Karnataka 2nd PUC Maths Important Questions

Question 1: Find the area of the triangle whose vertices are (-2, -3), (3, 2) and (-1, -8) by using the determinant method.

Solution:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

= (1 / 2) (30)
= 15 square units

Question 2: Write the simplest form of $\tan^{-1}$ (cosx - sinx) / (cosx + sinx), 0 < x < π / 2.

Solution:
$\tan^{-1}$ (cosx - sinx) / (cosx + sinx)
Divide throughout by cosx
= $\tan^{-1}$ [(cosx - sinx / cosx) / (cosx + sinx / cosx)]
= $\tan^{-1}$ [(cosx / cosx - sinx / cosx) / cosx / cosx + sinx / cosx)]
= $\tan^{-1}$ [(1 - tanx) / (1 + tanx)]
= $\tan^{-1}$ [(tan (**π** / 4) - tanx) / (1 + tan (**π** / 4) * tanx)]
= $\tan^{-1}$ [tan (**π** / 4 - x)]
= **π** / 4 - x

Question 3: Find dy / dx, if x^2 + xy + y^2 = 100.

Solution:
$x^2 + xy + y^2 = 100$
2x + y + x * (dy / dx) + 2y * (dy / dx) = 0
(2x + y) + (dy / dx) (x + 2y) = 0
(dy / dx) (x + 2y) = - (2x + y)
(dy / dx) = - (2x + y) / (x + 2y)

Question 4: Integrate $\frac{e^{\tan^{-1} x}}{1+x^2}$ with respect to x.

Solution:

Put $\tan^{-1} x = t$

$(1 / (1 + x^2)) = dt / dx$

$dx = (1 + x^2)\, dt$

$I = \int e^t / (1 + x^2) * (1 + x^2)\, dt$

$= \int e^t\, dt$

$= e^t + c$

$= e^{\tan^{-1} x} + c$

Question 5: Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a - b| is even}, is an equivalence relation.

Solution:

R = {(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (5, 1), (5, 3), (5, 5)}

$(a, a) \in R, \forall a \in A$

$\therefore$ R is Reflexive.

$(a, b) \in R$

$\Rightarrow (b, a) \in R$

$\therefore$ R is symmetric.

(a, b) and $(b, c) \in R$

$\Rightarrow (a, c) \in R$

$\therefore$ R is transitive.

$\therefore$ R is an equivalence relation.

Question 6: Find $\int x\, dx / (x + 1)(x + 2)$.

Solution:

$\int x\,dx / (x + 1)(x + 2)$

By using partial fractions method,

= A / (x + 1) + B / (x + 2)

x = A (x + 2) + B (x + 1)

Put x = - 2,

- 2 = A (- 2 + 2) + B (- 2 + 1)

- 2 = B (- 1)

B = 2

Put x = - 1,

- 1 = A (- 1 + 2) + B (- 1 + 1)

- 1 = A (1) + 0

A = - 1

$\int x\,dx / (x + 1)(x + 2) = -1 / (x + 1) + 2 / (x + 2)$

$= \int -1 / (x + 1)\,dx + 2 \int dx / (x + 2)$

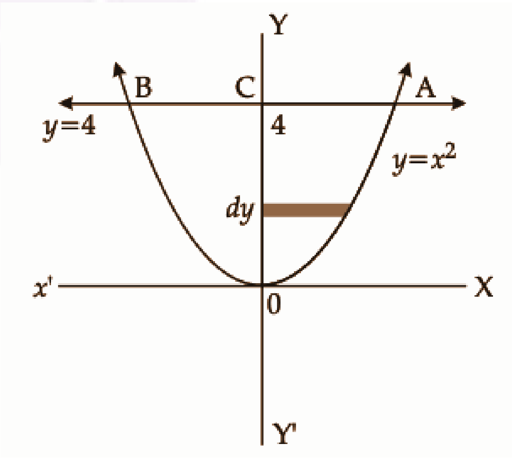
= - log |x + 1| + 2 log |x + 2| + c

= - log |x + 1| + log $|x + 2|^2$ + c

= log $|(x + 2)^2 / (x + 1)|$ + c

Question 7: Find the. area of the region bounded by the curve $y = x^2$ and the line y = 4.

