## Section - 1

**Question 1:** A current carrying wire heats a metal rod. The wire provides a constant power P to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as $T(t) = T_0 (1 + \beta t^{1/4})$ where β is a constant with appropriate dimension of temperature. The heat capacity of metal is:

(a) $\frac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$ (b) $\frac{4P(T(t)-T_0)^2}{\beta^4 T_0^3}$ (c) $\frac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$ (d) $\frac{4P(T(t)-T_0)}{\beta^4 T_0^2}$

**Solution:**

Heat capacity = dQ/dt

H = dQ/dT => dQ/dt = H . dT/dt

P = H . dT/dt

= $(H . T_0/4)\ \beta\ t^{-3/4}$

=> $4P/T_0\ \beta = Ht^{-3/4}$

$$t^{-3/4} = \left(\frac{T-T_0}{T_0\beta}\right)^3$$

$$H = \frac{4P(T-T_0)^3}{T_0^4\beta^4}$$

**Question 2:** In a capillary tube of radius 0.2 mm the water rises up to height of 7.5 cm with angle of contact equal to zero. If another capillary with same radius but of different material dipped in the same liquid. The height of water raised in capillary will be, if angle of contact becomes 60°.

(a) 7.5 cm (b) 15 cm (c) 3.75 cm (d) 30 cm

**Solution:**

T = Rhρg/2cosθ

h/cosθ = constant

7.5/1 = h'/(1/2)

=>h' = 3.75 cm

**Question 3:** A sample of $_{19}K^{40}$ disintegrates into two nuclei Ca & Ar with decay constant $\lambda_{Ca} = 4.5 \times 10^{-10}$ $S^{-1}$ and $\lambda_{Ar} = 0.5 \times 10^{-10}$ $S^{-1}$ respectively. The time after which 99% of $_{19}K^{40}$ gets decayed is:

(a) $6.2 \times 10^9$ sec (b) $9.2 \times 10^9$ sec (c) $7.2 \times 10^9$ sec (d) $4.2 \times 10^9$ sec

**Solution:**

$\lambda = \lambda$

$(1/100)\ N_0 = N_0\ e^{-\lambda t}$

$\ln(1/100) = -(\lambda_1 + \lambda_2)t + \ln 100 = +(\lambda_1 + \lambda_2)t^2$

$\Rightarrow (2.303 \times 2)/(5 \times 10^{-10}) = t$

Or $t = 9.2 \times 10^9$ sec

**Question 4:** Consider a spherical gaseous cloud of mass density $\rho(r)$ in a free space where r is the radial distance from its centre. The gaseous cloud is made of particles of equal mass m moving in circular orbits about their common centre with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant with time. The particle number density $n(r) = \rho(r)/m$ is:

(g = universal gravitational constant)

(a) $3K/\pi r^2 m^2 G$ (b) $K/2\pi r^2 m^2 G$ (c) $K/\pi r^2 m^2 G$ (d) $K/6\pi r^2 m^2 G$

**Solution:**

$GMm/r = mv^2/r = 2/r \times ½ \times mv^2 = 2k/r$

$M = 2kr/Gm$

$4\pi r^2\ dr\ \rho = 2k\ dr/Gm$

Or $\rho = k/2\pi G m r^2$

## Section - 2

**Question 5:** A thin spherical insulating shell of radius R caries a uniformly distributed charge such that the potential at its surface is $V_0$. A hole with small area $\alpha 4\pi R^2$ ($\alpha <<< 1$) is made in the shell without effecting the rest of the shell. Which one of the following is correct.

