Q1: A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is
(a) 20 RT
(b) 12 RT
(c) 4 RT
(d) 15 RT

Solution
$U = (f_1/2)n_1RT + (f_2/2)n_2RT$
U = (5/2)(3RT) + (3/2)(5RT)
U = 15RT
Answer: (d) 15 RT

Q2: An ideal gas is enclosed in a cylinder at a pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is 6×10^{-8} s. If the pressure is doubled and the temperature is increased to 500 K, the mean time between two successive collisions will be close to
(a) 4×10^{-8} s
(b) 3×10^{-6} s
(c) 0.5×10^{-8} s
(d) 2×10^{-7} s

Solution
Root mean square velocity

$$Vrms = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}}$$

Here,
R is the universal gas constant
M is the molar mass
P is the pressure due to gas
ρ is the density

$$\text{Vrms} \propto \sqrt{T}$$

$V_{rms} \propto$ mean free path/ time between successive collisions

And mean free path,

$$Y = \frac{kT}{\sqrt{2\pi\sigma^2 P}}$$

Vrms ∝ Y/b

Vrms $\propto T/\sqrt{p} \times\ t$ ----(1)

But Vrms $\propto \sqrt{T}$ -----(2)

$$\sqrt{T} \propto T/\sqrt{p} \times\ t$$

$$t \propto \sqrt{T}/p \quad \frac{t_2}{t_1} = \sqrt{\frac{T_2}{T_1} \times \frac{P_1}{P_2}} \quad \frac{t_2}{t_1}$$

$$= \sqrt{\frac{500}{300} \times \frac{P_1}{2P_1}} = \sqrt{\frac{5}{6}} \quad t_2 = \sqrt{\frac{5}{6}} t_1$$

$t_2 \approx 4 \times 10^{-8}$ s

Answer: (a) 4 x 10^{-8} s

Q3: The specific heats, C_P and C_V of gas of diatomic molecules, A, is given (in units of J mol^{-1} K^{-1}) by 29 and 22, respectively. Another gas of diatomic molecules, B, has the corresponding values 30 and 21. If they are treated as ideal gases, then

(a) A has a vibrational mode but B has none.
(b) A has one vibrational mode and B has two.
(c) A is rigid but B has a vibrational mode.
(d) Both A and B have a vibrational mode each.

Solution

Here Cp and Cv of A are 29 and 22 and Cp and Cv of B are 30 and 21.
γ = Cp/Cv = 1 + 2/f
For A, Cp/Cv = 1 + 2/f ⇒ f=6
Molecules A has 3 translational, 2 rotational and 1 vibrational degree of freedom
For B, Cp/Cv = 1 + 2/f ⇒ f=5
i.e., B has 3 translational and 2 rotational degrees of freedom.
Answer: (a) A has a vibrational mode but B has none.

Q4: Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume? (R = 8.3 J/mol K)

(a) 21.6 J/mol K
(b) 19.7 J/mol K
(c) 15.7 J/mol K
(d) 17.4 J/mol K

Solution

C_{v1} of helium = (3/2)R

C_{v2} of hydrogen = (5/2)R

$$C_v \text{ of mixture} = \frac{2\times\frac{3}{2}R+3\times\frac{5}{2}R}{(2+3)}$$

C_v of mixture = 17.4 J/mol K

Answer: (d) 17.4 J/mol K

Q5: The mass of a hydrogen molecule is 3.32×10^{-27}/ kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m s–1, then the pressure on the wall is nearly

(a) 2.35×10^3 N m^{-2}

(b) 4.70×10^3 N m^{-2}

(c) 2.35×10^2 N m^{-2}

(d) 4.70×10^2 N m^{-2}

Solution

As $p_i = p_f$

Net force on the wall,

F = dp/dt = $2np_f\cos45^0$= $2nmv\cos45^0$

Here, n is the number of hydrogen molecules striking per second.

Pressure = F/A = $(2nmv\cos45^0)$/Area

$$= \frac{2\times10^{23}\times3.32\times10^{-27}\times10^{3}\times(1/\sqrt{2})}{2\times10^{-4}} = 2.35 \times 10^3 \text{ N m}^{-2}$$

Answer: (c) 2.35×10^2 N m^{-2}

Q6: In an ideal gas at temperature T, the average force that a molecule applies on the walls of a closed container depends on T as T^q. A good estimate for q is

(a) 2

(b) 1

(c) 4/5

(d) 4/7

Solution

Pressure, P = ⅓(mN/V)V^2_{rms}

P = (mN)T/V

If the gas mass and temperature are constant then

$P \propto (V_{rms})^2 \propto T$

So, force ∝ $(V_{rms})^2 \propto T$
I.e., Value of q = 1
Answer: (b) 1

Q7: One kg of a diatomic gas is at a pressure of 8 × 10^4 N/m^2. The density of the gas is 4 kg/m^3. What is the energy of the gas due to its thermal motion?
(a) 3 × 10^4 J
(b) 5 × 10^4 J
(c) 6 × 10^4 J
(d) 7 × 10^4 J

Solution
The thermal energy or internal energy is 5 2 U RT for diatomic gases. (degree of freedom for diatomic gas = 5)
But PV = RT
V = mass/density = 1kg/(4 kg/m^3) = (¼)m^3
P = 8 x 10^4 N/m^2
U = (5/2) x 8 x 10^4 x ¼ = 5 x 10^4 J
Answer: (b) 5 × 10^4 J

