

**XII MATHEMATICS
QUESTION PAPER CODE 65/5/1
EXPECTED ANSWER/VALUE POINTS**

| Q. No. | VALUE POINTS | Marks |
|---|---|-------|
| SECTION - A | | |
| Question Numbers 1 to 20 carry 1 mark each. | | |
| Q. Nos. 1 to 10 are multiple choice questions of 1 mark each. Select the correct option: | | |
| 1 | If A is a square matrix of order 3, such that $A(\text{adj}A) = 10I$, then $ \text{adj}A $ is equal to (a) 1 (b) 10 (c) 100 (d) 101 Answer: (c) 100 | 1 |
| 2 | If A is a 3×3 matrix such that $ A = 8$, then $ 3A $ equals. (a) 8 (b) 24 (c) 72 (d) 216 Answer: (d) 216 | 1 |
| 3 | If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is equal to (a) $25y$ (b) $5y$ (c) $-25y$ (d) $15y$ Answer: (a) $25y$ | 1 |
| 4 | $\int x^2 e^{x^3} dx$ equals (a) $\frac{1}{3}e^{x^3} + C$ (b) $\frac{1}{3}e^{x^4} + C$ (c) $\frac{1}{2}e^{x^3} + C$ (d) $\frac{1}{2}e^{x^2} + C$ Answer: (a) $\frac{1}{3}e^{x^3} + C$ | 1 |
| 5 | If $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along three mutually perpendicular directions, then (a) $\hat{i} \cdot \hat{j} = 1$ (b) $\hat{i} \times \hat{j} = 1$ (c) $\hat{i} \cdot \hat{k} = 0$ (d) $\hat{i} \times \hat{k} = 0$ Answer: (c) $\hat{i} \cdot \hat{k} = 0$ | 1 |
| 6 | ABCD is a rhombus whose diagonals intersect at E. Then $\vec{EA} + \vec{EB} + \vec{EC} + \vec{ED}$ equals (a) $\vec{0}$ (b) \vec{AD} (c) $2\vec{BC}$ (d) $2\vec{AD}$ Answer: (a) $\vec{0}$ | 1 |
| 7 | The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{4-z}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2}$ are mutually perpendicular if the value of k is (a) $-\frac{2}{3}$ (b) $\frac{2}{3}$ (c) -2 (d) 2 Answer: (a) $-\frac{2}{3}$ | 1 |

| | | |
|--|---|---|
| 8 | <p>The graph of the inequality $2x+3y > 6$ is</p> <p>(a) half plane that contains the origin (b) half plane that neither contains the origin nor the points of the line $2x+3y = 6$ (c) whole XOY-plane excluding the points on the line $2x+3y = 6$ (d) entire XOY-plane</p> <div style="border: 1px solid black; padding: 5px; width: fit-content;"> Answer: (b) half plane that neither contains the origin nor the points of the line $2x+3y = 6$ </div> | 1 |
| 9 | <p>A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is</p> <p>(a) $\frac{1}{3}$ (b) $\frac{4}{13}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content;"> Answer: (c) $\frac{1}{4}$ </div> | 1 |
| 10 | <p>A die is thrown once. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then, $P(A \cup B)$ is</p> <p>(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) 0 (d) 1</p> <div style="border: 1px solid black; padding: 5px; width: fit-content;"> Answer: (d) 1 </div> | 1 |
| In Q. Nos. 11 to 15, fill in the blanks with correct word/sentence: | | |
| 11 | <p>A relation in a set A is called _____ relation, if each element of A is related to itself.</p> <div style="border: 1px solid black; padding: 5px; width: fit-content;"> Answer: reflexive </div> | 1 |
| 12 | <p>If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, then $A =$ _____</p> <div style="border: 1px solid black; padding: 5px; width: fit-content;"> Answer: $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$ </div> | 1 |
| 13 | <p>The least value of the function $f(x) = ax + \frac{b}{x}$ ($a > 0, b > 0, x > 0$) is _____.</p> <div style="border: 1px solid black; padding: 5px; width: fit-content;"> Answer: $2\sqrt{ab}$ </div> | 1 |
| 14 | <p>The integrating factor of the differential equation $x \frac{dy}{dx} + 2y = x^2$ is _____.</p> <div style="border: 1px solid black; padding: 5px; width: fit-content;"> Answer: x^2 </div> <p style="text-align: center;">OR</p> <p>The degree of the differential equation $1 + \left(\frac{dy}{dx}\right)^2 = x$ is _____.</p> <div style="border: 1px solid black; padding: 5px; width: fit-content;"> Answer: 2 </div> | 1 |