(a) The magnitude of Vector E at a point located on a line passing through the hole and shell's centre on a distance 2R

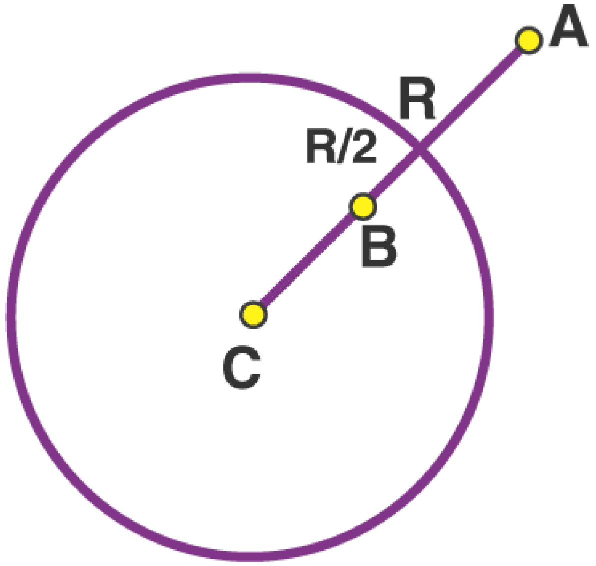
from the centre of spherical shell will be reduced by $\alpha V_0/2R$

(b) Potential at the centre of shell is reduced by $2\alpha V_0$ .

(c) The magnitude of $\vec{E}$ at the centre of shell reduced by $\alpha V_0/2R$

(d) The ratio of potential at the centre of the shell to that of the point at (1/2) R from centre towards the hole will be $(1-\alpha)/(1-2\alpha)$

**Solution:**
$dq = Q/4\pi R^2 - dA = Q\alpha$




$V_c = KQ/R - K\alpha Q/R = V_0(1- \alpha)$
$V_B = KQ/R - K\alpha Q/[R/2] = V_0(1 - 2\alpha)$
$V_A = KQ/(2R)^2 - K\alpha Q/R^2 = KQ/4R^2 - \alpha V_0/R$

Reduced by $\alpha V_0/R$

So $E@C = k\alpha Q/R^2 = \alpha V_0/R$

Increased by $2V_0/R$

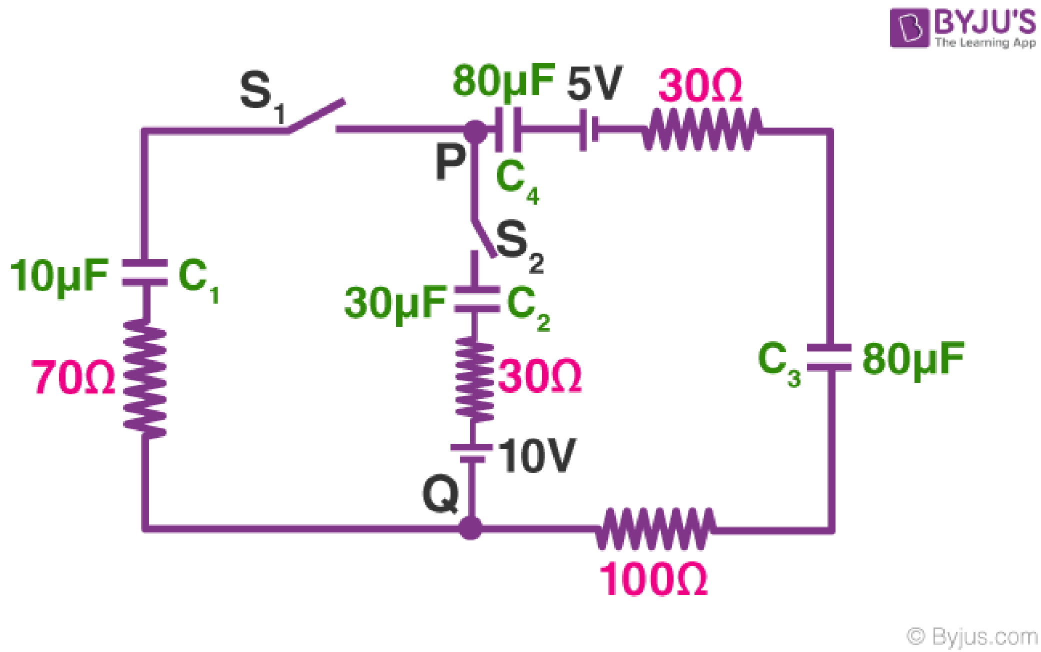
**Question 6:** A charged shell of radius R carries a total charge Q. Given φ as the flux of electric field through a closed cylindrical surface of height h, radius r & with its center same as that of the shell. Here center of cylinder is a point on the axis of the cylinder which is equidistant from its top & bottom surfaces. Which of the following are correct.

