CBSE Class 12 Maths Question Paper Solution 2020 Set 65/4/1

XII MATHEMATICS

QUESTION PAPER CODE 65/4/1

EXPECTED ANSWER/VALUE POINTS

Q. No.	VALUE POINTS	Marks
	SECTION - A	
	Question Numbers 1 to 20 carry 1 mark each.	
	Q. Nos. 1 to 10 are multiple choice questions of 1 mark each. Select the correct option:	
1	The value of $\sin^{-1}\left(\cos\frac{3\pi}{5}\right)$ is (a) $\frac{\pi}{10}$ (b) $\frac{3\pi}{5}$ (c) $\frac{-\pi}{10}$ (d) $\frac{-3\pi}{5}$ **Answer:** $(c)-\frac{\pi}{10}$	1
2	If $A=\begin{bmatrix}2 & -3 & 4\end{bmatrix}, B=\begin{bmatrix}3\\2\\2\end{bmatrix}, X=\begin{bmatrix}1 & 2 & 3\end{bmatrix}$ and $Y=\begin{bmatrix}2\\3\\4\end{bmatrix}$, then $AB+XY$ equals (a) $[28]$ (b) $[24]$ (c) 28 (d) 24 **Answer:** $(a)[28]$	1
3	If $\begin{vmatrix}2 & 3 & 2\\x & x & x\\4 & 9 & 1\end{vmatrix}+3=0$, then the value of x is (a) 3 (b) 0 (c) -1 (d) 1 **Answer:** $(c)-1$	1
4	$\int_0^{\frac{\pi}{8}}\tan^2(2x)\,dx$ is equal to (a) $\frac{4-\pi}{8}$ (b) $\frac{4+\pi}{8}$ (c) $\frac{4-\pi}{4}$ (d) $\frac{4-\pi}{2}$ **Answer:** $(a)\frac{4-\pi}{8}$	1
5	If $\vec{a}.\vec{b}=\frac{1}{2}\lvert\vec{a}\rvert\lvert\vec{b}\rvert$, then the angle between $\vec{a}$ and $\vec{b}$ is is (a) 0^0 (b) 30^0 (c) 60^0 (d) 90^0 **Answer:** $(c)60^0$	1

6	The two lines $x=ay+b, z=cy+d$; and $x=a'y+b', z=c'y+d'$ are perpendicular to each other, if (a) $\frac{a}{a'}+\frac{c}{c'}=1$ (b) $\frac{a}{a'}+\frac{c}{c'}=-1$ (c) $aa'+cc'=1$ (d) $aa'+cc'=-1$ **Answer:** $\boxed{(d)\, aa'+cc'=-1}$	1
7	The two planes $x-2y+4z=10$ and $18x+17y+kz=50$ are perpendicular, if k is equal to (a) -4 (b) 4 (c) 2 (d) -2 **Answer:** $\boxed{(b)\, 4}$	1
8	In a LPP, if the objective function $z=ax+by$ has the same maximum value on two corner points of a feasible region, then the number of points at which $z_{\max}$ occurs is (a) 0 (b) 2 (c) finite (d) infinite **Answer:** $\boxed{(d)\,\text{infinite}}$	1
9	From the set $\{1,2,3,4,5\}$, two numbers a and b $(a\neq b)$ are chosen at random. The probability that $\frac{a}{b}$ is an integer is (a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$ **Answer:** $\boxed{(b)\,\frac{1}{4}}$	1
10	A bag contains 3 white, 4 black and 2 red balls. If 2 balls are drawn at random (without replacement), then the probability that both the balls are white is (a) $\frac{1}{18}$ (b) $\frac{1}{36}$ (c) $\frac{1}{12}$ (d) $\frac{1}{24}$ **Answer:** $\boxed{(c)\,\frac{1}{12}}$	1
In Q. Nos. 11 to 15, fill in the blanks with correct word/sentence:		
11	If $f: R\rightarrow R$ be given by $f(x)=\left(3-x^3\right)^{\frac{1}{3}}$, then $fof(x)=$ __________. **Answer:** x	1
12	If $\begin{bmatrix} x+y & 7 \\ 9 & x-y \end{bmatrix}=\begin{bmatrix} 2 & 7 \\ 9 & 4 \end{bmatrix}$, then $x.y=$ __________. **Answer: - 3**	1
13	The number of points of discontinuity of f defined by $f(x)=\|x\|-\|x+1\|$ is	