| | | |
|---|---|--|
| 15 | <p>The vector equation of a line which passes through the points $(3, 4, -7)$ and $(1, -1, 6)$ is _____.</p> <p>Answer: $\vec{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda(-2\hat{i} - 5\hat{j} + 13\hat{k})$ Or, $\vec{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda(2\hat{i} + 5\hat{j} - 13\hat{k})$</p> <p style="text-align: center;">OR</p> <p>The line of shortest distance between two skew lines is _____ to both the lines.</p> <p>Answer: perpendicular</p> | 1 1 |
| Q. 16 to 20 are very short answer questions. | | |
| 16 | <p>Find the value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$.</p> <p>Answer: $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right] = -\sin^{-1}\left[\sin\left(2\pi + \frac{\pi}{8}\right)\right]$ $= -\frac{\pi}{8}$</p> | $\frac{1}{2}$ $\frac{1}{2}$ |
| 17 | <p>For $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, write A^{-1}.</p> <p>Answer: $A = 1$ $A^{-1} = \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$</p> | 1/2 1/2 |
| 18 | <p>If the function f defined as $f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & x \neq 3 \\ k, & x = 3 \end{cases}$ is continuous at $x = 3$, find the value of k.</p> <p>Answer: $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = 6, \therefore k = 6$</p> | $\frac{1}{2} + \frac{1}{2}$ |
| 19 | <p>If $f(x) = x^4 - 10$, then find the approximate value of $f(2.1)$.</p> <p>Answer: $f(2.1) \approx f(2) + (0.1)f'(2)$ $= 9.2$</p> <p style="text-align: center;">OR</p> <p>Find the slope of the tangent to the curve $y = 2\sin^2(3x)$ at $x = \frac{\pi}{6}$.</p> <p>Answer: $\frac{dy}{dx} = 6\sin 6x$ \therefore slope of tangent = 0</p> | 1/2 1/2 1/2 1/2 |
| 20 | <p>Find the value of $\int_1^4 x - 5 dx$.</p> <p>Answer: $\int_1^4 x - 5 dx = \int_1^4 (5 - x) dx = \frac{15}{2}$</p> | 1/2 + 1/2 |

