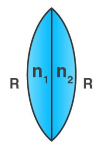
SECTION – 1

1. Consider two plane convex lens of same radius of curvature and refractive index n_1 and n_2 respectively. Now consider two cases:



Case – I: When $n_1 = n_2 = n$, then equivalent focal length of lens is f_0

Case – II: When $n_1 = n$, $n_2 = n + \Delta n$, then equivalent focal length of lens is $f = f_0 + \Delta f_0$

Then correct options are:

(a) If $\Delta n/n > 0$, then $\Delta f_0/f_0 < 0$

(b) $|\Delta f_0/f_0| < |\Delta n/n|$

(c) If n = 1.5, $\Delta n = 10^{-3}$ and f_0 = 20 cm then $|\Delta f_0|$ = 0.02 cm

(d)

Solution:

$$\frac{1}{f_1} = (n-1)\left(\frac{1}{f}\right) \Rightarrow \frac{1}{f_0} = \frac{2(n-1)}{R} \qquad \ldots (1)$$

$$\frac{1}{f_2} = (n+\Delta n-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)$$

$$\frac{1}{f+\Delta f_0} = \left(\frac{n-1}{R}\right)+(n+\Delta n-1)\left(\frac{1}{f}\right)$$

$$= \frac{2n+\Delta n-2}{R} \qquad \ldots (2)$$

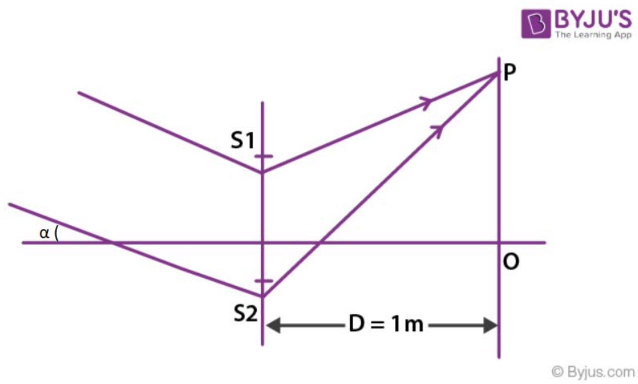
$$\left(\frac{f_0+\Delta f_0}{f_0}\right) = \frac{(2n-1)/R}{(2n+\Delta n-2)/R}$$

$$\frac{1+\Delta f_0}{f_0} = \frac{2(n-1)}{2n+\Delta n-2}$$

$$\Delta f_0 = -2\times 10^{-2}$$

A, C

2. In YDSE monochromatic light of wavelength 600 nm incident of slits as shown in figure.



If S_1S_2 = 3mm, OP = 11 mm then

(a) If $\alpha = \frac{0.36}{\pi}$ degree then destructive interfaces at point P.

(b) If $\alpha = \frac{0.36}{\pi}$ degree then constructive interfaces at point O.

(c) If α = 0 then constructive interfaces at O

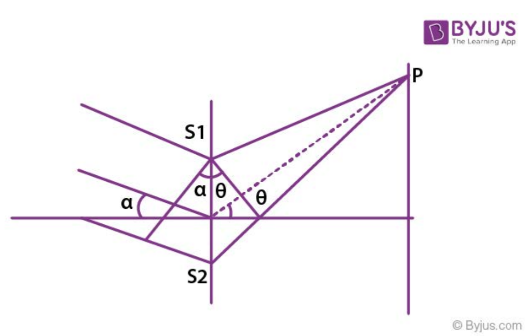
(d) Fringe width depends an α

Solution:

d = 3mm OP = 11 mm

$$\Delta x = d\sin\alpha + d\sin\theta$$

$$= d\alpha + \frac{dy}{D}$$



(A) $\Delta x = 3\times10^{-3}\times\frac{.36}{\pi}\times\frac{\pi}{180}+\frac{3\times11\times10^{-6}}{1}=3900$