	________. **Answer:** Zero	1
14	The slope of the tangent to the curve $y = x^3 - x$ at the point $(2,6)$ is ________. **Answer:** 11	1
	OR The rate of change of the area of a circle with respect to its radius r , when r=3 cm, is ________. **Answer:** $6\pi\ \text{cm}^2/\text{cm}$	1
15	If $\vec{a}$ is a non-zero vector, then $(\vec{a}.\hat{i})i+(\vec{a}.\hat{j})j+(\vec{a}.\hat{k})k$ equals ________. **Answer:** $\vec{a}$	1
	OR The projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$ is ________. **Answer:** 0	1
Q. 16 to 20 are very short answer questions.		
16	Find adj$A$, if $A=\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$. **Answer:** $\text{adj}A=\begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$ ($\frac{1}{2}$ mark for any two correct co-factors)	1
17	Find $\int \frac{2^{x+1}-5^{x-1}}{10^x}dx$ **Answer:** $I=\int\left(2(5^{-x})-\frac{1}{5}(2^{-x})\right)dx=-\frac{2}{5^x\log 5}+\frac{1}{5(2^x)\log 2}+C$	1
18	Evaluate $\int_0^{2\pi}\lvert\sin x\rvert dx$ **Answer:** $I=4\int_0^{\frac{\pi}{2}}\sin x\,dx=4$	$\frac{1}{2}+\frac{1}{2}$
19	If $\int_0^a \frac{dx}{1+4x^2}=\frac{\pi}{8}$, then find the value of a. **Answer:**	

	$\int_0^a \frac{dx}{(2x)^2+1}=\frac{\pi}{8}$ $\Rightarrow \frac{1}{2}\left[\tan^{-1}(2x)\right]_0^a=\frac{\pi}{8}$ $\Rightarrow a=\frac{1}{2}$ **OR** Find $\int \frac{dx}{\sqrt{x}+x}$ **Answer:** Put $\sqrt{x}=t$ $\therefore \int \frac{dx}{\sqrt{x}+x}=2\log\left(1+\sqrt{x}\right)+C$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
20	Show that the function $y=ax+2a^2$ is a solution of the differential equation $2\left(\frac{dy}{dx}\right)^2+x\left(\frac{dy}{dx}\right)-y=0$. **Answer:** $y=ax+2a^2 \Rightarrow \frac{dy}{dx}=a$ $\text{LHS}=2\left(\frac{dy}{dx}\right)^2+x\left(\frac{dy}{dx}\right)-y$ $=2(a)^2+x(a)-\left(ax+2a^2\right)=0=\text{RHS}$	$\frac{1}{2}$ $\frac{1}{2}$
SECTION – B		
Q. Nos. 21 to 26 carry 2 marks each.		
21	Check if the relation R on the set $A=\{1,2,3,4,5,6\}$ defined as $R=\{(x,y): y \text{ is divisible by } x\}$ is (i) symmetric (ii) transitive. **Answer:** (i) As $(2,4)\in R$ but $(4,2)\notin R \Rightarrow R$ is not symmetric. (ii) Let $(a,b)\in R$ and $(b,c)\in R$ $\Rightarrow b=\lambda a$ and $c=\mu b$ Now, $c=\mu b=\mu(\lambda a)\Rightarrow (a,c)\in R$ $\Rightarrow R$ is transitive. **OR** Prove that: $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ **Answer:**	1 1

