



6. The interval in which the function  $f$  given by  $f(x) = x^2e^{-x}$  is strictly increasing, is

- (A)  $(-\infty, \infty)$       (B)  $(-\infty, 0)$       (C)  $(2, \infty)$       (D)  $(0, 2)$

**Ans:** (D)  $(0, 2)$

1

7. If  $|\vec{a}| = 4$  and  $-3 \leq \lambda \leq 2$ , then  $|\lambda\vec{a}|$  lies in

- (A)  $[0, 12]$       (B)  $[2, 3]$       (C)  $[8, 12]$       (D)  $[-12, 8]$

**Ans:** (A)  $[0, 12]$

1

8. The vector  $3\hat{i} - \hat{j} + 2\hat{k}$ ,  $2\hat{i} + \hat{j} + 3\hat{k}$  and  $\hat{i} + \lambda\hat{j} - \hat{k}$  are coplanar if value of  $\lambda$  is

- (A)  $-2$       (B)  $0$   
(C)  $2$       (D) Any real number

**Ans:** (A)  $-2$

1

9. The area of a triangle formed by vertices O, A and B, where

$\vec{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$  and  $\vec{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$  is

- (A)  $3\sqrt{5}$  sq. units      (B)  $5\sqrt{5}$  sq. units  
(C)  $6\sqrt{5}$  sq. units      (D)  $4$  sq. units

**Ans:** (A)  $3\sqrt{5}$  sq. units

1

10. The coordinates of the foot of the perpendicular drawn from the point  $(2, -3, 4)$  on the  $y$ -axis is

- (A)  $(2, 3, 4)$       (B)  $(-2, -3, -4)$   
(C)  $(0, -3, 0)$       (D)  $(2, 0, 4)$

**Ans:** (C)  $(0, -3, 0)$

1

**Fill in the blanks in questions numbers 11 to 15.**

11. The range of the principal value branch of the function  $y = \sec^{-1} x$  is \_\_\_\_\_.

**Ans:**  $[0, \pi] - \left\{ \frac{\pi}{2} \right\}$

1

**OR**

The principal value of  $\cos^{-1}\left(-\frac{1}{2}\right)$  is \_\_\_\_\_.

**Ans:**  $\frac{2\pi}{3}$

1

12. Given a skew-symmetric matrix  $A = \begin{bmatrix} 0 & a & 1 \\ -1 & b & 1 \\ -1 & c & 0 \end{bmatrix}$ , the value of  $(a + b + c)^2$  is \_\_\_\_\_

**Ans: 0**

1

13. The distance between parallel planes  $2x + y - 2z - 6 = 0$  and  $4x + 2y - 4z = 0$  \_\_\_\_\_ units.

**Ans: 2**

1

**OR**

If  $P(1,0,-3)$  is the foot of the perpendicular from the origin to the plane, then the cartesian equation of the plane is \_\_\_\_\_

**Ans:  $x - 3z = 10$**

1

14. If the radius of the circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is \_\_\_\_\_

**Ans:  $\pi$  cm/s**

1

15. The corner points of the feasible region of an LPP are  $(0,0)$ ,  $(0,8)$ ,  $(2,7)$ ,  $(5,4)$  and  $(6,0)$ . The maximum profit  $P = 3x + 2y$  occurs at the point \_\_\_\_\_

**Ans:  $(5, 4)$**

1

**Question numbers 16 to 20 are very short answer type questions.**

16. Differentiate  $\sec^2(x^2)$  with respect to  $x^2$ .

**Ans:** Let  $x^2 = u$ , differentiating  $\sec^2 u$  w.r.t  $u$ , putting  $u = x^2$   
 $2\sec^2 x^2 \tan x^2$

1/2

1/2

**OR**

If  $y = f(x^2)$  and  $f'(x) = e^{\sqrt{x}}$ , then find  $\frac{dy}{dx}$ .

**Ans:**  $\frac{dy}{dx} = f'(x^2)2x$   
 $= 2xe^x$

1/2

1/2

17. Find the value of  $k$ , so that the function  $f(x) = \begin{cases} kx^2 + 5, & \text{if } x \leq 1 \\ 2, & \text{if } x > 1 \end{cases}$

is continuous at  $x = 1$ .