$$3900 = (2n-1)\frac{\lambda}{2} \Rightarrow n = 7$$

Dest

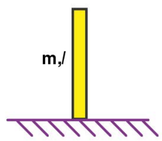
(B) $\Delta x = 3mm\times\frac{.36}{\pi}\times\frac{\pi}{180}=600\,nm$

$$600nm = n600nm$$

$\Rightarrow n = 1$ const

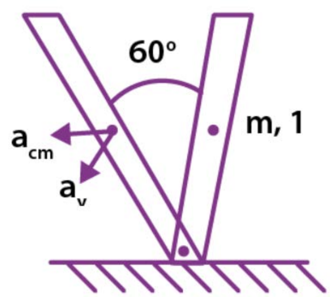
(C) $\alpha = 0 \;\; \Delta x = 0 \;\; \therefore$ const

3. A uniform rigid rod of mass m & length l is released from vertical position on rough surface with sufficient friction for lower end not to slip as shown in figure. When rod makes angle 60° with vertical then find correct alternative/s



(a) $\alpha = \dfrac{2g}{\ell}$ (b) $\omega = \sqrt{\dfrac{3g}{2\ell}}$ (c) $N = \dfrac{mg}{16}$ (d) $a_{radial} = \dfrac{3g}{4}$

Solution:



$$\Delta K + \Delta U = 0$$

$$\frac{1}{2}T_0\omega^2 = -\Delta U$$

$$\frac{1}{2}\frac{ml^2}{3}\omega^2 = -\left(-mg\frac{L}{4}\right)$$

$$\omega = \sqrt{\frac{3g}{u}}$$

$$a_{radial} = \frac{\omega^2\ell}{2} = \frac{3g}{u}\times\frac{\ell}{2} = \frac{3g}{4}$$

$$\tau = I_0\alpha$$

$$\alpha = \frac{mg\frac{l}{2}\sin 60}{l^2} = \frac{3\sqrt{3}g}{4l}$$

$$a_v = \left(\alpha \frac{l}{2}\right)\sin 60° + \omega^2 \frac{l}{2}\cos 60°$$

$$= \frac{3\sqrt{3}g}{8}\frac{\sqrt{3}}{2} + \frac{3g}{8}$$

$$mg - N = ma_v$$

$$N = \frac{mg}{16}$$

4. Monoatomic gas A having 5 mole is mixed with diatomic gas B having 1 mole in container of volume V_0. Now the volume of mixture is compressed to $\frac{V_0}{4}$ by adiabatic process. Initial pressure and temperature of gas mixture is P_0 and T_0. [given $2^{3.2} = 9.2$]

Choose correct option:

(a) $\gamma_{mix} = 1.6$

(b) Final pressure is between $9P_0$ and $10P_0$

(c) $|W.D| = 13RT_0$

(d) Average Translational kinetic energy

Solution:

$$V_{mix} = \frac{n_1C_{P_1} + n_2l_{P_2}}{n_1C_{V_1} + n_2C_{V_2}} = \frac{8}{5}$$

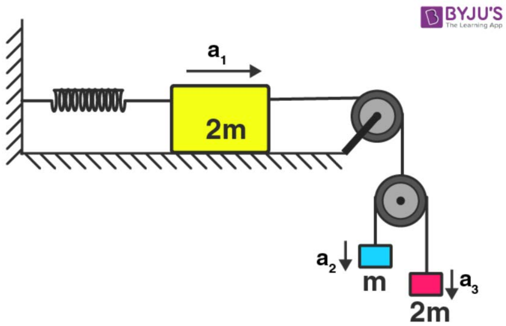
$$W = \frac{P_1V_1 - P_2V_2}{V - 1}$$

$$P_0V_0^{815} = P_2\left(\frac{V_0}{9}\right)^{8/5}$$

$$P_2 = 9.2P_0$$

$$\omega = \frac{\left(P_0V_0 - 92P_0\frac{V_0}{4}\right)}{3/5} = -13RT_0$$

5. The given arrangement is released from rest when spring is in natural length. Maximum extension in spring during the motion is x_0. a_1, a_2 and a_3 are accelerations of the blocks. Make the correct options



