JEE Main Physics Previous Year Questions With Solutions on "Simple Harmonic Motion"

Q1. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in second is

(a) 8π/3

(b) 4π/3

(c) 3π/8

(d) 7π/3

Solution

In SHM, speed v = $v = \omega\sqrt{A^2 - x^2}$

Acceleration a = – ω^2x

As $|v| = |a|$

ω =3/4⇒T = 2π/ω = 8π/3

Answer: (a) 8π/3

Q2: A cylindrical plastic bottle of negligible mass is filled with 310 mL of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency. If the radius of the bottle is 2.5 cm then ω is close to (density of water = 10^3 kg/m^3)

(a) 2.50 rad s^{-1}

(b) 3.75 rad s^{-1}

(c) 5.00 rad s^{-1}

(d) 1.25 rad s^{-1}

Solution

Restoring force due to pressing the bottle with amount x,

F = -(ρAx)g

a = -(ρAg/m)x

Therefore, ω^2 = ρAg/m = [ρ(πr^2)g]/m

$$\omega = \sqrt{\frac{10^3 \times \pi \times (2.5 \times 10^{-2})^2 \times 10}{310 \times 10^{-3}}} = 7.95 \text{ rad/s}$$

None of the options given are correct

Q3: A particle undergoing simple harmonic motion has time-dependent displacement given by x(t) = Asin(πt/90). The ratio of kinetic to potential energy of this particle at t = 210 s will be

(a) 1/ 3

(b) 2

(c) 1

(d) 3

Solution

The maximum kinetic energy of the particle is (½)($A^2\omega^2$)

The potential energy of the particle at any time t is (½)($x^2\omega^2$)

Using energy conservation

$$\Rightarrow \frac{KE}{PE} = \frac{KE_{max}}{PE} - 1$$

$$\frac{KE}{PE} = \frac{\frac{1}{2}mA^2\omega^2}{\frac{1}{2}m\omega^2x^2} - 1$$

$$= \frac{1}{sin(2\pi+\pi/3)^2} - 1 = \frac{1}{3}$$

Answer: (a) 1/ 3

Q4: A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then

(a) $K_2 = K_1$

(b) $K_2 = K_1/2$

(c) $K_2 = 2K_1$

(d) $K_2 = K_1/4$

Solution

K_1 = (½)mv^2_{max}= (½)$mA^2\omega_1^2$———-(1)

K_2 = (½)mv^2_{max}= (½)$mA_2^2\omega_2^2$———-(2)

Here, A_2 = A1

From (1) and (2)

$(K_1/K_2) = (\omega_2^2/\omega_1^2)$

$(K_1/K_2) = l_1/l_2 = l_1/2l_1 \Rightarrow K_2 = K_1/2$

Answer: (b) $K_2 = K_1/2$

Q5: A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by

(a) 10^{-5} rad/s

(b) 10^{-1} rad/s

(c) 1 rad/s

(d) 10^{-3} rad/s

Solution

Angular frequency of the pendulum

$$\omega = \sqrt{\frac{g_{eff}}{l}}$$

$$(\Delta\omega/\omega) = (½)(\Delta g_{eff}/g_{eff})$$

$$(\Delta\omega/\omega) = (½)(2A\omega^2_s/g) = (½)(2 \times (1)^2 \times (10)^{-2}/10) = 10^{-3}$$

Answer: (d) 10^{-3} rad/s

Q6: A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to 1/1000 of the original amplitude is close to

(a) 100 s

(b) 10 s

(c) 20 s

(d) 50 s

Solution

Frequency of damped oscillation, f = 5 Hz

For A=A/2,t_1 =2s

Also, A = $A_0e^{-(b/2m)t}$ or ½ = $e^{-(b/2m)2}$

b/m = In 2

For A = A/1000, t_2 = ?

1/1000 = $e^{-(b/2m)t2}$

Or 10^{-3} = $e^{-(b/2m)t2}$

$(b/2m)t_2$ = 3In 10 or t_2= 6In10/In2

t_2= 20 s

Answer: (c) 20 s

Q7: A resonance tube is old and has a jagged end. It is still used in the laboratory to determine the velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark, near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment, is close to

(a) 335 m s^{-1}

(b) 322 m s^{-1}

(c) 328 m s^{-1}

(d) 341 m s^{-1}

Solution

Due to the jagged end

$v/(4(11- x) \times 10^{-2}) = 512$————(1)

$v/(4(27- x) \times 10^{-12}) = 256$————-(2)

From (1) and (2)

$2(11 – x) = (27 -x) \Rightarrow x = -5$ cm

From equa (1)

$v/(4 \times 16 \times 10^{-2}) = 512$

$v = 328$ ms^{-1}

Answer: (c) 328 m s^{-1}

Q8: Two simple harmonic motions, as shown here, are at right angles. They are combined to form Lissajous figures.

$x(t) = A \sin(at + \delta)$

$y(t) = B \sin(bt)$

Identify the correct match below.