**Ans:** L.H.L. is  $k + 5$   
 getting  $k = -3$

1/2

1/2

18. Evaluate:  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos^2 x dx$ .

**Ans:** Let  $f(x) = x \cos^2 x$   $f(-x) = -f(x)$  or  $f$  is odd 1/2

$\therefore \int_{-\pi/2}^{\pi/2} x \cos^2 x dx = 0$  1/2

19. Find the general solution of the differential equation  $e^{y-x} \frac{dy}{dx} = 1$ .

**Ans:** Given differential equation is  $e^y dy = e^x dx$  1/2

Integrating to get  $e^y = e^x + C$  1/2

20. Find the coordinates of the point where the line  $\frac{x-1}{3} = \frac{y+4}{7} = \frac{z+4}{2}$  cuts the line  $xy$ -plane.

**Ans:** Putting  $z = 0$  in given equation gives  $\frac{x-1}{3} = \frac{y+4}{7} = 2$  1/2

Coordinates of required point are  $(7, 10, 0)$  1/2

### SECTION-B

**Question numbers 21 to 26 carry 2 marks each.**

21. If  $A = \begin{bmatrix} -3 & 2 \\ 1 & -1 \end{bmatrix}$  and  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , find scalar  $k$  so that  $A^2 + I = kA$ .

**Ans:**  $A^2 = \begin{bmatrix} 11 & -8 \\ -4 & 3 \end{bmatrix}$  1

$A^2 + I = kA \Rightarrow \begin{bmatrix} 12 & -8 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} -3k & 2k \\ k & -k \end{bmatrix}$  1/2

$k = -4$  1/2

22. If  $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$ , find  $f'\left(\frac{\pi}{3}\right)$ .

**Ans:**  $f(x) = \sqrt{\frac{\frac{1}{\cos x} - 1}{\frac{1}{\cos x} + 1}} = \tan \frac{x}{2}$  1

$f'(x) = \frac{1}{2} \sec^2 \frac{x}{2}$  1/2

$f'\left(\frac{\pi}{3}\right) = \frac{2}{3}$  1/2

**OR**

Find  $f'(x)$  if  $f(x) = (\tan x)^{\tan x}$ .

**Ans:** Taking log on both sides.  $\log f(x) = \tan x \log \tan x$  1/2

differentiating to get  $\frac{f'(x)}{f(x)} = \sec^2 x + \sec^2 x \log \tan x$  1

Thus,  $f'(x) = (\tan x)^{\tan x} \cdot \sec^2 x (1 + \log \tan x)$  1/2

23. Find  $\int \frac{\tan^3 x}{\cos^3 x} dx$ .

**Ans:** Given Integral is  $I = \int \frac{\sin^3 x}{\cos^6 x} dx$  1/2

Put  $\cos x = t$

$\sin x dx = -dt$  1/2

$$= \int \left[ \frac{-1}{t^6} + \frac{1}{t^4} \right] dt$$

$$= \frac{t^{-5}}{5} - \frac{t^{-3}}{3} + c$$
 1/2

$$= \frac{1}{5(\cos x)^5} - \frac{1}{3(\cos x)^3} + c \text{ or } \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + c$$
 1/2

24. Find a vector  $\vec{r}$  equally inclined to the three axes and whose magnitude is  $3\sqrt{3}$  units.

**Ans:** Let the vector  $\vec{r} = a\hat{i} + a\hat{j} + a\hat{k}$  1/2

$\therefore \sqrt{3a^2} = 3\sqrt{3}$  1

required vector is  $3\hat{i} + 3\hat{j} + 3\hat{k}$  or  $-3\hat{i} - 3\hat{j} - 3\hat{k}$  1/2

**OR**

Find the angle between unit vectors  $\vec{a}$  and  $\vec{b}$  so that  $\sqrt{3}\vec{a} - \vec{b}$  is also a unit vector.

**Ans:** Using  $|\sqrt{3}\vec{a} - \vec{b}| = 1$  i.e.  $|\sqrt{3}\vec{a} - \vec{b}|^2 = 1$  1/2

& getting  $\vec{a} \cdot \vec{b} = \frac{\sqrt{3}}{2}$  1

getting angle  $\frac{\pi}{6}$  or  $30^\circ$  1/2

25. Find the points of intersection of the line  $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$  and the plane  $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$ .

**Ans:** Any point on line is  $(2 + 3\lambda, -1 + 4\lambda, 2 + 2\lambda)$  1

Putting in equation of plane to get  $\lambda = 0$  1/2

Thus point of intersection is  $(2, -1, 2)$  1/2

26. A purse contains 3 silver and 6 copper coins and a second purse contains 4 silver and 3 copper coins. If a coin is drawn at random from one of the two purses, find the probability that it is a silver coin.