Solution:



The area enclosed by $y = x^2$ and the line y = 4 is given by

Area BOAB = 2 * area of OACO

$= 2 \int_0^4 x \, dy$
$= 2 \int_0^4 \sqrt{y} \, dy$
$= 2 [y^{3/2} / (3 / 2)]_0^4$
$= (4 / 3) [y^{3/2}]_0^4$
$= (4 / 3) [4^{3/2} - 0^{3/2}]$
$= (4 / 3) [8 - 0]$
= 32 / 3 square units

Question 8: A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag, which is found to be red. Find the probability that the ball is drawn from the first bag.

Solution:
$P (E_1) = 1 / 2$
$P (E_2) = 1 / 2$
$P (A / E_1) = 1 / 2$
$P (A / E_2) = 1 / 4$
$P (E_1 / A) = [P (E_1) * P (A / E_1)] / [(P (E_1) * P (A / E_1)) + (P (E_2) * P (A / E_2))]$
$= [(1 / 2) * (1 / 2)] / [(1 / 2) * (1 / 2) + (1 / 2) * (1 / 4)]$
= 2 / 3

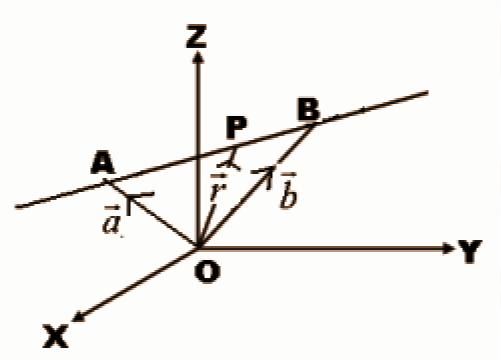
Question 9: Sand is pouring from a pipe at a rate of 12 cubic cm l s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

Solution:
$dV / dt = 12 \text{ cm}^3/\text{sec}$
Height of the cone = (1 / 6) of the radius of the base of the cone
Volume of the cone = $(1 / 3) \pi r^2 h$
$= (1 / 3) \pi (6h)^2 h$ [h = r / 6]
$= 12\pi h^3$
$dV / dt = d (12\pi h^3) / dt$

$12 = 12\pi \cdot 3h^2 (dh / dt)$
$1 = \pi * 3 (4)^2 (dh / dt)$
$1 / 48\pi = dh / dt$
$dh / dt = 1 / 48\pi$
$= 1 / (48 * (22 / 7))$
$= 7 / (48 * 22)$
$= 0.0066$ cm/sec

Question 10: Derive the equation of the line in space passing through two given points both in vector and Cartesian form.

Solution:



Let a, b and r be the position vectors of the two points A (x_1, y_1, z_1) is (x_2, y_2, z_2) and p (x, y, z) respectively.
AP = OP - OA = r - a
AB = OB - OA = b - a
The point p will lie on the line AB if and only if AP and AB are collinear.
AP = λ AB
(r - a) = λ (b - a)
r = a + λ (b - a) is the vector equation of the line passing through two points.
Let r = xi + yj + zk, a = $x_1i + y_1j + z_1k$, b = $x_2i + y_2j + z_2k$, r = a + λ (b - a)
$xi + yj + zk = x_1i + y_1j + z_1k + \lambda ((x_2 - x_1) i + (y_2 - y_1) j + (z_2 - z_1) k)$
$= [x_1 + \lambda (x_2 - x_1)] i + [y_1 + \lambda (y_2 - y_1)] j + [z_1 + \lambda (z_2 - z_1)] k$
$x = x_1 + \lambda (x_2 - x_1)$
$x - x_1 = \lambda (x_2 - x_1)$
$\lambda = (x - x_1) / (x_2 - x_1)$
$y = y_1 + \lambda (y_2 - y_1)$

$y - y_1 = \lambda (y_2 - y_1)$
$\lambda = (y - y_1) / (y_2 - y_1)$
$z = z_1 + \lambda (z_2 - z_1)$
$z - z_1 = \lambda (z_2 - z_1)$
$\lambda = (z - z_1) / (z_2 - z_1)$
Hence the equation of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $(x - x_1) / (x_2 - x_1) = (y - y_1) / (y_2 - y_1) = (z - z_1) / (z_2 - z_1)$.

