\sqrt{m^2+n^2}$ **1**

10. In Figure 3, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If $\angle QPR = 90°$, then length of PQ is

(a) 3 cm

(b) 4 cm

(c) 2 cm

(d) $2\sqrt{2}$ cm

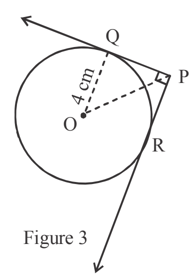


Figure 3

Ans: (b) 4 cm **1**

Fill in the blanks in question numbers 11 to 15.

11. The probability of an event that is sure to happen, is __________.

Ans: 1 **1**

12. Simplest form of $\frac{1+\tan^2 A}{1+\cot^2 A}$ is __________.

Ans: $\tan^2 A$ **1**

13. AOBC is a rectangle whose three vertices are A(0, –3), O(0, 0) and B(4, 0). The length of its diagonal is __________.

Ans: 5 units **1**

14. In the formula $\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$, $u_i =$ __________.

Ans: $\frac{x_i - a}{h}$ **1**

15. All concentric circles are __________ to each other.

Ans: similar **1**

Answer the following question numbers 16 to 20.

16. Find the sum of the first 100 natural numbers.

Ans: $\frac{100}{2}[2 + 99] = 5050$ **1/2+1/2**

17. In Figure 4, the angle of elevation of the top of a tower from a point C on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of tower.

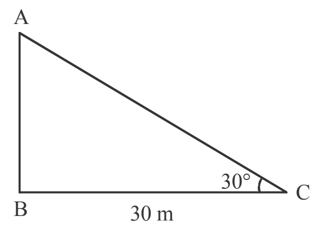


Figure 4

Ans: $\frac{AB}{30} = \frac{1}{\sqrt{3}} \Rightarrow AB = \frac{30}{\sqrt{3}}$ m or $10\sqrt{3}$ m **1/2+1/2**

18. The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, find the other.

Ans: $\frac{182 \times 13}{26} = 91$ **1/2+1/2**

19. Form a quadratic polynomial, the sum and product of whose zeros are (–3) and 2 respectively.

Ans: $x^2 + 3x + 2$ **1**

OR

Can $(x^2 – 1)$ be a remainder while dividing $x^4 – 3x^2 + 5x – 9$ by $(x^2 + 3)$? Justify your answer with reasons.

Ans: No, degree of remainder < degree of divisor **1**

20. Evaluate : $\frac{2 \tan 45° \times \cos 60°}{\sin 30°}$

Ans: $\frac{2 \times 1 \times \frac{1}{2}}{\frac{1}{2}} = 2$ **1/2+1/2**

SECTION – B

Question numbers 21 to 26 carry 2 marks each.

21. In the given Figure 5, DE || AC and DF || AE.

Prove that $\frac{BF}{EF} = \frac{BE}{EC}$.

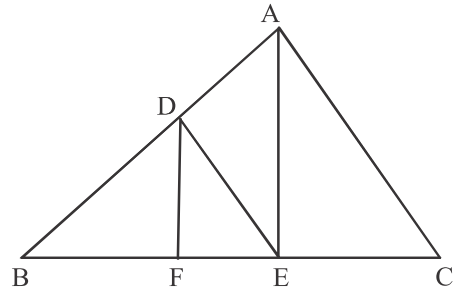


Figure 5

Ans: In ΔABE, $DF \parallel AE$, $\therefore \frac{BD}{AD} = \frac{BF}{FE}$ … (i) **1**

In ΔABC, $DE \parallel AC$, $\therefore \frac{BD}{AD} = \frac{BE}{EC}$ … (ii) **1/2**

From (i) and (ii) $\frac{BF}{FE} = \frac{BE}{EC}$ **1/2**

22. Show that $5+2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number.

Ans: Let us assume that $5+2\sqrt{7}$ is not an irrational number.

$\therefore$ $5+2\sqrt{7}$ is a rational number p i.e. $5+2\sqrt{7} = p$ **1**

$\Rightarrow \sqrt{7} = \frac{p-5}{2}$ **1/2**

Which is a contradiction as RHS is a rational but LHS is irrational.