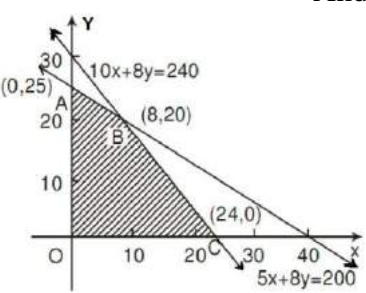
SECTION – B

Q. Nos. 21 to 26 carry 2 marks each.

| | | |
|----|--|---|
| 21 | <p>If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, then show that $(f \circ f)(x) = x$, for all $x \neq \frac{2}{3}$. Also, write inverse of f.</p> <p>Answer:</p> $(f \circ f)(x) = f\left(\frac{4x+3}{6x-4}\right) = \frac{4\left(\frac{4x+3}{6x-4}\right) + 3}{6\left(\frac{4x+3}{6x-4}\right) - 4} = \frac{34x}{34} = x$ <p>Now, $(f \circ f)(x) = x \Rightarrow f^{-1} = f$ or $f^{-1}(x) = \frac{4x+3}{6x-4}$</p> <p style="text-align: center;">OR</p> <p>Check if the relation R in the set \mathbf{R} of real numbers defined as $R = \{(a, b) : a < b\}$ is (i) symmetric, (ii) transitive.</p> <p>Answer:</p> <p>(i) $1, 2 \in \mathbf{R}$ such that $1 < 2 \Rightarrow (1, 2) \in R$, but since 2 is not less than 1 $\Rightarrow (2, 1) \notin R$. Hence R is not symmetric.</p> <p>(ii) Let $(a, b) \in R$ and $(b, c) \in R, \therefore a < b$ and $b < c$ $\Rightarrow a < c \Rightarrow (a, c) \in R \therefore R$ is transitive.</p> | <p>$1 \frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p> |
| 22 | <p>Find $\int \frac{x}{x^2+3x+2} dx$</p> <p>Answer:</p> $\int \frac{x}{x^2+3x+2} dx = \int \frac{x}{(x+1)(x+2)} dx = \int \left(\frac{-1}{x+1} + \frac{2}{x+2} \right) dx$ $= -\log x+1 + 2\log x+2 + C$ | <p>1</p> <p>1</p> |
| 23 | <p>If $x = a \cos \theta; y = b \sin \theta$, then find $\frac{d^2y}{dx^2}$.</p> <p>Answer:</p> $\frac{dx}{d\theta} = -a \sin \theta, \frac{dy}{d\theta} = b \cos \theta \Rightarrow \frac{dy}{dx} = -\frac{b}{a} \cot \theta$ $\frac{d^2y}{dx^2} = \frac{b}{a} \operatorname{cosec}^2 \theta \left(\frac{-1}{a \sin \theta} \right) = -\frac{b}{a^2} \operatorname{cosec}^3 \theta$ <p style="text-align: center;">OR</p> <p>Find the differential of $\sin^2 x$ w.r.t. $e^{\cos x}$.</p> <p>Answer:</p> <p>Let $y = \sin^2 x$ and $z = e^{\cos x} \therefore \frac{dy}{dx} = 2 \sin x \cos x$ and $\frac{dz}{dx} = -\sin x e^{\cos x}$</p> $\therefore \frac{dy}{dz} = \frac{2 \sin x \cos x}{-\sin x e^{\cos x}} = \frac{-2 \cos x}{e^{\cos x}} \text{ or } -2 \cos x e^{-\cos x}$ | <p>$\frac{1}{2} + \frac{1}{2}$</p> <p>$\frac{1}{2} + \frac{1}{2}$</p> <p>$\frac{1}{2} + \frac{1}{2}$</p> |

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| 24 | <p>Evaluate $\int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2} \right] e^{2x} dx$</p> <p>Answer:</p> <p>Put $2x = t, \therefore dx = \frac{1}{2} dt$</p> <p>$\therefore I = \int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2} \right] e^{2x} dx = \int_2^4 \left[\frac{1}{t} - \frac{1}{t^2} \right] e^t dt$</p> <p>$= \left[\frac{1}{t} e^t \right]_2^4 = \frac{e^4}{4} - \frac{e^2}{2}$</p> | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$ |
| 25 | <p>Find the value of $\int_0^1 x(1-x)^n dx$.</p> <p>Answer:</p> <p>$\int_0^1 x(1-x)^n dx = \int_0^1 (1-x)(1-1+x)^n dx = \int_0^1 (x^n - x^{n+1}) dx$</p> <p>$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2} \right]_0^1 = \frac{1}{n+1} - \frac{1}{n+2}$ or $\frac{1}{(n+1)(n+2)}$</p> | 1 $\frac{1}{2} + \frac{1}{2}$ |
| 26 | <p>Given two independent events A and B such that $P(A) = 0.3$ and $P(B) = 0.6$, find $P(A' \cap B')$.</p> <p>Answer:</p> <p>$P(A' \cap B') = P(A')P(B')$</p> <p>$= (0.7)(0.4) = 0.28$</p> | 1 1 |
| SECTION - C | | |
| Q. Nos. 27 to 32 carry 4 marks each. | | |
| 27 | <p>Solve for $x : \sin^{-1}(1-x) - 2 \sin^{-1} x = \frac{\pi}{2}$.</p> <p>Answer:</p> <p>$\sin^{-1}(1-x) - 2 \sin^{-1} x = \frac{\pi}{2} \Rightarrow (1-x) = \sin \left(\frac{\pi}{2} + 2 \sin^{-1} x \right)$</p> <p>$\Rightarrow (1-x) = \cos(2 \sin^{-1} x) \Rightarrow 1-x = 1-2x^2$</p> <p>$\therefore 2x^2 - x = 0 \Rightarrow x = 0, x = \frac{1}{2}$</p> <p>since $x = \frac{1}{2}$ does not satisfy the given equation</p> <p>$\therefore x = 0$ is the required solution.</p> | $\frac{1}{2}$ 1 1 $\frac{1}{2}$ |
| 28 | <p>If $y = (\log x)^x + x^{\log x}$, then find $\frac{dy}{dx}$</p> | |