	$$\text{LHS}=\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}$$ $$=\frac{9}{4}\left[\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right]=\frac{9}{4}\cos^{-1}\frac{1}{3}$$ $$=\frac{9}{4}\sin^{-1}\left(\sqrt{1-\left(\frac{1}{3}\right)^2}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)=\text{RHS}$$	1 1
22	Find the value of $\frac{dy}{dx}$ at $\theta=\frac{\pi}{3}$, if $x=\cos\theta-\cos 2\theta$, $y=\sin\theta-\sin 2\theta$. **Answer:** $\frac{dx}{d\theta}=-\sin\theta+2\sin 2\theta$ $\frac{dy}{d\theta}=\cos\theta-2\cos 2\theta$ $\therefore \frac{dy}{dx}=\frac{\cos\theta-2\cos 2\theta}{-\sin\theta+2\sin 2\theta}$ $\therefore \left.\frac{dy}{dx}\right\|_{\theta=\frac{\pi}{3}}=\sqrt{3}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
23	Show that the function f defined by $f(x)=(x-1)e^x+1$ an increasing function for all $x>0$. **Answer:** $f'(x)=xe^x$ Now $x>0$ and $e^x>0$ for all x $\therefore f'(x)>0 \Rightarrow f$ is an increasing function.	1 $\frac{1}{2}$ $\frac{1}{2}$
24	Find $\|\vec{a}\|$ and $\|\vec{b}\|$, if $\|\vec{a}\|=2\|\vec{b}\|$ and $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=12$. **Answer:** $(\vec{a}+\vec{b})(\vec{a}-\vec{b})=12 \Rightarrow \|\vec{a}\|^2-\|\vec{b}\|^2=12$ $\Rightarrow 3\|\vec{b}\|^2=12 \Rightarrow \|\vec{b}\|=2$ Now, $\|\vec{a}\|^2=12+\|\vec{b}\|^2=16 \Rightarrow \|\vec{a}\|=4$ **OR** Find the unit vector perpendicular to each of the vectors $\vec{a}=4\hat{i}+3\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-\hat{j}+2\hat{k}$ **Answer:**	1 1

	$\vec{a}\times\vec{b}=7\hat{i}-6\hat{j}-10\hat{k}$ and $\lvert\vec{a}\times\vec{b}\rvert=\sqrt{185}$ Required unit vector $=\frac{1}{\sqrt{185}}\left(7\hat{i}-6\hat{j}-10\hat{k}\right)$	$1+\frac{1}{2}$ $\frac{1}{2}$
25	Find the equation of the plane with intercept 3 on the y-axis and parallel to xz – plane. **Answer:** Let required plane parallel to xz -plane is $y=k$ Given y -intercept is 3 $\Rightarrow k=3$ $\Rightarrow$ Equation of required plane is $y=3$	1 1
26	Find $\left[P(B\mid A)+P(A\mid B)\right]$, if $P(A)=\frac{3}{10}, P(B)=\frac{2}{5}$ and $P(A\cup B)=\frac{3}{5}$. **Answer:** $P(A\cap B)=\frac{3}{10}+\frac{2}{5}-\frac{3}{5}=\frac{1}{10}$ Now, $P(B\mid A)+P(A\mid B)$ $=\frac{P(A\cap B)}{P(A)}+\frac{P(A\cap B)}{P(B)}$ $=\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$	$\frac{1}{2}$ $\frac{1}{2}$ 1
	SECTION – C	
	Q. Nos. 27 to 32 carry 4 marks each.	
27	Prove that the relation R on Z, defined by $R=\{(x,y):(x-y)\text{ is divisible by }5\}$ is an equivalence relation. **Answer:** For reflexive $x-x=0$, for every $x\in Z$ is divisible by 5 $\Rightarrow (x,x)\in R$ For symmetric $(x,y)\in R\Rightarrow x-y$ is divisible by $5\Rightarrow y-x$ is divisible by 5 $\Rightarrow (y,x)\in R\Rightarrow R$ is symmetric. For transitive Let $(x,y)\in R$ and $(y,z)\in R$ $(x,y)\in R\Rightarrow x-y=5\lambda \quad ...(i)$ $(y,z)\in R\Rightarrow y-z=5\mu \quad ...(ii)$ adding (i) and (ii), $x-z=5(\lambda+\mu)=5k$ $\Rightarrow (x,z)\in R\Rightarrow R$ is transitive. Hence R is an equivalence relation.	1 1 2
28	If $y=\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$, then show that $\frac{dy}{dx}=\frac{-1}{2\sqrt{1-x^2}}$	