(a) If $h > 2R$ and $r > R$ then $\varphi = Q/\epsilon_o$
(b) If $h > 2R$ and $r = 4R/5$ then $\varphi = Q/5\epsilon_o$
(c) If $h < 8R/5$ and $r = 3R/5$ then $\varphi = 0$
(d) If $h > 2R$ and $r = 3R/5$ then $\varphi = Q/5\epsilon_o$

**Solution:**
Case 1: If $h > 2R$
$Q = Q/E0$

Case 2: $h = 8R/5$
$r = 3R/5$
Using Gauss law concept ACD are correct

**Question 7:** Which statements is/are correct:




(a) At time t = 0, the $S_1$ is closed instantaneous current in the closed circuit will be 25 mA
(b) The key $S_1$ is kept closed for long time such that capacitors are fully charged. Now key $S_2$ is closed at this
time the instantaneous current across 30Ω resistor between P & Q will be 0.2A.
(c) If key $S_1$ is kept closed for long time such that capacitors are fully charged the voltage across $C_1$ will be 4V.
(d) If $S_1$ is kept closed for long time such that capacitors are fully charged the voltage difference between P & Q will be 10V.

**Solution:**
When $S_1$ is closed then charge on capacitor is zero.
While replacing all capacitors with wire, we have

$i = t/[70+100+30] = 5/200 = 25$ mA

When circuit is in steady state, $q/10 + q/80 + q/80 - 5 = 0$

or $q = 40\ \mu C$

Therefore, voltage across $C_1$ = 40/10 = 4 volt

Just after closing of $S_2$, charge on capacitor remain same, KVL

-10+30x+40/10+y×70=0

or 30x + 70y = 6

Again, -40/80 + 5 + (x-y)30 - 40/80 + (x - y) 100 - 10 + 30x = 0

=> 160x - 130y - 6 = 0

Solving both the equations, we have x = 0.05 amp and y = 96/1510

Option (b) and (c) are correct.

**Question 8:** A galvanometer of resistance 10 ohm and maximum current of 2μA is converted into voltmeter of range 100 mV
and when converted into ammeter then range is 1mA. When these voltmeter and ammeter are connected by a
(ideal) battery in series with a resistance of R = 1000 Ω, then
(a) Measured value of R is between 978Ω and 996Ω
(b) Resistance of voltmeter 105Ω
(c) Shunt resistance is 20mΩ
(d) If the ideal battery is replaced by non-ideal battery with internal resistance of 5Ω then R will be > 1000 Ω