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| | <p>Answer:</p> $y = (\log x)^x + x^{\log x} = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ $\therefore \log u = x \log(\log x) \text{ and } \log v = (\log x)^2$ $\frac{du}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x) \right] \text{ and } \frac{dv}{dx} = x^{\log x} \cdot \frac{2 \log x}{x}$ $\Rightarrow \frac{dy}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x) \right] + x^{\log x} \cdot \frac{2 \log x}{x}$ | <p>1</p> <p>$\frac{1}{2}$</p> <p>1+1</p> <p>$\frac{1}{2}$</p> |
| 29 | <p>Solve the differential equation: $x \sin\left(\frac{y}{x}\right) \frac{dy}{dx} + x - y \sin\left(\frac{y}{x}\right) = 0$, given that $x = 1$ when $y = \frac{\pi}{2}$.</p> <p>Answer:</p> <p>Given differential equation gives $\frac{dy}{dx} = \frac{y \sin\left(\frac{y}{x}\right) - x}{x \sin\left(\frac{y}{x}\right)}$</p> $\frac{y}{x} = v \Rightarrow y = vx \text{ and } \frac{dy}{dx} = v + x \frac{dv}{dx}$ $\therefore v + x \frac{dv}{dx} = \frac{v \sin v - 1}{\sin v} \Rightarrow x \frac{dv}{dx} = -\frac{1}{\sin v}$ $\Rightarrow \int \sin v \, dv = -\int \frac{1}{x} \, dx$ $\therefore \cos v = \log x + C \text{ or } \cos\left(\frac{y}{x}\right) = \log x + C$ <p>Given $x = 1$ when $y = \frac{\pi}{2} \Rightarrow C = 0$</p> $\therefore \cos\left(\frac{y}{x}\right) = \log x \text{ is the required solution.}$ | <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> |
| 30 | <p>If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.</p> <p>Answer:</p> <p>Diagonal vectors are: $\vec{a} + \vec{b} = 3\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{a} - \vec{b} = -\hat{i} - 2\hat{j} + 8\hat{k}$ (or, $\vec{b} - \vec{a} = \hat{i} + 2\hat{j} - 8\hat{k}$)</p> $\therefore \text{unit vectors are } \frac{(\vec{a} + \vec{b})}{ \vec{a} + \vec{b} } = \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k} \text{ and } \frac{(\vec{a} - \vec{b})}{ \vec{a} - \vec{b} } = -\frac{1}{\sqrt{69}}\hat{i} - \frac{2}{\sqrt{69}}\hat{j} + \frac{8}{\sqrt{69}}\hat{k}$ <p style="text-align: center;">OR</p> <p>Using vectors, find the area of the triangle ABC with vertices A (1, 2, 3), B(2, -1, 4) and C(4, 5, -1).</p> | <p>1+1</p> <p>1+1</p> |