(a) $a_2 - a_1 = a_1 - a_3$

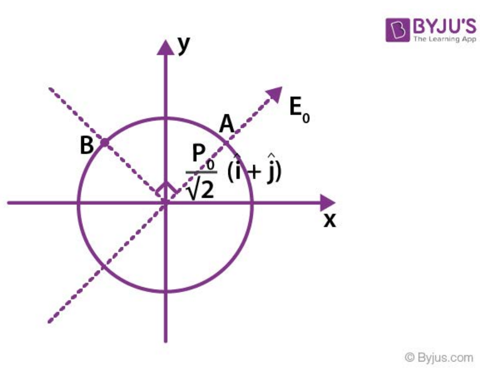
(b) $x_0 = \dfrac{4mg}{3k}$

(c) Velocity of 2m connected to spring when elongation is $\dfrac{x_0}{2}$ is $v = \dfrac{x_0}{2}\sqrt{\dfrac{3k}{14m}}$

(d) Acceleration a_1 at $\dfrac{x_0}{4}$ is $\dfrac{3kx_0}{42m}$

Solution:

6. A dipole of Dipole moment $p = \dfrac{p_0}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$. is placed at origin. Now a uniform external electric field at magnitude E_0 is applied along direction of dipole. Two points A and B are lying on a equipotential surface of radius R centered at origin. A is along axial position of dipole and B is along equatorial position. There correct option are:



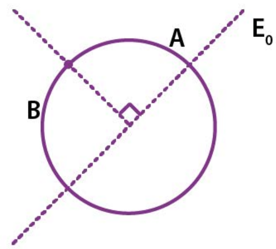
(a) Net electric field at point A is $3E_0$

(b) Net electric field at point B is Zero

(c) Radius of equatorial surface $R=\left(\frac{kp_0}{E_0}\right)^{1/3}$ (d) Radius of equatorial surface $R=\left(\frac{\sqrt{2}kp_0}{E_0}\right)^{1/3}$

Solution:

$$P=\frac{P_0}{\sqrt{2}}(x+1)$$



$$\frac{KP_0}{r^3}=E_0$$

$$(E_A)_{net}=\frac{2KP_0}{r^3}+E_0=3E_0$$

$$(E_B)_{net}=0$$

7. A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from state n = 1 to n = 4. Immediately after electron jumps to n = m state by emitting a photon of wavelength λ_e. Let change in momentum of atom due to the absorption and the emission are ΔP_a and Δp_e respectively. If $\lambda_a/\lambda_e=1/5$. Which of the following is correct

(a) m = 2

(b) $\Delta P_a/P_e = 1/2$

(c) λ_e = 418 nm

(d) Ratio of K.E. of electron in the state n = m to n = 1 is ¼.

Solution:

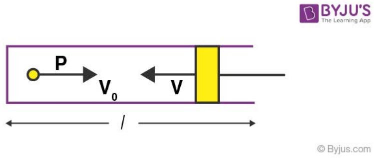
$$\frac{\lambda_a}{\lambda_c}=\frac{E_4-E_1}{E_4-E_m}=\frac{\left(1-\frac{1}{16}\right)}{\left(\frac{1}{m^2}-\frac{1}{16}\right)}=\frac{1}{5}$$

$$\Rightarrow m=2$$

$$\lambda_c=\frac{12400\times 4}{13.6}=3647$$

$$\frac{K_2}{K_1} = \frac{1^2}{2^2} = \frac{1}{4}$$

8. In a cylinder a heavy piston is moving with speed v as shown diagram and gas is filled inside it. A gas molecule is moving with speed v_0 towards moving piston. Then which of the following is correct (Assume v <<<< $v_0 \frac{\Delta\ell}{\ell}$) and collision is elastic)



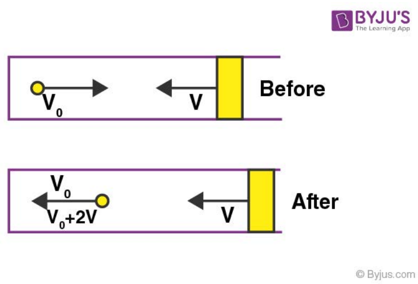
(a) change in speed after collision is 2V