	Answer: Put $x=\cos 2\theta \Rightarrow \theta=\frac{1}{2}\cos^{-1}x$ $\therefore y=\sin^{-1}\left(\frac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{2}\right)=\sin^{-1}\left(\sin\left(\frac{\pi}{4}+\theta\right)\right)$ $\Rightarrow y=\frac{\pi}{4}+\theta=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x$ $\Rightarrow \frac{dy}{dx}=\frac{-1}{2\sqrt{1-x^2}}$ **OR** Verify the Rolle's Theorem for the function $f(x)=e^x\cos x$ in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. **Answer:** f is continuous in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. f is differentiable in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ with $f'(x)=e^x(\cos x-\sin x)$ Also, $f\left(\frac{-\pi}{2}\right)=f\left(\frac{\pi}{2}\right)=0$ All conditions of Rolle's Theorem are satisfied. So, there exist $c\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ such that $f'(c)=0\Rightarrow e^c(\cos c-\sin c)=0$ $\Rightarrow c=\frac{\pi}{4}\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$	1 2 $\frac{1}{2}$ $\frac{1}{2}$ 2 1 1
29	Evaluate: $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$ **Answer:** $I=\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx \quad ...(i)$ $\Rightarrow I=\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx=\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}dx \quad ...(ii)$ Adding (i) and (ii) $2I=\int_0^{\pi}\frac{\pi\sin x}{1+\cos^2 x}dx$ Putting $\cos x=t$ gives $I=\frac{\pi}{2}\int_{-1}^{1}\frac{dt}{1+t^2}$ $\Rightarrow I=\frac{\pi}{2}\left[\tan^{-1}t\right]_{-1}^{1}=\frac{\pi^2}{4}$	1 1 1 1
30	For the differential equation given below, find a particular solution satisfying the given condition. $(x+1)\frac{dy}{dx}=2e^{-y}+1$; $y=0$ when $x=0$	

Answer:

Given differential equation can be written as

$$\frac{dy}{2e^{-y}+1}=\frac{dx}{x+1}$$ 1

$$\Rightarrow \int \frac{e^{y}}{2+e^{y}}dy=\int \frac{dx}{x+1}$$

$$\Rightarrow \log\left|2+e^{y}\right|=\log\left|x+1\right|+\log C$$ 2

$$\Rightarrow 2+e^{y}=C\left(x+1\right)$$

when $x=0,\ y=0 \Rightarrow C=3$ $\frac{1}{2}$

$\therefore$ Required solution is $2+e^{y}=3\left(x+1\right)$ or $e^{y}=3x+1$ $\frac{1}{2}$

31 A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for at least 5 hours a day . He produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

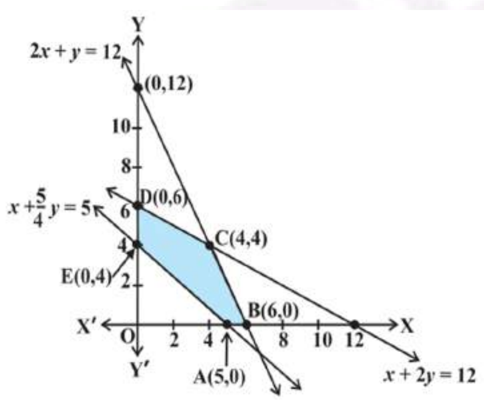
Items	Number of hours required on Machines		
	I	II	III
M	1	2	1
N	2	1	1.25

He makes a profit of ₹ 600 and ₹ 400 on items M and N respectively. How many of each item should he produce so as to maximize his profit assuming that she can sell all the items that he produced? What will be the maximum profit?