Hence $5+2\sqrt{7}$ can not be rational, so irrational. **1/2**

OR

Check whether 12^n can end with the digit 0 for any natural number n.

Ans: Prime factors of 12 are $2 \times 2 \times 3$ **1**

Since 5 is not a factor, so 12^n can not end with 0. **1**

23. If A, B and C are interior angles of a ΔABC, then show that

$\cot\left(\frac{B+C}{2}\right) = \tan\left(\frac{A}{2}\right)$.

Ans: $A + B + C = 180°$, $\therefore \frac{B+C}{2} = 90° - \frac{A}{2}$ **1**

$\therefore \cot\left(\frac{B+C}{2}\right) = \cot\left(90° - \frac{A}{2}\right) = \tan\frac{A}{2}$ **1**

24. In Figure 6, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = BC + AD.

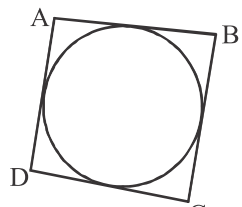
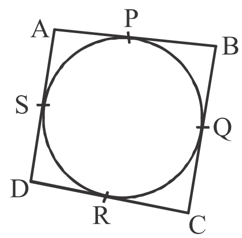


Figure 6

Ans: Let the circle touches the sides AB, BC, CD and AD at P, Q, R and S respectively. **1/2**



$$\therefore \left.\begin{aligned} AP &= AS \\ BP &= BQ \\ DR &= DS \\ CR &= CQ \end{aligned}\right\}$$ **1**

adding, we get (AP + BP) + (DR + CR) = (AS + DS) + (BQ + CQ) **1/2**

$\therefore$ AB + CD = BC + AD

OR

In Figure 7, find the perimeter of ΔABC, if AP = 12 cm.

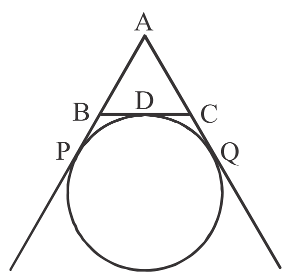


Figure 7

Ans:
$$\left.\begin{aligned} AP &= AB + BP = AB + BD \\ AQ &= AC + CQ = AC + CD \end{aligned}\right\}$$ **1**

$\Rightarrow$ AP + AQ = AB + AC + (BD + CD) = AB + AC + BC

But AP = AQ $\therefore$ 2 AP = Perimeter of ABC **1/2**

$\therefore$ Perimeter = 2(12) = 24 cm **1/2**

25. Find the mode of the following distribution:

Marks:	0-10	10-20	20-30	30-40	40-50	50-60
Number of Students:	4	6	7	12	5	6

Ans: Modal Group : 30 – 40 **1/2**

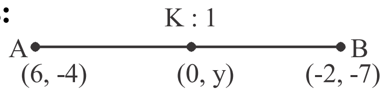
$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h = 30 + \frac{5}{12} \times 10$$ **1**

$= 34.17$ **1/2**

26.	2 cubes, each of volume 125 cm^3, are joined end to end. Find the surface area of the resulting cuboid.	
	Ans: Side of cube = $(125)^{1/3} = 5$ cm	1/2
	$\therefore$ Dimensions of cuboid : 10, 5, 5	1/2
	S.A = 2(50 + 25 + 50) = 250 cm^2	1
	SECTION – C	
	Question numbers 27 to 34 carry 3 marks each.	
27.	A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.	
	Ans: Let the fraction be $\frac{x}{y}$	1/2
	$\therefore \frac{x-1}{y} = \frac{1}{3}, \frac{x}{y+8} = \frac{1}{4}$	1/2+1/2
	$\Rightarrow 3x - y = 3$, $4x - y = 8$	1/2
	Solving to get x = 5, y = 12 $\therefore$ Fraction is $\frac{5}{12}$	1
	OR	
	The present age of a father is three years more than three times the age of his son. Three years hence the father's age will be 10 years more than twice the age of the son. Determine their present ages.	
	Ans: Let the present age of son be x years	
	$\therefore$ Father's present age = (3x + 3) years.	1
	3 years hence, Son's age = (x + 3) years and father's age = (3x + 6) years	1/2
	$\therefore \quad 3x + 6 = 2(x + 3) + 10$	1
	$\Rightarrow \quad x = 10 \quad \therefore$ Son's age = 10 years, Father's age = 33 years	1/2
28.	Use Euclid Division Lemma to show that the square of any positive integer is either of the form 3q or 3q + 1 for some integer q.	
	Ans: Any positive integer 'n' can be of the form 3m, 3m + 1, 3m + 2 (for some integer m)	1
	$\therefore \quad n^2 = (3m)^2 = 9m^2 = 3(3m^2) = 3q$, or $\quad n^2 = (3m + 1)^2 = 9m^2 + 6m + 1 = 3(3m^2 + 2m) + 1 = 3q + 1$, or $\quad n^2 = (3m + 2)^2 = 9m^2 + 12m + 3 + 1$ $= 3(3m^2 + 4m + 1) + 1 = 3q + 1$	$1\frac{1}{2}$
	Hence square of any positive integer is either of the form 3q or 3q + 1 for some integer q.	1/2