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|----------------------|--|--|--|--|---|---|--|--------|--|--|--|--|--|--|----------------------|----------------------|----------------------|---------------------|--|---------|---|--------------------|--------------------|-------------------|--|----------------------|-------------------------------------|--|--|--|--|---|
| | <p>Answer:</p> $\text{area of } \Delta ABC = \frac{1}{2} \text{cross product of any two side vectors} $ $\vec{AB} = \hat{i} - 3\hat{j} + \hat{k} \text{ and } \vec{BC} = 2\hat{i} + 6\hat{j} - 5\hat{k}$ $\vec{AB} \times \vec{BC} = 9\hat{i} + 7\hat{j} + 12\hat{k}$ $\therefore \text{area of } \Delta ABC = \frac{1}{2} \sqrt{81 + 49 + 144} = \frac{1}{2} \sqrt{274}$ | <p>1</p> <p>$\frac{1}{2} + \frac{1}{2}$</p> <p>1</p> <p>1</p> | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 31 | <p>A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A requires 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. Given that total time for cutting is 3 hours 20 minutes and for assembling 4 hours. The profit for type A souvenir is ₹100 each and for type B souvenir, profit is ₹120 each. How many souvenirs of each type should the company manufacture in order to maximize the profit? Formulate the problem as an LPP and solve it graphically.</p> <p>Answer: Let the company manufacture 'x' number of souvenirs of Type A And, 'y' number of souvenirs of Type B</p>  <p>\therefore LPP is: Maximise $P = 100x + 120y$ subject to $5x + 8y \leq 200$ $10x + 8y \leq 240$ $x \geq 0, y \geq 0$</p> <p>Correct Graph</p> <p>$P(A) = ₹ 3,000$ $P(B) = ₹ 3,200$ (Max.) $P(C) = ₹ 2,400$</p> <p>\therefore For Maximum profit, No. of souvenirs of Type A = 8 No. of souvenirs of Type B = 20</p> | <p>$\frac{1}{2}$</p> <p>1</p> <p>$1\frac{1}{2}$</p> <p>1</p> | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 32 | <p>Three rotten apples are mixed with seven fresh apples. Find the probability distribution of the number of rotten apples, if three apples are drawn one by one with replacement. Find the mean of the number of rotten apples.</p> <p>Answer: Let X represents the number of rotten apples drawn.</p> <table border="0" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;">X :</td> <td style="width: 20%;">0</td> <td style="width: 20%;">1</td> <td style="width: 20%;">2</td> <td style="width: 20%;">3</td> <td></td> </tr> <tr> <td>P(X) :</td> <td>$\frac{7}{10} \cdot \frac{7}{10} \cdot \frac{7}{10}$</td> <td>$3 \cdot \frac{7}{10} \cdot \frac{7}{10} \cdot \frac{3}{10}$</td> <td>$3 \cdot \frac{7}{10} \cdot \frac{3}{10} \cdot \frac{3}{10}$</td> <td>$\frac{3}{10} \cdot \frac{3}{10} \cdot \frac{3}{10}$</td> <td></td> </tr> <tr> <td></td> <td>$= \frac{343}{1000}$</td> <td>$= \frac{441}{1000}$</td> <td>$= \frac{189}{1000}$</td> <td>$= \frac{27}{1000}$</td> <td></td> </tr> <tr> <td>X.P(X):</td> <td>0</td> <td>$\frac{441}{1000}$</td> <td>$\frac{378}{1000}$</td> <td>$\frac{81}{1000}$</td> <td></td> </tr> <tr> <td>Mean = $\sum X P(X)$</td> <td colspan="4">$= \frac{900}{1000} = \frac{9}{10}$</td> <td></td> </tr> </table> | X : | 0 | 1 | 2 | 3 | | P(X) : | $\frac{7}{10} \cdot \frac{7}{10} \cdot \frac{7}{10}$ | $3 \cdot \frac{7}{10} \cdot \frac{7}{10} \cdot \frac{3}{10}$ | $3 \cdot \frac{7}{10} \cdot \frac{3}{10} \cdot \frac{3}{10}$ | $\frac{3}{10} \cdot \frac{3}{10} \cdot \frac{3}{10}$ | | | $= \frac{343}{1000}$ | $= \frac{441}{1000}$ | $= \frac{189}{1000}$ | $= \frac{27}{1000}$ | | X.P(X): | 0 | $\frac{441}{1000}$ | $\frac{378}{1000}$ | $\frac{81}{1000}$ | | Mean = $\sum X P(X)$ | $= \frac{900}{1000} = \frac{9}{10}$ | | | | | <p>$\frac{1}{2}$</p> <p>2</p> <p>1</p> <p>$\frac{1}{2}$</p> |
| X : | 0 | 1 | 2 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| P(X) : | $\frac{7}{10} \cdot \frac{7}{10} \cdot \frac{7}{10}$ | $3 \cdot \frac{7}{10} \cdot \frac{7}{10} \cdot \frac{3}{10}$ | $3 \cdot \frac{7}{10} \cdot \frac{3}{10} \cdot \frac{3}{10}$ | $\frac{3}{10} \cdot \frac{3}{10} \cdot \frac{3}{10}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | $= \frac{343}{1000}$ | $= \frac{441}{1000}$ | $= \frac{189}{1000}$ | $= \frac{27}{1000}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| X.P(X): | 0 | $\frac{441}{1000}$ | $\frac{378}{1000}$ | $\frac{81}{1000}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Mean = $\sum X P(X)$ | $= \frac{900}{1000} = \frac{9}{10}$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