**Solution:**
$V = 100 \times 10^{-3}$ V

$V = I_g(R_g + R)$

$= 10^{-1}/[2\times10^{-6}]$

$= (R_g + R)^{R_V}$

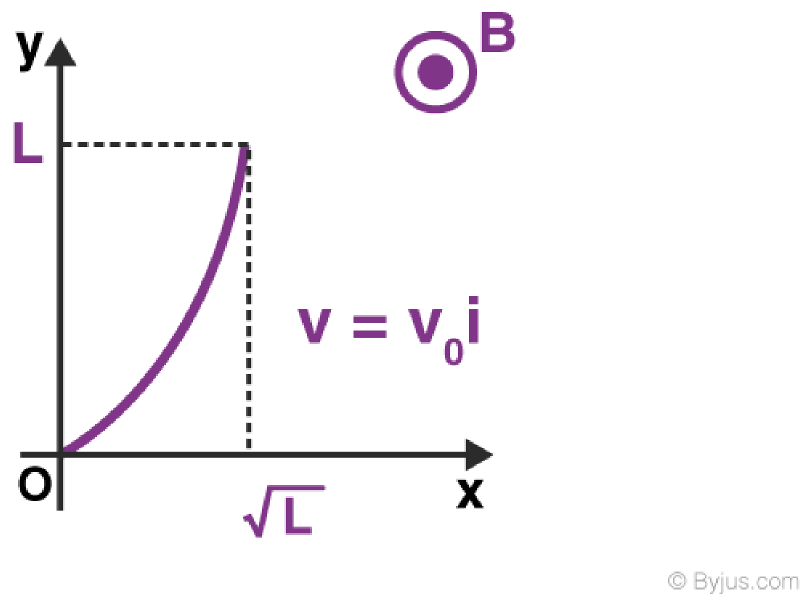
$R_V = 5 \times 10^4$

$$S = \left(\frac{10}{\frac{10^{-3}}{2\times10^{-3}} - 1}\right) = 20\,m\Omega$$

**Question 9:** Conducting wire of parabolic shape, initially y = $x^2$ is moving with velocity $\vec{v} = v_0\hat{i}$ in a non-uniform magnetic field

$$\vec{B} = B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$$

as shown in figure. If $V_0$, $B_0$, L and B are +ve constants and Δφ is potential difference develop between the ends of wire, then correct statement(s) is/are




(a) |Δϕ| = (1/2) $B_0V_0L$ for β = 0
(b) |Δϕ| = (4/3) $B_0V_0L$ for β = 2
(c) |Δϕ| is proportional to the length of wire projected on y-axis.
(d) |Δϕ| remains same if the parabolic wire is replaced by a straight wire, y = x, initially of length √2l

**Solution:**

$$d\varepsilon = BV_0 dy$$

$$= B_0 \left\{1 + \left(\frac{y}{2}\right)^{\beta}\right\} V_0 \, dy$$

$$\varepsilon = B_0 \int_0^{L} 4 + \left(\frac{y}{L}\right)^{\beta} \Bigg\} V_0 \, dy$$

$$= B_0 \, V_0 L \left(1 + \frac{1}{\beta + 1}\right)$$

Let $\beta = 0 \quad \varepsilon = 2B_0 V_0 L$

$$\beta = 2 \quad \varepsilon = B_0 V_0 L \left(1 + \frac{1}{3}\right)$$

$$= \frac{4}{3} B_0 V_0 L$$

B, C is correct

D is also correct because projection of wire on y axis is same.

**Question 10:** If in a hypothetical system if the angular momentum and mass are dimensionless. Then which of the following is true.

(a) The linear momentum varies as $L^{-1}$

(b) The energy varies as $L^{-2}$

(c) The power varies as $L^{-4}$

(d) The force varies as $L^{-5}$

**Solution:**

$[M] = [M^0L^0T^0]$

$[J] = [ML^2T^{-1}]$

$[ML^2T^{-1}] = [M^0L^0T^0]$

$\Rightarrow [L^2] = [T]$

Momentum:

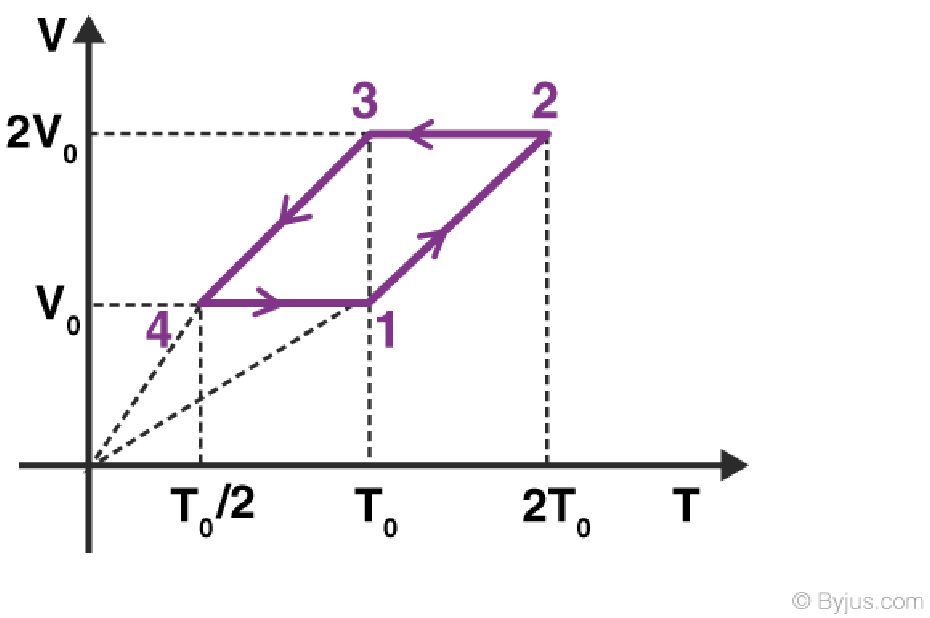
$[P] = [MLT^{-2} \, . \, LT^{-1}] = [ML^2T^{-3}] = [L^2L^{-6}] = [L^{-4}]$

$[E] = [MLT^{-2} \, . \, L] = [L^2L^{-4}] = [L^{-2}]$

$[F] = [LL^{-4}] = [L^{-3}]$

Options A,B,C are correct.

**Question 11:** V – T diagram for n mol monoatomic gas is given below:




Choose the correct statement:

(a) $\left|\frac{\Delta Q_{1\to 2}}{\Delta Q_{3\to 4}}\right| = \frac{1}{2}$

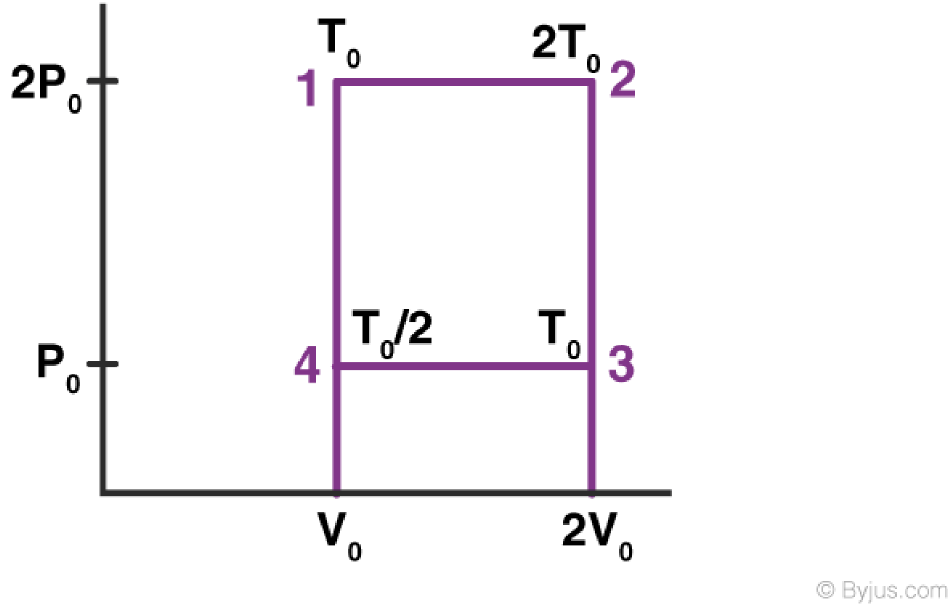
(b) $\left|\frac{\Delta Q_{1\to 2}}{\Delta Q_{2\to 3}}\right| = \frac{5}{3}$

(c) Work done in cyclic process is $\Delta W = nRT_0/2$

(d) There are only adiabatic and isochoric processes are involved.