29. Find the ratio in which the y-axis divides the line segment joining the points (6, –4) and (–2, –7). Also find the point of intersection.

Ans:



Let the point P(0, y) on y-axis divides the line segment AB in K : 1 — **1**

$$\therefore \quad 0 = \frac{-2K+6}{K+1} \Rightarrow K = 3 \quad \therefore \text{ Ratio is } 3 : 1$$ **1**

$$\text{Also, } y = \frac{3(-7)+1(-4)}{3+1} = \frac{-25}{4} \therefore \text{ Point of intersection is } \left(0, \frac{-25}{4}\right)$$ **1**

OR

Show that the points (7, 10), (–2, 5) and (3, –4) are vertices of an isosceles right triangle.

Ans: Let the points be A(7, 10), B(–2, 5) and C(3, –4)

$$AB = \sqrt{(-2-7)^2 + (5-10)^2} = \sqrt{106}$$ **1**

$$BC = \sqrt{(3+2)^2 + (-4-5)^2} = \sqrt{106}$$ **1/2**

$$AC = \sqrt{(3-7)^2 + (-4-10)^2} = \sqrt{212}$$ **1/2**

$AB = BC$ and $AC^2 = AB^2 + BC^2$

Hence ABC is isosceles right triangle. — **1**

30. Prove that: $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Ans: $\text{LHS} = \sqrt{\dfrac{1+\sin A}{1-\sin A} \cdot \dfrac{1+\sin A}{1+\sin A}}$ — **1**

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \frac{1+\sin A}{\cos A}$$ **$1+\frac{1}{2}$**

$$= \sec A + \tan A$$ **1/2**

31. For an A.P., it is given that the first term (a) = 5, common difference (d) = 3, and the n^{th} term (a_n) = 50. Find n and sum of first n terms (S_n) of the A.P.

Ans: $50 = 5 + (n-1)3 \Rightarrow n = 16$ — **$1+\frac{1}{2}$**

$$S_{16} = \frac{16}{2}[10 + 15 \times 3] = 440$$ **$1+\frac{1}{2}$**

32. Construct a ΔABC with sides BC = 6 cm, AB = 5 cm and $\angle ABC = 60°$.

Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC.

Ans: Constructing ΔABC with given dimensions — **1**

Constructing the similar triangle. — **2**

OR

Draw a circle of radius 3.5 cm. Take a point P outside the circle at a distance of 7 cm from the centre of the circle and construct a pair of tangents to the circle from that point.

Ans: Drawing a circle of radius 3.5 cm and centre O, and taking a point P such that OP = 7 cm **1**

Constructing two tangents. **2**

33. Read the following passage and answer the questions given at the end:

Diwali Fair

A game in booth at Diwali fair involves using of spinner first. Then, if the spinner stops at an even number, the player is allowed to pick a marble from bag. The spinner and the marbles in the bag are represented in Figure-8

Prizes are given, when a black marble is picked. Shweta plays the game once.