**Ans:**  $E_1$  : coin is drawn from purse 1.

$E_2$  : coin is drawn from purse 2. 1/2

A : Silver coin is drawn

$P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}, P\left(\frac{A}{E_1}\right) = \frac{3}{9}, P\left(\frac{A}{E_2}\right) = \frac{4}{7}$  1

$P(A) = \frac{1}{2} \times \frac{3}{9} + \frac{1}{2} \times \frac{4}{7}$   
 $= \frac{19}{42}$  1/2

**SECTION-C**

**Question numbers 27 to 32 carry 4 marks each.**

27. Check whether the relation R in the set N of natural numbers given by

$R = \{(a, b) : a \text{ is divisor of } b\}$

is reflexive, symmetric or transitive. Also determine whether R is an equivalence relation.

**Ans:** For reflexive

Let  $a \in N$  clearly a divides a  $\therefore (a, a) \in R$

$\therefore R$  is reflexive 1

For symmetric

$(1, 2) \in R$  but  $(2, 1) \notin R$

1

$\therefore R$  is not symmetric

For transitive

Let  $(a, b), (b, c) \in R$

$\therefore a$  divides  $b$  and  $b$  divides  $c$

$\Rightarrow a$  divides  $c \quad \therefore (a, c) \in R$

1

$R$  is transitive

As  $R$  is not symmetric  $\therefore$  It is not an equivalence relation

1

**OR**

Prove that  $\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} = \frac{1}{2} \sin^{-1} \left( \frac{4}{5} \right)$ .

**Ans:** LHS =  $\tan^{-1} \left( \frac{\frac{1}{4} + \frac{2}{9}}{1 - \frac{1}{4} \cdot \frac{2}{9}} \right) = \tan^{-1} \frac{1}{2}$

2

$$= \frac{1}{2} \cdot 2 \tan^{-1} \frac{1}{2}$$

1

$$= \frac{1}{2} \sin^{-1} \left( \frac{2 \times \frac{1}{2}}{1 + \frac{1}{4}} \right) = \frac{1}{2} \sin^{-1} \left( \frac{4}{5} \right)$$

1

= RHS

28. If  $\tan^{-1} \left( \frac{y}{x} \right) = \log \sqrt{x^2 + y^2}$ , prove that  $\frac{dy}{dx} = \frac{x+y}{x-y}$ .

**Ans:** Differentiating both sides w.r.t.  $x$  to get

$$\frac{1}{1 + \frac{y^2}{x^2}} \cdot \frac{x \frac{dy}{dx} - y}{x^2} = \frac{1}{\sqrt{x^2 + y^2}} \cdot \frac{2x + 2y \frac{dy}{dx}}{2\sqrt{x^2 + y^2}}$$

$1 \frac{1}{2} + 1 \frac{1}{2}$

Simplyfying we get  $x \frac{dy}{dx} - y = x + y \frac{dy}{dx}$

$\frac{1}{2}$

getting  $\frac{dy}{dx} = \frac{x+y}{x-y}$

$\frac{1}{2}$

**OR**

If  $y = e^{a \cos^{-1} x}$ ,  $-1 < x < 1$ , then show that  $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$

**Ans:**  $\frac{dy}{dx} = \frac{-ae^{a\cos^{-1}x}}{\sqrt{1-x^2}}$  1

$$\sqrt{1-x^2} \frac{dy}{dx} = -ae^{a\cos^{-1}x} \quad \frac{1}{2}$$

Differentiating again & getting

$$\sqrt{1-x^2} \frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}} \cdot \frac{dy}{dx} = \frac{a^2 e^{a\cos^{-1}x}}{\sqrt{1-x^2}} \quad 2$$

$$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0 \quad \frac{1}{2}$$

29. Find  $\int \frac{x^3+1}{x^3-x} dx$

**Ans:** Writing Integral as  $I = \int \frac{x^3 - x + x + 1}{x^3 - x} dx$  1

$$= \int 1 dx + \int \frac{1}{x(x-1)} dx \quad 1$$

$$= x + \int \left( \frac{1}{x-1} - \frac{1}{x} \right) dx \quad 1$$

$$= x + \log|x-1| - \log|x| + c \quad 1$$

30. Solve the following differential equation:

$$(1+e^{y/x}) dy + e^{y/x} \left(1 - \frac{y}{x}\right) dx = 0 \quad (x \neq 0)$$