**Solution:**

Corresponding PV entraps



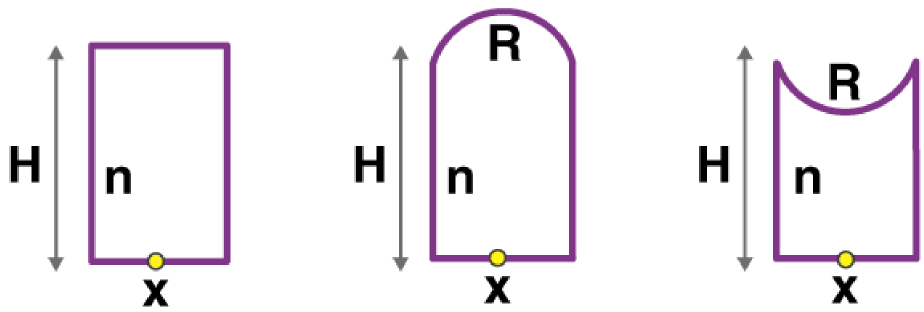

(a) $\left|\frac{\Delta Q_{1-2}}{\Delta Q_{3\to4}}\right| = \left[\frac{NC_P\Delta T_{1\to2}}{NC_P\Delta T_{3-4}}\right] = \frac{T_0}{T_0/2} = 2$

(b) $\left|\frac{\Delta Q_{1-2}}{\Delta Q_{2\to3}}\right| = \left|\frac{NC_P\Delta T_{1\to2}}{NC_V\Delta T_{2\to3}}\right| = \frac{C_P}{C_V} = \frac{5}{3}$

(c) $\omega = amu = P_0V_0$

(d) There are no adiabatic process is involved.
Only Options (a) and (b) are correct.

**Question 12:** Apparent depth for point object x in all three cases are H1, H2 & H3 respectively when seen from below given H
= 30 cm, n = 1.5 & R = 3m, then




(a) $0.8 < H_2 - H_1 < 0.9$
(b) $H_2 > H_1$
(c) $H_2 > H_3$
(d) $H_3 > H_1$

**Solution:**

**Case I**

$$\frac{n_2}{n_1} = \frac{d^1}{d} \Rightarrow \frac{1}{n} = \frac{d^1}{(3Q)}$$

$$d^1 = \frac{30}{3} \times 2 = 20cm$$

**Case II**

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{1^2}$$

$$\frac{1}{-v_2} - \frac{3}{-2\times 30} = \frac{1-3/2}{-300}$$

$$H_2 = 20.684$$

**Case III**

$$V_3 = 19.354$$

## Section – 3

**Question 13:**
Consider the following nuclear fission reaction

$${}_{88}Ra^{226} \longrightarrow {}_{86}Rn^{222} + {}_{2}He^{4} + Q.$$

In this fission reaction. Kinetic energy of α-particle emitted is 4.44 MeV. Find the energy emitted as γ – radiation in keV in this reaction.
m($_{88}Ra^{226}$) = 226.005 amu
m($_{2}He^{4}$) = 4.000 amu

**Solution:**
Δm = 0.005 amu

$$\frac{K_\alpha}{K_{Rn}} = \frac{m_{Rn}}{m_\alpha}$$

$$K_{Rn} = \frac{m_\alpha}{m_{Rn}} K_\alpha$$

$$= \frac{4}{222} \times 4.44 = 0.08\,MeV$$

Energy of γ – photon

$$= 4.655 - (4.44 + 0.08)$$

$$= 0.135 \text{ MeV}$$

**Question 14:** N dielectrics are introduced in series in a capacitor of thickness D. Each dielectric have width d = D/N &
dielectric constant of mth dielectric is given by $K_m = K(1 + m/N)$ : [N >> $10^3$, Area of plates = A]
Net capacitance is given by [K$\epsilon_o$A]/[αDln2]. Find value of α.

