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Meghalaya Board Class 12 Maths Question Paper 2020

HS/XII/A.Sc.Com/M/OC/20

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MATHEMATICS

(Old Course)

Full Marks : 100

Time : 3 hours

General Instructions :

(i) Write all the answers in the Answer Script.

(ii) The question paper consists of three Sections—A, B and C.

(iii) Section—A consists of 15 questions, carrying *2* marks each.

(iv) Section—B consists of 10 questions, carrying *4* marks each, out of which 3 questions have internal choices.

(v) Section—C has 5 questions, carrying *6* marks each, out of which 2 questions have internal choices.

SECTION—A

1. Let S be the set of all real numbers and let R be a relation in S defined by

$$R \equiv \{(a,\ b) : (1 + ab) > 0\}$$

Show that R is reflexive and symmetric. 2

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2. Let

$$R = \left\{ \left(a, \frac{1}{a} \right) : a \in N \text{ and } 1 \le a \le 5 \right\}$$

List the elements of the above relation. Find the domain and range. 2

3. Show that the function $f : \mathbb{R} \to \mathbb{R} : f(x) = |x|$ is neither one-one nor onto. 2

4. Show that the operation $*$ on $\mathbb{Z}$, defined by

$$a * b = a + b + 1 \quad \forall\, a, b \in \mathbb{Z}$$

satisfies *(a)* closure property and *(b)* commutative law. 2

5. Construct a 3×2 matrix whose elements are given by $a_{ij} = i + 2j$. 2

6. Let

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & -5 & 7 \end{bmatrix}$$

Verify that $(A')' = A$. 2

7. Show that $f(x) = x^3$ is continuous at the point $x = 2$. 2

8. Differentiate $y = \tan\sqrt{x}$ with respect to *x*. 2

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9. If $x^3 + y^3 = 3axy$, find $\frac{dy}{dx}$. 2

10. Find $\frac{dy}{dx}$, when $x = at^2$, $y = 2at$. 2

11. A stone is dropped into a quiet lake and the waves move in circles. If the radius of a circular wave increases at the rate of 4 cm/s, find the rate of increase in its area at the instant when its radius is 10 cm. 2

12. Evaluate : 2

$$\int x^2 e^x dx$$

13. Let

$$\bar{a} = 3\hat{i} + 2\hat{j} \quad \text{and} \quad \bar{b} = 2\hat{i} + 3\hat{j}$$

Is $|\bar{a}| = |\bar{b}|$? Is $\bar{a} = \bar{b}$? 2

14. If

$$\bar{a} = 3\hat{i} + \hat{j} - 4\hat{k} \quad \text{and} \quad \bar{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$$

find $\bar{a} \times \bar{b}$ and $|\bar{a} \times \bar{b}|$. 2

15. If E_1 and E_2 are two independent events such that $P(E_1) = 0{\cdot}35$ and $P(E_1 \cup E_2) = 0{\cdot}60$, find $P(E_2)$. 2

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SECTION—B

16. Express the matrix

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

as the sum of a symmetric and a skew-symmetric matrix. 4

Or

If

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

verify that $A^2 - 4A - I = 0$ and hence find A^{-1}. 4

17. Show that the semivertical angle of a right circular cone of given surface area and maximum volume is

$$\sin^{-1}\left(\frac{1}{3}\right)$$ 4

18. Verify Rolle's theorem for the function $f(x) = e^x \cos x$ in

$$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$ 4

Or

Using Lagrange's mean value theorem, find a point on the curve $y = \sqrt{x-2}$, defined in the interval $[2, 3]$, where the tangent is parallel to the chord joining the end points of the curve. 4

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19. *(a)* The side of a square is increasing at the rate of 0·2 cm/s. Find the rate of increase of the perimeter of the square. 2

(b) Find the approximate value of the cube root of 127. 2

20. Water is leaking from a conical funnel at the rate of 5 cm^3/s. If the radius of the base of the funnel is 5 cm and its altitude is 10 cm, find the rate at which the water level is dropping when it is 2·5 cm from the top. 4

21. *(a)* If $y = e^x(\sin x + \cos x)$, prove that

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$ 2

(b) Solve the differential equation

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$$ 2

22. *(a)* Evaluate : 2

$$\int_0^a \frac{dx}{\sqrt{ax - x^2}}$$

(b) Evaluate : 2

$$\int_0^{\sqrt{2}} \sqrt{2 - x^2}\, dx$$

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Or

Evaluate : 4

$$\int_0^{\pi} \frac{x}{(a^2 \cos^2 x + b^2 \sin^2 x)} dx$$

23. Find the intervals in which the function

$$f(x) = -2x^3 - 9x^2 - 12x + 1$$

is *(a)* strictly increasing and *(b)* strictly decreasing. 4

24. A line passes through the point (3, 4, 5) and is parallel to the vector $(2\hat{i} + 2\hat{j} - 3\hat{k})$. Find the equations of the line in the vector as well as Cartesian forms. 4

25. Find the equations of the normals to the curve $3x^2 - y^2 = 8$ parallel to the line $x + 3y = 4$. 4

SECTION—C

26. Using matrices, solve the following system of equations : 6

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$
$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$
$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

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Or

(a) Without expanding the determinants, prove that

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$ 3

(b) Solve for x : 3

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

27. Evaluate

$$\int_a^b \sin x \, dx$$

from the first principle. 6

Or

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$. 6

28. *(a)* Find the angle between the lines whose direction ratios are 2, 3, 6 and 1, 2, 2. 2

(b) Show that the lines

$$\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$$

intersect each other. Also, find the point of their intersection. 4

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29. A factory has three machines X, Y and Z, producing 1000 bolts, 2000 bolts and 3000 bolts per day respectively. The machine X produces 1% defective bolts, Y produces 1·5% defective bolts and Z produces 2% defective bolts. At the end of the day, a bolt is drawn at random and it is found to be defective. What is the probability that this defective bolt has been produced by machine X? 6

30. A firm manufactures two types of products A and B and sells them at a profit of ₹ 5 per unit of type A and ₹ 3 per unit of type B. Each product is processed on two machines M_1 and M_2. One unit of type A requires one minute of processing time on M_1 and two minutes of processing time on M_2, whereas one unit of type B requires one minute of processing time on M_1 and one minute of processing time on M_2. Machines M_1 and M_2 are respectively available for at most 5 hours and 6 hours in a day. Find out how many units of each type of products should the firm produce a day in order to maximize the profit. Solve the problem graphically. 6

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