**Ans:** Writing given differential equation as  $\frac{dy}{dx} = \frac{e^{y/x}(y/x-1)}{1+e^{y/x}}$  1

Putting  $y = vx$  &  $\frac{dy}{dx} = v + x \frac{dv}{dx}$  1

to get  $x \frac{dv}{dx} = \frac{-(e^v+v)}{1+e^v}$  1

$$\Rightarrow \int \frac{e^v+1}{e^v+v} dv = -\int \frac{dx}{x}$$

$$\log|e^v+v| = -\log|x| + \log c \quad 1$$

$$e^{y/x} + \frac{y}{x} = \frac{c}{x} \quad \text{or} \quad x e^{y/x} + y = c \quad \frac{1}{2}$$



31. Find the shortest distance between the lines

$$\vec{r} = 2\hat{i} - \hat{j} + \hat{k} + \lambda(3\hat{i} - 2\hat{j} + 5\hat{k})$$

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \mu(4\hat{i} - \hat{j} + 3\hat{k})$$

**Ans:**  $\vec{a}_2 - \vec{a}_1 = \hat{i} + 3\hat{j} - 5\hat{k}$

1

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 5 \\ 4 & -1 & 3 \end{vmatrix} = -\hat{i} + 11\hat{j} + 5\hat{k}$$

2

$$\begin{aligned} \text{Using, shortest distance} &= \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \\ &= \frac{7}{\sqrt{147}} \text{ units or } \frac{1}{\sqrt{3}} \text{ units} \end{aligned}$$

1

32. A cotton industry manufactures pedestal lamps and wooden shades. Both the products require machine time as well as craftsman time in the making. The number of hour(s) required for producing 1 unit of each and the corresponding profit is given in the following table

Items	Machine time	Craftsman time	Profit (in ₹)
Pedestal lamp	1.5 hours	3 hours	30
Wooden shades	3 hours	1 hour	20

In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsman time.

Assuming that all items manufactured are sold, how should the manufacture schedule his daily production in order to maximize the profit? Formulate it as an LPP and solve it graphically.

**Ans:** Let number of pedestal lamps = x  
number of wooden shades = y

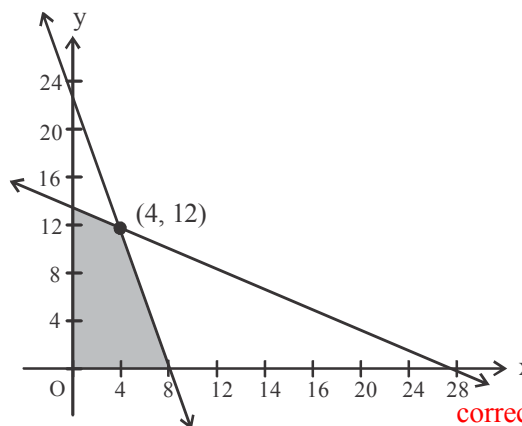
Maximize Profit  $Z = 30x + 20y$

Subject to constraints

$$1.5x + 3y \leq 42$$

$$3x + y \leq 24$$

$$x \geq 0, y \geq 0$$



$\frac{1}{2}$

$1\frac{1}{2}$

correct graph 1

getting corners points & values of Z

$$(0, 0) \quad 0$$

$$(8, 0) \quad 240$$

$$(4, 12) \quad 360$$

$$(0, 14) \quad 280$$

1/2

Maximum profit = ₹ 360 where  $x = 4, y = 12$

1/2

### SECTION-D

Question numbers 33 to 36 carry 6 marks each.

33.  $A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$ , find  $A^{-1}$  and use it to solve the following system of equations:

$$5x - y + 4z = 5$$

$$2x + 3y + 5z = 2$$

$$5x - 2y + 6z = -1$$

Ans:  $|A| = 51$

1

Cofactors:  $A_{11} = 28 \quad A_{12} = 13 \quad A_{13} = -19$   
 $A_{21} = -2 \quad A_{22} = 10 \quad A_{23} = 5$   
 $A_{31} = -17 \quad A_{32} = -17 \quad A_{33} = 17$

2

$$A^{-1} = \frac{1}{51} \begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}$$

1

Given system is  $AX = B$  where  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$   $B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$

1/2

$$\Rightarrow X = A^{-1}B = \frac{1}{51} \begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$$

1

$$x = 3, y = 2, z = -2$$

1/2

OR

If  $x, y, z$  are different and  $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ , then using properties of determinants

show that  $1 + xyz = 0$ .

**Ans:** Writing L.H.S as  $\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$  1

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$
 1

getting

$$(1 + xyz) \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} = 0$$
 2

Applying  $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$$(1 + xyz) \begin{vmatrix} x & x^2 & 1 \\ y-x & (y+x)(y-x) & 0 \\ z-x & (z+x)(z-x) & 0 \end{vmatrix} = 0$$
 1

Expanding and simplifying to get

$$\Rightarrow (1 + xyz) (x - y) (y - z) (z - x) = 0$$
 1/2

As  $x, y, z$  are different 1/2

$$\therefore 1 + xyz = 0$$

34. Amongst all open (from the top) right circular cylindrical boxes of volume  $125\pi \text{ cm}^3$ , find the dimensions of the box which has the least surface area.