Parameters	Curve
a) $A = B, a = b; \delta = \pi/2$	Line
b) $A \neq B, a = b; \delta = 0$	Parabola
c) $A = B, a = 2b; \delta = \pi/2$	Circle
c) $A \neq B, a = b; \delta = \pi/2$	Ellipse

Solution

x = Asin(at + δ)

y = ABsin(bt)

If a = b

x = A [sin(at).cosδ + cos(at).sinδ)

Sin (at) = y/B

$$\frac{x}{A} = \frac{y}{A}cos\delta + \sqrt{1 - \frac{y^2}{B^2}}sin\delta$$

$$\left(\frac{x}{A} - \frac{y}{B}cos\delta\right)^2 = \left(1 - \frac{y^2}{B^2}\right)sin^2\delta$$

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}.cos\delta = sin^2\delta$$

If δ = π /2, $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ (ellipse)

For A = B, a =2b and δ =π /2

$x^2 + y^2 = A^2$ (Circle)

For A ≠ B, a =b and δ = 0 (Parabola)

For A≠ B, a =b and δ =π /2 (Ellipse)

Q9: In an engine, the piston undergoes vertical simple harmonic motion with amplitude 7 cm. A washer rests on top of the piston and moves with it. The motor speed is slowly increased. The frequency of the piston at which the washer no longer stays in contact with the piston is close to

(a) 0.7 Hz

(b) 1.9 Hz

(c) 1.2 Hz

(d) 0.1 Hz

Solution

The amplitude of S.H.M., A = 7 cm = 0.07 m

As the washer does not stay in contact with the piston, at some particular frequency i.e. normal force on the washer = 0

Maximum acceleration of washer = $A\omega^2$ = g

$$\omega = \sqrt{\frac{g}{A}} = \sqrt{\frac{10}{0.07}} = \sqrt{\frac{1000}{7}}$$

Frequency of the piston, $v = \omega/2\pi = \frac{1}{2\pi}\sqrt{\frac{1000}{7}} = 1.9$ Hz

Answer: (b) 1.9 Hz

Q10: A toy-car, blowing its horn, is moving with a steady speed of 5 m/s, away from a wall. An observer, towards whom the toy car is moving, is able to hear 5 beats per second. If the velocity of sound in air is 340 m/s, the frequency of the horn of the toy car is close to

(a) 680 Hz

(b) 510 Hz

(c) 340 Hz

(d) 170 Hz

Solution

v_{air} = 340 m/s, v = 5m/s

$f_1 – f_2$ = 5 beats per second

Apparently frequency heard by the observer on reflection from the wall,

f_1 = [v/(v-vs)]f = [340/(340-5)]f = (340/335)f

f_2 = [v/(v+vs)]f = [340/(340+5)]f = (340/345)f

Since $f_1 – f_2 = 5$

[(340/335) – (340/345)]f = 5 ⇒ f = (5/340)([335 x 345]/10) = 169.96 Hz = 170 Hz

Answer: (d) 170 Hz

Q11: A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of10^{12} s^{-1}. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver 108 and Avogadro number 6.02 × 10^{23} gm $mole^{-1}$)

(a) 6.4 N m^{-1}

(b) 7.1 N m^{-1}

(c) 2.2 N m^{-1}

(d) 5.5 N m^{-1}

Solution

Frequency of a particle executing SHM,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$k = 4\pi^2 \times f^2 \times m$

Here, $f = 10^{12}$ s-1, m= [108/(6.02 x 10^{23})] x 10^{-3} Kg

$k = 4 \times (3.14)^2 \times (10^{12})^2 \times [108 \times 10^{-3}/6.02 \times 10^{23}] = 7.1\ Nm^{-1}$

Answer:(b) 7.1 N m^{-1}

Q12: The ratio of maximum acceleration to maximum velocity in a simple harmonic motion is 10 s^{-1}. At, t = 0 the displacement is 5 m. What is the maximum acceleration? The initial phase is π/4

(a) 500√2 m/s^2

(b) 500 m/s^2

(c) 750√2 m/s^2

(d) 750 m/s^2

Solution

For simple harmonic motion,

Maximum acceleration/Maximum velocity = 10⇒ $\omega^2 a/\omega a$ =10 or ω=10

At t = 0; displacement, x = 5

x = a sin (ωt + Φ)

5 = asin(0 + π/4) or 5 = asin(π/4) or a = 5√2 m

Maximum acceleration = $\omega^2 a = 10^2 \times 5\sqrt{2} = 500\sqrt{2}\ ms^{-2}$

Answer:(a) 500√2 m/s^2

Q13: A child swinging on a swing in sitting position, stands up, then the time period of the swing will

(a) increase

(b) decrease

(c) remains the same

(d) increases if the child is long and decreases if the child is short

Solution

The time period will decrease.

When the child stands up, the centre of gravity is shifted upwards and so the length of swing decreases. T= 2π√l/g

Answer: (b) decrease

Q14: The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is (a) 11%

(b) 21%

(c) 42%

(d) 10%.

Solution

Let the lengths of pendulum be (100*l*) and (121*l*)

$$\frac{T'}{T} = \sqrt{\frac{121}{100}} = 11/10$$

Fractional change = (T' – T)/T = (11 – 10)/10 = 1/10

Percentage change = 10%.

Answer: (d) 10%.

Q15. In a simple harmonic oscillator, at the mean position

(a) kinetic energy is minimum, potential energy is maximum

(b) both kinetic and potential energies are maximum

(c) kinetic energy is maximum, potential energy is minimum

(d) both kinetic and potential energies are minimum

Answer: (b) When the temperature increases, l increases. Hence, the frequency decreases.