OR

In a shop X, 30 tins of ghee of type A and 40 tins of ghee of type B which look alike, are kept for sale. While in a shop Y, similar 50 tins of ghee of type A and 60 tins of ghee of type B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type B. Find the probability that it is purchased from shop Y.

Answer:

E_1 :selecting shop X

E_2 :selecting shop Y

A :purchased tin is of type B

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$P(A|E_1) = \frac{4}{7}, P(A|E_2) = \frac{6}{11}$$

$$P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$\begin{aligned} &= \frac{\frac{1}{2} \cdot \frac{6}{11}}{\frac{1}{2} \cdot \frac{4}{7} + \frac{1}{2} \cdot \frac{6}{11}} \\ &= \frac{21}{43} \end{aligned}$$

$\frac{1}{2}$

1

2

$\frac{1}{2}$

SECTION – D

Q. Nos. 33 to 36 carry 6 marks each.

33 Find the vector and Cartesian equations of the line which is perpendicular to the lines with equations $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point (1, 1, 1). Also find the angle between the given lines.

Answer:

Let equation of required line is $\frac{x-1}{a} = \frac{y-1}{b} = \frac{z-1}{c}$ (i)

Since this line is perpendicular to $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$,

$$a + 2b + 4c = 0 \quad \text{.....(ii)}$$

$$2a + 3b + 4c = 0 \quad \text{.....(iii)}$$

Solving (ii) and (iii), $\frac{a}{-4} = \frac{b}{4} = \frac{c}{-1}$

\therefore DR's of line in cartesian form is : -4, 4, -1

Equation of line in Cartesian form is: $\frac{x-1}{-4} = \frac{y-1}{4} = \frac{z-1}{-1}$

Vector form of line is $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(-4\hat{i} + 4\hat{j} - \hat{k})$

Let θ be the angle between given lines.

$$\cos \theta = \frac{1(2) + 2(3) + 4(4)}{\sqrt{1+4+16} \sqrt{4+9+16}} = \frac{24}{\sqrt{21} \sqrt{29}} \quad \therefore \theta = \cos^{-1} \left(\frac{24}{\sqrt{21} \sqrt{29}} \right)$$

$\frac{1}{2}$

1

1

1

1

$1 + \frac{1}{2}$

34 Using integration find the area of the region bounded between the two circles $x^2 + y^2 = 9$ and $(x-3)^2 + y^2 = 9$.

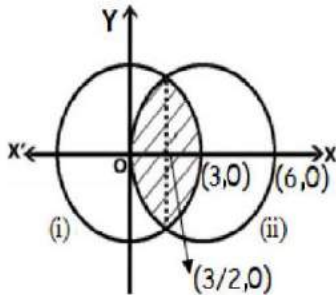
Answer:

Correct Figure

1

Point of intersection of, $x^2 + y^2 = 9; (x-3)^2 + y^2 = 9 \Rightarrow (x-3)^2 - x^2 = 0 \Rightarrow x = \frac{3}{2}$

1



$$\begin{aligned} \text{Required area} &= 2 \left[\int_0^{\frac{3}{2}} \sqrt{9 - (x-3)^2} dx + \int_{\frac{3}{2}}^3 \sqrt{9 - x^2} dx \right] \\ &= 4 \left[\int_{\frac{3}{2}}^3 \sqrt{9 - x^2} dx \right] \end{aligned}$$

$1\frac{1}{2}$

$$= 4 \left[\frac{x}{2} \sqrt{9 - x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} \right]_{\frac{3}{2}}^3 = \left(6\pi - \frac{9\sqrt{3}}{2} \right)$$

$1\frac{1}{2} + 1$

OR

Evaluate the following integral as the limit of sums $\int_1^4 (x^2 - x) dx$.