(b) change is speed after collision is $2v_0 \frac{\Delta\ell}{\ell}$

(c) rate of collision is $\frac{V}{\ell}$

(d) When piston is at $\frac{\ell}{2}$ its kinetic energy will be four times

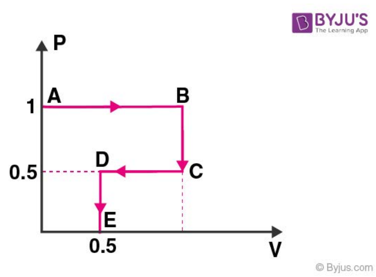
Solution:



Change in speed is $(2V + V_0 - V_0) = 2V$

SECTION – 2

9. If $f = \alpha y\hat{i} + 2\alpha x\hat{j}$ calculate the work done if a particle moves along path as shown in diagram.



Solution:

$$d\omega = \alpha y dx + 2\alpha x\, dx$$

$$\omega_{A\to B} = \int \alpha y\, dx = \alpha 1 \int_0^1 dx = \alpha$$

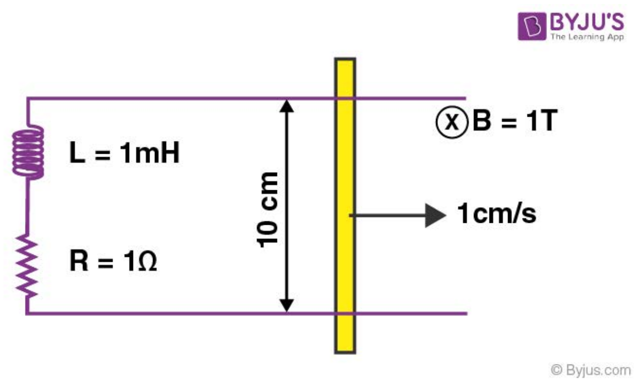
$$\omega_{B\to C} = 2\alpha 1\int_{1}^{0.5} dy = -\alpha$$

$$\omega_{C\to D} = \int_{1}^{0.5} \alpha y\, dx = -\frac{\alpha}{4}$$

$$\omega_{D\to E} = 2\times\alpha\int_{0.5}^{0} x\, dy = -\frac{\alpha}{2}$$

$$\omega = -3/4$$

10. In a given circuit inductor of L = 1mH and resistance R = 1Ω are connected in series to ends of two parallel conducting rods as shown. Now a rod of length 10 cm is moved with constant velocity of 1 cm/s in magnetic field B = 1T. If rod starts moving at t = 0 then current in circuit after 1 millisecond is $x\times 10^{-3} A.$ Then value of x is: (given $e^{-1} = 0.37$)



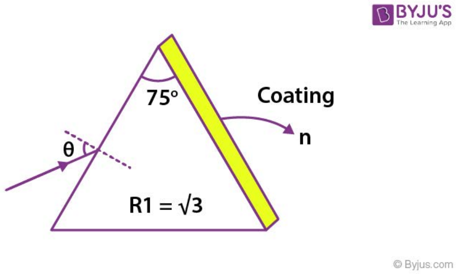
Solution:

$$e = (V\times B)\,dl = 10^{-3} vdf$$

$$i = 10^{-3}\left(1-e^{-1}\right)$$

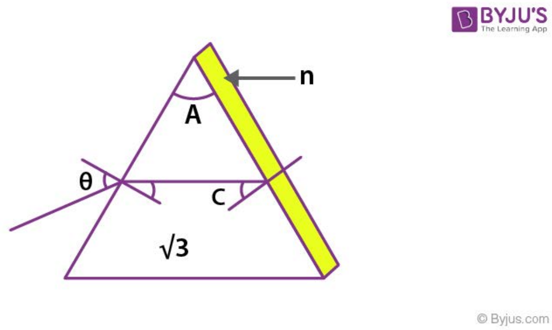
$$i = 0.63 mA$$

11. A prism is shown in the figure with prism angle 75° and refractive index $\sqrt{3}.$ A light ray incidents on a surface at incident angle θ. Other face is coated with a medium of refractive index n. For $\theta \le 60°$ ray suffers total internal reflection find value of n^2.