Q8: A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio C_P /C_V of the mixture is
(a) 1.4
(b) 1.54
(c) 1.59
(d) 1.62

Solution
For 16g of helium, n_1 = 16/4 = 4
For 16g of oxygen, n_2 = 16/32 = ½
For a mixture of gases
C_v =(n_1Cv_1 + n_2Cv_2)/(n_1 + n_2) where C_v = (f/2)R
C_p =(n_1Cp_1 + n_2Cp_2)/(n_1 + n_2) where C_p = ([f/2] + 1)R
For helium, f = 3 ,n_1 = 4
For oxygen, f =5, n_2 = ½

$$\frac{Cp}{Cv} = \frac{(4\times\frac{5}{2}R)+(\frac{1}{2}\times\frac{7}{2}R)}{(4\times\frac{3}{2}R)+(\frac{1}{2}\times\frac{5}{2}R)} = \frac{47}{29} = 1.62$$

Answer: (d) 1.62

Q9: Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will
(a) increase
(b) decrease
(c) remain the same
d) decrease for some, while the increase for others.
Solution
It is the relative velocities between molecules that are important. Root mean square velocities are different from lateral translation.
Answer: (c) remain the same

Q10: At what temperature is the root mean square velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C?
(a) 80 K
(b) –73 K
(c) 3 K
(d) 20 K.

Solution

$v_{rms} = \sqrt{RT/M}$ $(V_{rms})O_2 = (V_{rms})H_2$

$$\sqrt{\frac{273+47}{32}} = \sqrt{\frac{T}{2}}$$

T = 20 K

Answer: (d) 20 K

Q11:The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to
[Boltzmann constant, $k_B = 1.38 \times 10^{-23}$ J/K,
Avogadro Number, $N_A = 6.02 \times 10^{26}$/ kg,
Radius of Earth: 6.4×10^6 m,
Gravitational acceleration on Earth = 10 m s^{-2}]
(a) 10^4 K
(b) 800 K
(c) 650 K
(d) 3×10^5 K

Solution

The escape speed of the molecule, $v_e = \sqrt{2gR}$

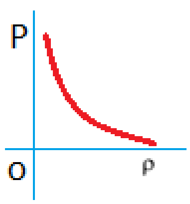
Root mean square velocity, $v_{rms} = \sqrt{\frac{3(k_B N)T)}{m}}$

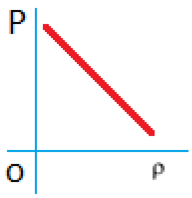
So, for $v_e = v_{rms} \Rightarrow 2gR = 3(k_BN)T/m$

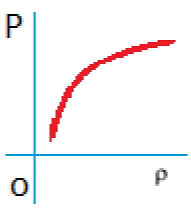
$T = 2gRm/3k_BN = (2 \times 10 \times 6.4 \times 10^6 \times 2 \times 10^{-3})/(3 \times 1.38 \times 10^{-23} \times 6.02 \times 10^{23}) = 10^4$ K

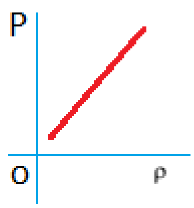
Answer: (a) 10^4 K

Q12: Which of the following shows the correct relationship between the pressure 'P' and density of an ideal gas at constant temperature?

a. 

b. 

c. 

d. 

Solution

Ideal gas equation, PV = nRT

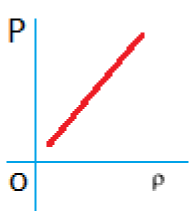
As the temperature is constant.

PV = constant

$P(m/\rho)$ = constant

$P \propto \rho$ (for given m)

Answer:

d. 

Q13: The value closest to the thermal velocity of a Helium atom at room temperature (300 K) in m s^{-1} is [$k_B = 1.4 \times 10^{-23}$ J/K; $m_{He} = 7 \times 10^{-27}$ kg]

(a) 1.3×10^3
(b) 1.3×10^5
(c) 1.3×10^2
(d) 1.3×10^4

Solution

$(3/2)k_BT = ½mv^2$

$$v = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3\times1.4\times10^{-23}\times300}{7\times10^{-27}}}$$

$= 1.3 \times 10^3\ ms^{-1}$

Answer:(a) 1.3×10^3

Q14: An ideal gas has molecules with 5 degrees of freedom. The ratio of specific heats at constant pressure (C_P) and at constant volume (C_V) is
(a) 6
(b) 7/2
(c) 5/2
(d) 7/5

Solution
An ideal gas has molecules with 5 degrees of freedom, then
Cv = (5/2)R and Cp = (7/2)R
Cp/Cv = (7/2)R/(5/2)R
Cp/Cv = 7/5
Answer: (d) 7/5

Q15: N moles of a diatomic gas in a cylinder is at a temperature T. Heat is supplied to the cylinder such that the temperature remains constant but n moles of the diatomic gas get converted into monoatomic gas. What is the change in the total kinetic energy of the gas?
(a) 0
(b) (5/2)nRT
(c) (½)nRT
(d) (3/2)nRT

Solution
Initial kinetic energy of the system K_i = (5/2)RTN
Final kinetic energy of the system K_f = (5/2)RT(N-n) + (3/2)RT(2n)
$K_f - K_i = \Delta K$ = nRT(3 - 5/2) = (½)nRT
Answer: (c) (½)nRT