**Solution:**
x/m = Δ/N

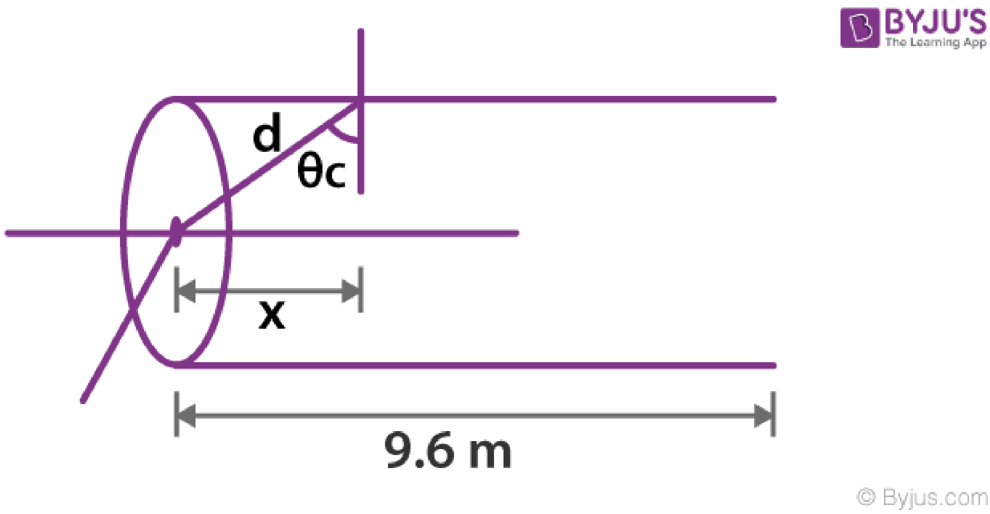
$$d\frac{1}{C} = \frac{dx}{Km\varepsilon_0 A} = \frac{dx}{K\varepsilon_0 A\left(H\frac{m}{N}\right)}$$

$$\int d\frac{1}{C} = \int_0^D \frac{dx}{K\varepsilon_0 A\left(H\frac{x}{D}\right)}$$

Integrating we get
$C_{eq}$ = [K$\epsilon_o$A]/[Dln2]
Therefore, α = 1

**Question 15:** If at angle θ the light takes maximum time to travel in optical fiber. Then the maximum time is x ($10^{-8}$), calculate x.