Answer:

Let x units of item M and y units of item N are produced.

For correct graph : $1\frac{1}{2}$ marks



Maximize $Z=600x+400y$

subject to

$x+2y\le 12$

$2x+y\le 12$

$x+1.25y\ge 5$

$x\ge 0,\ y\ge 0$ $1\frac{1}{2}$

Corner points values:

$Z_{A(5,0)}=3000,\ Z_{B(6,0)}=3600$

$Z_{C(4,4)}=4000,\ Z_{D(0,6)}=2400$

$Z_{E(0,4)}=1600$ $\frac{1}{2}$

$\therefore$ 4 units each of M and N must be produced to get maximum profit of Rs. 4,000 $\frac{1}{2}$

32 A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails. Hence find the mean of the number of tails.

Answer:

$P(\text{Head})=\frac{3}{4}, P(\text{Tail})=\frac{1}{4}$ 1

Let X = number of tails. Clearly X can be $0, 1, 2$ $\frac{1}{2}$

Probability distribution is given by

X	0	1	2
$P(X)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

$1\frac{1}{2}$

$\text{Mean} = \sum X.P(X) = \frac{1}{2}$ 1

OR

Suppose that 5 men out of 100 and 25 women out of 1000 are good orators. Assuming that there are equal number of men and women, find the probability of choosing a good orator.

Answer:

Let M be an event of choosing a man and N be an event of choosing a women. $\frac{1}{2}$

A be an event of choosing a good orator.

$P(M)=P(W)=\frac{1}{2};$

$P(A|M)=\frac{5}{100}=\frac{1}{20}, P(A|W)=\frac{25}{1000}=\frac{1}{40}$ 2

$P(A)=P(A|M).P(M)+P(A|W).P(W)$

$=\frac{1}{20}\times\frac{1}{2}+\frac{1}{40}\times\frac{1}{2}=\frac{3}{80}$ $1+\frac{1}{2}$

SECTION – D

Q. Nos. 33 to 36 carry 6 marks each.

33 Using properties of determinants prove that: $\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix} = a^3+b^3+c^3-3abc$.

Answer:

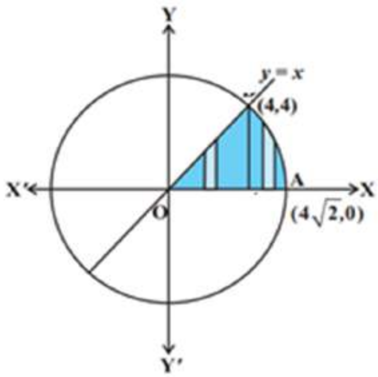
$$\text{LHS} = \Delta = \begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix}$$

$C_3 \to C_3 + C_2$

$$\Delta = \begin{vmatrix} a-b & b+c & a+b+c \\ b-c & c+a & a+b+c \\ c-a & a+b & a+b+c \end{vmatrix}$$ 1

	taking $(a+b+c)$ common from C_3 and applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$ $\Delta = (a+b+c)\begin{vmatrix} a-2b+c & b-a & 0 \\ b-2c+a & c-b & 0 \\ c-a & a+b & 1 \end{vmatrix}$ expanding along C_3, $\Delta = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ $= a^3+b^3+c^3-3abc = \text{RHS}$ **OR** If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that $A^3 - 4A^2 - 3A + 11I = O$. Hence find A^{-1}. **Answer:** $A^2 = \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$ $A^3 = \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix}$ $LHS = A^3 - 4A^2 - 3A + 11I$ $= \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix} - 4\begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix} - 3\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$ Now, $A^{-1} = -\frac{1}{11}(A^2 - 4A - 3I)$ $= -\frac{1}{11}\begin{bmatrix} 2 & -5 & -3 \\ -7 & 1 & 5 \\ 4 & 1 & -6 \end{bmatrix}$	1+1+1 1 1 1 1 2 1 1
34	Find the intervals on which the function $f(x) = (x-1)^3 (x-2)^2$ is (a) strictly increasing (b) strictly decreasing.	