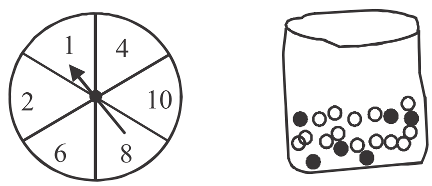


Figure 8

(i) What is the probability that she will be allowed to pick a marble from the bag?

(ii) Suppose she is allowed to pick a marble from the bag, what is the probability of getting a prize, when it is given that the bag contains 20 balls out of which 6 are black?

Ans: (i) P(she will be allowed to pick a marble) = $\frac{5}{6}$ **$1\frac{1}{2}$**

(ii) P(getting a prize) = $\frac{6}{20}$ or $\frac{3}{10}$ **$1\frac{1}{2}$**

Both answers $\frac{6}{20}$ or $\frac{0}{20}$ for part (ii) in Q33 are to be treated correct as the bag contains marbles only.

34. In Figure-9, a square OPQR is inscribed in a quadrant OAQB of a circle. If the radius of the circle is $6\sqrt{2}$ cm, find the area of shaded region.

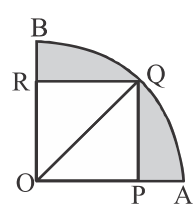


Figure 9

Ans: Let side of square be 'a' cm $\therefore$ $a^2 + a^2 = \left(6\sqrt{2}\right)^2$ $\Rightarrow$ a = 6 cm **1**

$\therefore$ Area of shaded region $= \pi r^2 \dfrac{90}{360} - a^2 = \dfrac{22}{7} \times \left(6\sqrt{2}\right)^2 \cdot \dfrac{1}{4} - 36$ **$1 + \frac{1}{2}$**

$= \dfrac{396 - 252}{7} = \dfrac{144}{7} \text{cm}^2$ or 20.57 cm^2 **1/2**

SECTION – D

Question numbers 35 to 40 carry 4 marks each.

35. Obtain other zeroes of the polynomial

$$P(x) = 2x^4 - x^3 - 11x^2 + 5x + 5$$

If two of its zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

Ans: Since $\sqrt{5}$ and $-\sqrt{5}$ are zeroes of p(x), so $\left(x - \sqrt{5}\right)$ and $\left(x + \sqrt{5}\right)$ are factors of p(x). Thus $(x^2 - 5)$ is a factor of p(x). **1**

$\left(2x^4 - x^3 - 11x^2 + 5x + 5\right) \div \left(x^2 - 5\right) = 2x^2 - x - 1$ **$1\frac{1}{2}$**

$2x^2 - x - 1 = (2x + 1)(x - 1)$ **1**

$\therefore$ Other zeroes of p(x) are $1, -\dfrac{1}{2}$ **1/2**

OR

What minimum must be added to $2x^3 - 3x^2 + 6x + 7$ so that the resulting polynomial will be divisible by $x^2 - 4x + 8$?

Ans:

$$\begin{array}{r|l} & 2x + 5 \\ x^2 - 4x + 8 & 2x^3 - 3x^2 + 6x + 7 \\ - & \underline{2x^3 \underset{+}{-} 8x^2 \underset{-}{+} 16x} \\ & \quad 5x^2 - 10x + 7 \\ - & \quad \underline{5x^2 \underset{+}{-} 20x \underset{-}{+} 40} \\ & \qquad 10x - 33 \end{array}$$

3

$\therefore$ We have to add $(33 - 10x)$ **1**

36. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Ans: For correct Given, To Prove, Constructions and figure **$\frac{1}{2} \times 4 = 2$**

For correct proof **2**

37. Sum of the areas of 2 squares is 544 m^2. If the difference of their perimeter is 32 m, find the sides of two squares.