**Solution:**




$1.5 \sin \theta_C = 1.44 \sin 90^0$

$\sin \theta_C = 24/25$

and $\sin \theta_C = x/d$

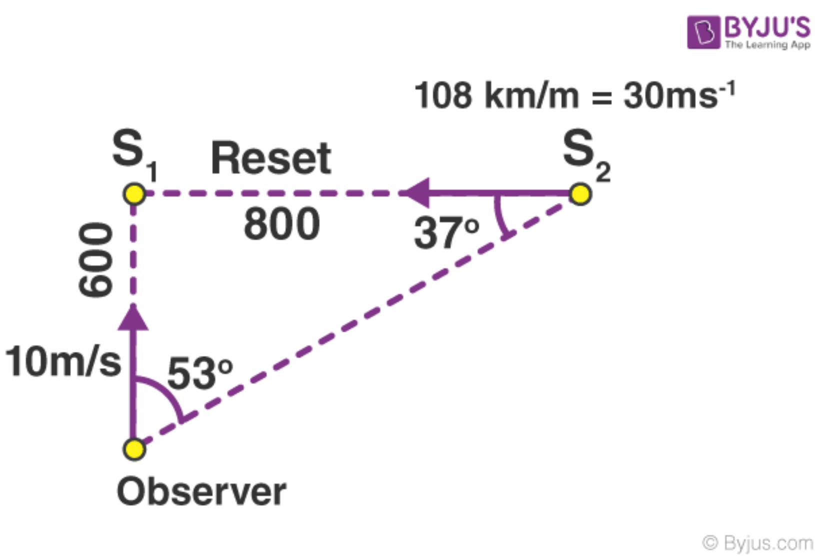
=> $d = 25x/24$

Now,

$t = L/(C/n_2) = 5 \times 10^8$ s

Putting: $C = 3 \times 10^8$, $n_2 = 1.5$ and $L = 10$

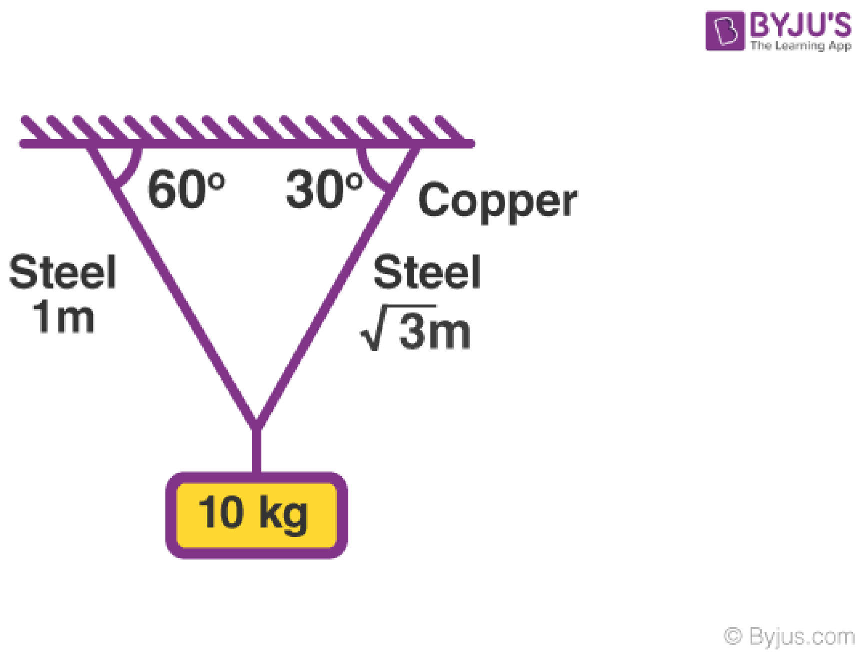
**Question 16:** The source $S_1$ is at rest. The observer and the source $S_2$ are moving towards $S_1$ as shown in figure. The roof beats observed by the observer if both sources have frequency 120 Hz and speed of sound 330 m/s in is



**Solution:**

$$f_b = 120\left(\frac{330+10\cos 53}{330-30\cos 37}\right)-\left(\frac{330+10}{330}\right) \quad = 8.128 \text{ Hz}$$

**Question 17:** A weight of 100 N is suspended by two wires made by steel and copper as shown in figure length of steel wire is 1 m and copper wire is √3 m. Find ratio of change in length of copper wire ($\Delta l_0$) to change in length of steel wire ($\Delta l_s$). Given Young's modulus: $Y_{steel} = 2 \times 10^{11}$ N/m$^2$, $Y_{copper} = 1 \times 10^{11}$ N/m$^2$.




**Solution:**

$$\frac{T_S}{Z} = T_C\frac{\sqrt{3}}{2}$$

$$T_S = \sqrt{3}\,T_C$$

$$\frac{\Delta\lambda_C}{\Delta\lambda_S} = \left(\frac{T_C}{T_S}\right)\left(\frac{L_C}{L_S}\right)\frac{Y_S}{Y_C}$$

$$= \left(\frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{1}\right)\frac{2\times10^{11}}{1\times10^{11}} = 2$$

**Question 18:** An optical bench, to measure the focal length of lens, is 1.5 m long and on the bench marks are with spacing 1/4 cm. Now a lens is placed at 75 cm and pin type object is placed at 45 cm marks on the bench. If its image is formed at 135 cm find maximum possible error in calculation of focal length.

**Solution:**
V = 30 cm and dv = 0.5 cm
V = 60 cm and dv = 0.5 cm

Now, using lens formula,

1/v - 1/u = 1/f
1/60 + 1/30 = 1/f
=> f = 20 cm

$$=> \frac{-dv}{v^2} + \frac{-du}{u^2} = \frac{-df}{f^2}$$

$$=> \frac{df}{f} \times 100 = f[\frac{dv}{v^2} + \frac{du}{u^2}]$$

$$= (20/2)[1/602 + 1/302]$$

Multiply by 100 both the sides, we get

df/f = 1.38 and 1.39