Question 11: Solve the following system of linear equations by matrix method.
$x - y + 2z = 7$
$3x + 4y - 5z = -5$
$2x - y + 3z = 12$

Solution:

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

AX = B
$AA^{-1}X = A^{-1}B$
$IX = A^{-1}B$
$X = A^{-1}B$

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{vmatrix}$$

$= 1 \cdot 4 \cdot 3 + (-1) \cdot (-5) \cdot 2 + 2 \cdot 3 \cdot (-1) - 2 \cdot 4 \cdot 2 - 1 \cdot (-5) \cdot (-1) - (-1) \cdot 3 \cdot 3$
$= 12 + 10 - 6 - 16 - 5 + 9$
$= 4$

Matrix of cofactors = $$\mathbf{C} = \begin{pmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & 7 \end{pmatrix}$$

Transposed matrix of cofactors = $C^T = \begin{pmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{pmatrix}$

$A^{-1} = C^T / |A| = \begin{pmatrix} \frac{7}{4} & \frac{1}{4} & -\frac{3}{4} \\ -\frac{19}{4} & -\frac{1}{4} & \frac{11}{4} \\ -\frac{11}{4} & -\frac{1}{4} & \frac{7}{4} \end{pmatrix}$

$X = A^{-1}B$

$= \begin{pmatrix} \frac{7}{4} & \frac{1}{4} & -\frac{3}{4} \\ -\frac{19}{4} & -\frac{1}{4} & \frac{11}{4} \\ -\frac{11}{4} & -\frac{1}{4} & \frac{7}{4} \end{pmatrix} \cdot \begin{pmatrix} 7 \\ -5 \\ 12 \end{pmatrix} = \begin{pmatrix} \frac{7}{4}\cdot 7 + \frac{1}{4}\cdot(-5) + \left(-\frac{3}{4}\right)\cdot 12 \\ \left(-\frac{19}{4}\right)\cdot 7 + \left(-\frac{1}{4}\right)\cdot(-5) + \frac{11}{4}\cdot 12 \\ \left(-\frac{11}{4}\right)\cdot 7 + \left(-\frac{1}{4}\right)\cdot(-5) + \frac{7}{4}\cdot 12 \end{pmatrix} = \begin{pmatrix} \frac{49}{4} - \frac{5}{4} - 9 \\ -\frac{133}{4} + \frac{5}{4} + 33 \\ -\frac{77}{4} + \frac{5}{4} + 21 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

x = 2, y = 1, z = 3

Question 12: Define collinear vectors.

Solution:
The two vectors are said to be collinear if they lie on the same parallel lines.

Question 13: Find the direction cosines of a line which makes equal angles with the positive coordinate axes.

Solution:
Let the direction cosines of the line make an angle α with each of the coordinate axes.

l = cos α, m = cos β, n = cos **γ**

It is given that they make equal angles with the positive coordinate axis.

α = β = **γ**

$l^2 + m^2 + n^2 = 1$

$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

$\cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1$

$3 \cos^2 \alpha = 1$

$\cos^2 \alpha = 1 / 3$

$\cos \alpha = \sqrt{1 / 3}$

$\cos \alpha = \pm 1 / \sqrt{3}$

The direction cosines are $l = \pm 1 / \sqrt{3}$, $m = \pm 1 / \sqrt{3}$, $n = \pm 1 / \sqrt{3}$.

Question 14: Find the approximate change in the volume of a cube of side x metres caused by increasing the size by 3%.

Solution:

The volume of a cube (V) of side x is given by $V = x^3$.

$dV = (dV / dx) \Delta x$

$= (3x^2) \Delta x$

$= (3x^2) (0.03x)$

$= 0.09 x^3$

Question 15: Find the probability distribution of the number of heads in two tosses of a coin.