**Ans:** Let radius =  $r$  & height =  $h$

$$\pi r^2 h = 125\pi \quad \text{or} \quad h = \frac{125}{r^2}$$
 1

$$\text{Surface Area, } S = 2\pi r h + \pi r^2$$

$$S = \frac{250\pi}{r} + \pi r^2$$
 1

$$\frac{dS}{dr} = \frac{-250\pi}{r^2} + 2\pi r \quad 1$$

$$\frac{dS}{dr} = 0 \Rightarrow r = 5 \quad 1/2+1$$

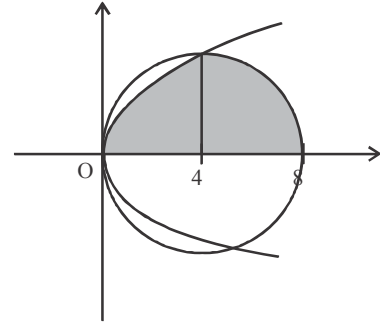
$$\frac{d^2S}{dr^2} = \frac{500\pi}{r^3} + 2\pi > 0 \text{ so } s \text{ is least} \quad 1$$

when  $r = 5 \text{ cm}$  , gives  $h = 5 \text{ cm}$  1/2

35. Using integration, find the area lying above x-axis and included between the circle  $x^2 + y^2 = 8x$  and inside the parabola  $y^2 = 4x$ .

**Ans.** Correct figure 1

x-coordinate of point of intersection is 4, 0 1



$$\text{Required Area} = \int_0^4 2\sqrt{x} \, dx + \int_4^8 \sqrt{16 - (x-4)^2} \, dx \quad 2$$

$$= \frac{4}{3} x^{\frac{3}{2}} \Big|_0^4 + \frac{x-4}{2} \sqrt{16 - (x-4)^2} + 8 \sin^{-1} \frac{x-4}{4} \Big|_4^8 \quad 1$$

$$= \frac{32}{3} + 4\pi \quad 1$$

**OR**

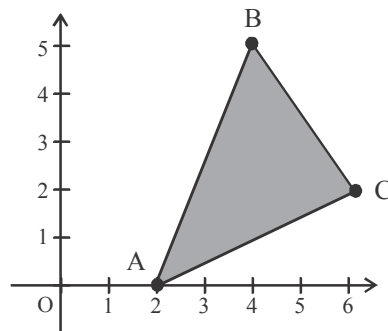
Using the method of integration, find the area of the  $\Delta ABC$ , coordinates of whose vertices are  $A(2, 0)$ ,  $B(4, 5)$  and  $C(6, 3)$ .

**Ans.** Correct figure 1

$$\text{Equation of } AB : y = \frac{5}{2}(x-2) \quad 1/2$$

$$\text{Equation of } BC : y = 9 - x \quad 1/2$$

$$\text{Equation of } AC : y = \frac{3}{4}(x-2) \quad 1/2$$



$$\text{Required Area} = \frac{5}{2} \int_2^4 (x-2) \, dx + \int_4^6 (9-x) \, dx - \frac{3}{4} \int_2^6 (x-2) \, dx \quad 2$$

$$= \frac{5}{2} \frac{(x-2)^2}{2} \Big|_2^4 + \frac{(9-x)^2}{-2} \Big|_4^6 - \frac{3}{4} \frac{(x-2)^2}{2} \Big|_2^6 \quad 1$$

$$= 7 \quad 1/2$$

36. Find the probability distribution of the random variable  $X$ , which denotes the number of doublets in four throws of a pair of dice. Hence, find the mean of the number of doublets ( $X$ ).

**Ans.** Let  $X$  denote the number of doublets  $P(\text{doublet}) = \frac{1}{6}$ ,  $P(\text{not a doublet}) = \frac{5}{6}$  1

$X$	0	1	2	3	4	1
$P(X)$	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$	$2\frac{1}{2}$

$X \cdot P(X)$	0	$\frac{500}{1296}$	$\frac{300}{1296}$	$\frac{60}{1296}$	$\frac{4}{1296}$	1
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$$\text{mean} = \sum X \cdot P(X) = \frac{864}{1296} \text{ or } \frac{2}{3} \quad \text{1/2}$$


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