Ans: Let 'a' and 'b' be the sides of two squares, with a > b.

then $a^2 + b^2 = 544$ and $4a - 4b = 32$ — $1\frac{1}{2}$

or $a - b = 8 \quad \therefore a = b + 8$

$\therefore (b+8)^2 + b^2 = 544 \quad \Rightarrow 2b^2 + 16b - 480 = 0$ — 1

$\therefore b^2 + 8b - 240 = 0 \quad \Rightarrow (b+20)(b-12) = 0 \quad \Rightarrow b = 12$ — 1

$b = 12$ m $\Rightarrow a = 12 + 8 = 20$ m — 1/2

OR

A motorboat whose speed is 18 km/hr in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Ans: Let speed of the stream be x km/h

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$ — 2

$\Rightarrow 24(2x) = 324 - x^2$ or $x^2 + 48x - 324 = 0$ — 1

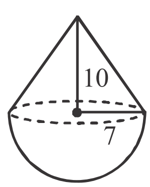
$\Rightarrow (x+54)(x-6) = 0 \Rightarrow x = 6$ — 1

$\therefore$ Speed of the stream = 6 km/h

38. A solid toy in the form of a hemisphere surmounted by a right circular cone of same radius. The height of the cone is 10 cm and the radius of its base is 7 cm. Determine the volume of the toy. Also find the area of the colored sheet required to cover the toy.

(Use $\pi = \frac{22}{7}$ and $\sqrt{149} = 12.2$)

Ans: Volume of toy $= \frac{2}{3}\pi(7)^3 + \frac{1}{3}\pi(7)^2 \times 10\,\text{cm}^3$ — 1



$= \frac{1}{3} \times \frac{22}{7} \times 49\,(14 + 10) = 1232\,\text{cm}^3$ — 1

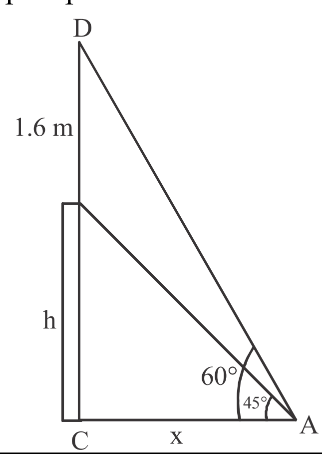
Area of Sheet = Surface area $= 2\pi(7)^2 + \pi(7)\sqrt{10^2 + 7^2}$ — 1

$= 308 + 22 \times 12.2 = 576.4\text{cm}^2$ — 1

39. A statue 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statute is 60° and from the same point the angle of elevation of the top of pedestal id 45°. Find the height of the pedestal. (Use $\sqrt{3} = 1.73$)

Ans: For correct figure. — 1

Let h m be the height of pedestal



Then from figure, $\frac{h}{x} = \tan 45° = 1$

and $\frac{h+1.6}{x} = \tan 60° = \sqrt{3}$ — 1 + 1

$\Rightarrow \frac{h+1.6}{h} = \sqrt{3} \Rightarrow \left(\sqrt{3} - 1\right)h = 1.6$ — 1/2

$$\Rightarrow h = \frac{160}{73} = 2.19 \text{ m (approx)}$$ 1/2

40. For the following data, draw a 'less than' ogive and hence find the median of the distribution.

Age (In years):	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of persons:	5	15	20	25	15	11	9

Ans: The points to be plotted for less than ogive are

(10, 5), (20, 20), (30, 40), (40, 65), (50 , 80), (60, 91), (70, 100) 2

Drawing the ogive $1\frac{1}{2}$

Getting median = 34 (approx) 1/2

OR

The distribution given below shows that the number of wickets taken by bowler in one-day cricket matches. Find the mean and the median of the number of wickets taken.

Number of wickets :	20-60	60-100	100-140	140-180	180-220	230-260
Number of bowlers :	7	5	16	12	2	3

Ans:

No. of wickets :	20-60	60-100	100-140	140-180	180-220	220-260	Sum	
(f_i) No. of bowlers :	7	5	16	12	2	3	45	
x_i	40	80	120	160	200	240		
u_i	−2	−1	0	1	2	3		1/2
$f_i x_i$	−14	−5	0	12	4	9	6	1/2
cf	7	12	28	40	42	45		1/2

$$\text{Mean} = a + \frac{\sum f_i u_i}{\sum f_i} \times h = 120 + \frac{6 \times 40}{45} = 125.33$$ $1\frac{1}{2}$

$$\text{Median} = l + \frac{\frac{N}{2} - c}{f} \times h = 100 + \frac{22.5 - 12}{16} \times 40 = 126.25$$ 1