Answer:

$$\int_1^4 (x^2 - x) dx = \lim_{h \rightarrow 0} h \left[f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h) \right]$$

1

where $f(x) = (x^2 - x)$ and $nh = 3$

1

$$\therefore \int_1^4 (x^2 - x) dx$$

$$= \lim_{h \rightarrow 0} h \left[(1-1) + (1+h^2 + 2h - h - 1) + (1+4h^2 + 4h - 2h - 1) + \dots + (1+(n-1)^2 h^2 + 2(n-1)h - (n-1)h - 1) \right]$$

2

$$= \lim_{h \rightarrow 0} h \left[h^2 (1^2 + 2^2 + 3^2 + \dots + (n-1)^2) + h(1+2+3+\dots+(n-1)) \right]$$

1

$$= \lim_{h \rightarrow 0} \left[\frac{nh(nh-h)(2nh-h)}{6} + \frac{(nh(nh-h))}{2} \right] = 9 + \frac{9}{2} = \frac{27}{2}$$

1

35 Find the minimum value of $(ax + by)$, where $xy = c^2$.

Answer:

Let $S = ax + by$, where $y = \frac{c^2}{x} \therefore S = ax + \frac{bc^2}{x}$

1

$$\frac{dS}{dx} = a - \frac{bc^2}{x^2}$$

1

$$\frac{dS}{dx} = 0 \Rightarrow x^2 = \frac{bc^2}{a} \text{ or } x = \sqrt{\frac{b}{a}} \cdot c$$

$1\frac{1}{2}$

$$\frac{d^2S}{dx^2} \Big|_{x=\sqrt{\frac{b}{a}} \cdot c} = \frac{2bc^2}{x^3} = 2bc^2 \left[\sqrt{\frac{a}{b}} \frac{1}{c} \right]^3 > 0 \text{ for } a, b, c > 0 \text{ and } x = \sqrt{\frac{b}{a}} \cdot c$$

$1\frac{1}{2}$

| | | |
|----|---|--|
| | $\therefore \text{minimum value} = a\sqrt{\frac{b}{a}} \cdot c + b \cdot \frac{c^2}{c} \sqrt{\frac{a}{b}} = 2\sqrt{ab} \cdot c$ | 1 |
| 36 | <p>If a, b, c are $p^{\text{th}}, q^{\text{th}},$ and r^{th} terms respectively of a G.P, then prove that</p> $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$ <p>Answer: $a = AR^{p-1}, b = AR^{q-1}, c = AR^{r-1}$</p> $\therefore \Delta = \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix} = \log A \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log R \begin{vmatrix} p & p & 1 \\ q & q & 1 \\ r & r & 1 \end{vmatrix} - \log R \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix}$ $= 0 + 0 + 0 = 0$ <p style="text-align: center;">OR</p> <p>If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}.</p> <p>Using A^{-1}, solve the following system of equations: $2x - 3y + 5z = 11$ $3x + 2y - 4z = -5$ $x + y - 2z = -3$</p> <p>Answer: $A = 2(0) + 3(-2) + 5(1) = -1$</p> $\Rightarrow A^{-1} = - \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \text{ (1 mark for any 4 correct co-factors)}$ <p>Given equations can be written as $AX = B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$</p> $\therefore X = A^{-1}B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\Rightarrow x = 1, y = 2, z = 3$ | <p>$1\frac{1}{2}$</p> <p>1+1+1+1</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>2</p> <p>1</p> <p>1</p> <p>1</p> |