Solution:

When a coin is tossed twice, the number of heads maybe 0, 1, 2.

Sample space = S = {HH, HT, TH, TT}

X	0	1	2
P (X)	1 / 4	2 / 4	1 /4

Question 16: Form the differential equation of the family of circles having a centre on y-axis and radius 3 units.

Solution:

The required equation of the circle is $(x - 0)^2 + (y - k)^2 = 3^2$ ---- (1)

$x^2 + y^2 + k^2 - 2yk = 9$ [k is any value]
On differentiating,
$2x + 2y \cdot y_1 + 0 - 2k\, y_1 = 0$
$x + yy_1 - ky_1 = 0$
$x + (y - k)\, y_1 = 0$
$y - k = - x / y_1$ ---- (2)
Put (2) in (1)
$x^2 + (-x / y_1) = 9$
$x^2 + x^2 / y_1 = 9$
$x^2 (y_1^2 + 1) = 9y_1^2$ is the required differential equation.

Question 17: If A = $\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ then show that $A^3 - 23A - 40I = 0$.

Solution:

$$A^2 = A \times A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6+12 & 2-4+6 & 3+2+3 \\ 3-6+4 & 6+4+2 & 9-2+1 \\ 4+6+4 & 8-4+2 & 12+2+1 \end{bmatrix} = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$\therefore A^3 = A^2 \times A = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 19+12+32 & 38+8+16 & 57+4+8 \\ 1+36+32 & 2-24+16 & 3+12+8 \\ 14+18+60 & 28-12+30 & 42+6+15 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

Now, LHS = A3 – 23A – 40I

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - 23\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - \begin{bmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{bmatrix} - \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} 63-23-40 & 46-46+0 & 69-69+0 \\ 69-69-0 & -6+46-40 & 23-23+0 \\ 92-92-0 & 46-46+0 & 63-23+40 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 = \text{RHS.}$$

Question 18: Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8$, $x \in [-4, 2]$.

Solution:

Since the given function is a polynomial function, it is continuous at [- 4, 2].

f ‘ (x) = 2x + 2

The given function is differentiable at [- 4, 2].

f (- 4) = 16 - 8 - 8 = 0

f (2) = 4 + 4 - 8 = 0

f (− 4) = f (2) at $x \in [-4, 2]$

By Rolles’ theorem, there exists a real valued function $c \in [-4, 2]$

f ‘ (c) = 0

2c + 2 = 0

2c = - 2

$c = -1 \in [-4, 2]$

Thus Rolle’s theorem is verified.

Question 19: A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall, at a rate of 2cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall?

Solution:

Let y m be the height of the wall at which the ladder touches.

Let the foot of the ladder be x m away from the wall.

By Pythagoras theorem,

$x^2 + y^2 = 25$

$y = \sqrt{25 - x^2}$

The rate of change of the height (y) with respect to time (t) is given by

$dy / dt = [-x / \sqrt{25 - x^2}] (dx / dt)$

It is given that dx / dt = 2cm/sec.

$dy / dt = -2x / \sqrt{25 - x^2}$

When x = 4m,

$dy / dt = [-2 * 4] / [\sqrt{25 - 4^2}]$

= 8 / 3 cm/sec

Question 20: Find the two positive numbers whose sum is 15 and the sum of whose squares is minimum.

Solution:

Let the first number be x.

The sum of two numbers is 15.

⇒ First number + second number = 15

⇒ x + second number = 15

⇒ Second number = 15 - x

f (x) shows the sum of the squares of the number.

⇒ $f(x) = x^2 + (15 - x)^2 = 2x^2 - 30x + 225$

By differentiating with respect to x,

$f'(x) = 4x - 30$

For maximum or minimum, $f'(x) = 0$.

⇒ $4x - 30 = 0$

⇒ $x = 7.5$

Again differentiating $f'(x)$ with respect to x,

$f''(x) = 4$

At $x = 7.5$, $f''(x)$ = positive.

Thus, f (x) is minimum at $x = 7.5$.

Hence, the first number is 7.5 and the second number is 15 - 7.5 = 7.5.