		Marks
	Answer:	
	$f(x)=(x-1)^3(x-2)^2$	
	$\Rightarrow f'(x)=(x-1)^2(x-2)(5x-8)$	2
	(a) for strictly increasing, $f'(x)>0$	
	$\Rightarrow (x-1)^2(x-2)(5x-8)>0$	1
	$\Rightarrow (x-2)(5x-8)>0 \qquad (\text{as } x\neq 1)$ $\Rightarrow x\in\left(-\infty,\frac{8}{5}\right)\cup(2,\infty) \qquad (\text{as } x\neq 1)$ $\therefore x\in(-\infty,1)\cup\left(1,\frac{8}{5}\right)\cup(2,\infty)$	$1\frac{1}{2}$
	(b) for strictly decreasing, $f'(x)<0$	
	$\Rightarrow x\in\left(\frac{8}{5},2\right)$	$1\frac{1}{2}$
	OR	
	Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its side. Also, find the maximum volume.	
	Answer: Let sides of a rectangle are x cm and y cm.	
	Given $2x+2y=36 \Rightarrow y=18-x$	1
	Volume, $V=\pi x^2 y=\pi x^2(18-x)=\pi\left(18x^2-x^3\right)$	1
	$\Rightarrow \frac{dV}{dx}=\pi\left(36x-3x^2\right)$	1
	For maxima/minima, put $\frac{dV}{dx}=0$	
	$\Rightarrow x=12\text{ cm} \quad (\because x\neq 0)$	1
	Again, $\frac{d^2V}{dx^2}=\pi(36-6x)$ $\Rightarrow \left.\frac{d^2V}{dx^2}\right\|_{x=12\text{ cm}}=-36\pi<0$	1
	$\therefore$ Volume is maximum when $x=12$ cm.	
	Also, $y=(18-x)\text{ cm}=6\text{ cm}$	
	Dimension of rectangle are $12\text{ cm}\times 6\text{cm}$ Maximum volume $=\pi x^2 y=864\pi\text{ cm}^3$	$\frac{1}{2}+\frac{1}{2}$

35	Find the area of the region lying in the first quadrant and enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$. **Answer:** Point of intersection of $y = x$ and $x^2 + y^2 = 32$ in first quadrant is $(4,4)$.  Correct figure $\text{Area} = \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$ $= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x}{2}\sqrt{32 - x^2} + 16\sin^{-1}\left(\frac{x}{4\sqrt{2}}\right)\right]_4^{4\sqrt{2}}$ $= 8 + (4\pi - 8) = 4\pi$	1 1 1 2 1
36	Show that the lines $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{b} + \mu\vec{a}$ are coplanar and the plane containing them is given by $\vec{r}.(\vec{a} \times \vec{b}) = 0$. **Answer:** Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are coplanar if $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = 0$ Now, $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)$ $= (\vec{b} - \vec{a}).(\vec{b} \times \vec{a})$ $\quad (\because \vec{a}_1 = \vec{a}, \vec{b}_1 = \vec{b}, \vec{a}_2 = \vec{b}, \vec{b}_2 = \vec{a})$ $= \vec{b}.(\vec{b} \times \vec{a}) - \vec{a}.(\vec{b} \times \vec{a})$ $= [\vec{b} \quad \vec{b} \quad \vec{a}] - [\vec{a} \quad \vec{b} \quad \vec{a}] = 0$ Hence the lines are coplanar. Equation of plane containing them is given by, $(\vec{r} - \vec{a}).(\vec{b} \times \vec{a}) = 0$ $\quad [\because (\vec{r} - \vec{a}).\vec{n} = 0]$ $\Rightarrow \vec{r}.(\vec{b} \times \vec{a}) - \vec{a}.(\vec{b} \times \vec{a}) = 0$ $\Rightarrow \vec{r}.(\vec{b} \times \vec{a}) = 0$ $\quad (\because [\vec{a} \quad \vec{b} \quad \vec{a}] = 0)$	4 2