Solution:

$$\sin\theta = \frac{n}{\sqrt{3}}$$



$$\sin\theta = \sqrt{3}\sin(75 - C)$$

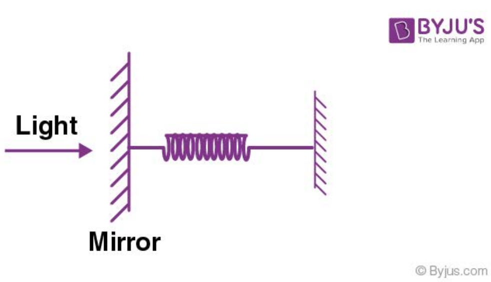
$$@\theta = 60 \ \underline{T2R}$$

$$\sin 60 = \sqrt{3}\sin(75 - C)$$

C = (45°)

$$\frac{n}{\sqrt{3}} = \frac{1}{\sqrt{2}} \ \ n = \frac{\sqrt{3}}{\sqrt{2}} \ n^2 = 1.5$$

12. Perfectly reflecting mirror of mass M mounted on a spring constitute a spring mass system of angular frequency Ω such that $\frac{4\pi M\Omega}{h} = 10^{24} m^{-2}$ where h is plank constant. N photons of wavelength $\lambda = 8\pi \times 10^{-6}$ m strikes the mirror simultaneously at normal incidence such that the mirror gets displaced by 1 μm. If the value of N is $x \times 10^{12}$, then find the value of x.



Solution:

Photons are reflected

$\therefore \quad MV = \dfrac{2Nh}{\lambda}$ mean

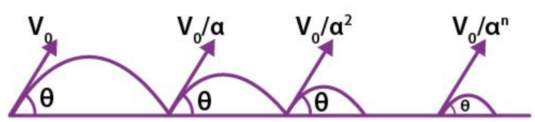
$V_{mean} = \omega A$ $A = 1$ min

$N = \dfrac{m\omega\left(10^{-6}\right)\lambda}{2h}$

$N = \dfrac{4\pi M\omega}{h} \times 10^{-12}$

$\therefore X = 1$

13. A particle is projected with speed v_0 at an angle $\theta\ (\theta \neq 90°)$ with horizontal and it bounce at same angle with horizontal. If average velocity of journey is 0.8 v_0 where v_0 is average velocity of first projectile then α is.

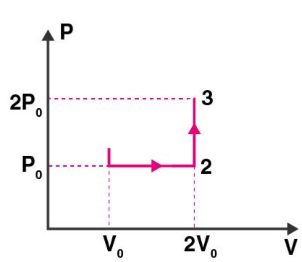


14.

Match the column

A sample of monoatomic gas undergoes different thermodynamic process. Q = Heat given to the gas, W = Work done by the gas, U = Change in internal energy of the gas.

15. The sample of monoatomic gas undergoes a process as represented by P – V graph (if $P_0V_0 = 1/3\ RT_0$) then



(P) $W_{1\to 2} = 1/3RT_0$ (Q) $Q_{1\to 2\to 3} = 11/6RT_0$ (R) $U_{1\to 2} = RT_0/2$ (S) $W_{1\to 2\to 3} = 1/3RT_0$

Which of the following options are correct

(a) P, Q, R, S are correct
(b) Only P, Q are correct
(c) Only R, S are correct
(d) Only P, R, S correct

SOLUTION:

$$\omega_{1-2} = P_0V_0 = \frac{1}{3}RT_0$$

$$Q_{2\to3} = nC_V\Delta T = \frac{f}{2}2V_0P_0 = RT_0$$

$$Q_{1\to2} = nC_P\Delta T = \frac{5}{6}RT_0$$

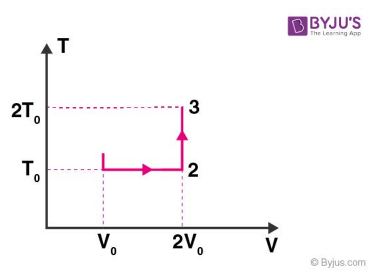
$$\omega = \frac{1}{3}RT_0$$

$$U_{1-2} = nC_V\Delta T$$

$$= n\frac{3}{2}RT_0$$

$$= \frac{RT_0}{2}$$

16. A sample of monoatomic gas undergoes a process as represented by T – V graph (if $P_0V_0 = 1/3\ RT_0$) then



(P) $W_{1\to2} = \frac{1}{3}RT_0\ell n2$

(Q) $Q_{1\to2\to3} = \frac{RT_0}{6}\left(2\ell n(2)+3\right)$

(R) $U_{1\to2} = 0$

(S) $W_{1\to2\to3} = \frac{RT_0}{3}\ell n2$

Which of the following option are correct:

(a) P, Q are incorrect
(b) R, S are incorrect
(c) P, Q, S are incorrect
(d) none of these

Solution:

$$\omega_{1-2} = nRT_0 \ln 2$$

$$Q_{1-2-3} = Q_{12} + Q_{23}$$

$$= d\omega_{12} + dU_{2-3}$$

$$= \frac{RT_0}{3} \ell n_{\,2} + n\frac{f}{2}RT_0$$

$$= \frac{RT_0}{3} \ell n_{\,2} + \frac{1}{3}\frac{3}{2}RT_0$$

$$\omega_{1-2-3} = \frac{1}{3}R_0T_0 \;\ell n_{\,2}$$

17. Length of string of a musical instrument is varied from L_o to $2L_o$ in 4 different cases. Wire is made of different materials of mass per unit length μ, 2μ, 3μ, 4μ respectively. For first case (string – 1) length is L_o, Tension is T_o then fundamental frequency is f_o, for second case length of the string is $\frac{3L_o}{2}$ (3^{rd} Harmonic), for third case length of the string is $\frac{5L_o}{4}$ (5^{th} Harmonic) and for the fourth case length of the string is $\frac{7L_o}{4}$ (14^{th} harmonic). If frequency of all is same then tension in strings in terms of T_o will be:

(a) String – 1	(P) T_o
(b) String – 2	(Q) $\frac{T_o}{\sqrt{2}}$
(c) String – 3	(R) $\frac{T_o}{2}$
(d) String – 4	(S) $\frac{T_o}{16}$
	(T) $\frac{3T_o}{16}$

Solution:

(1) $f_1 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

(2) $L = \frac{3L_0}{2}$

$$f_2 = \frac{3}{2\frac{3L_0}{2}}\sqrt{\frac{T_2}{2\mu}}$$

$$T_2 = \frac{T_0}{2}$$

(3) $L = \frac{5L_0}{4}$

$$T_3 = \frac{T_0}{16}$$

(4) Similarly $T_4 = \frac{T_0}{16}$

SECTION – 3

18. The free length of all four string is varied from L_0 to $2L_0$. Find the maximum fundamental frequency of 1, 2, 3, 4 in terms of f_0 (tension is same in all strings)

(a) String – 1	(P) 1
(b) String – 2	(Q) $\frac{1}{2}$
(c) String – 3	(R) $\frac{1}{\sqrt{2}}$
(d) String – 4	(S) $\frac{1}{\sqrt{3}}$
	(T) $\frac{1}{16}$
	(U) $\frac{3}{16}$

Solution:

(1) $f_1 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

(2) $f_2 = \frac{1}{L_0}\sqrt{\frac{T_2}{2\mu}} = \frac{f_0}{\sqrt{2}}$

$f_3 = \frac{1}{L_0}\sqrt{\frac{T_2}{3\mu}} = \frac{f_0}{\sqrt{3}}$

$f_4 = \frac{1}{L_0}\sqrt{\frac{T_2}{4\mu}} = \frac{f_0}{2}$