Exercise 12.1

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1. Evaluate:
(i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $(1/2)^{-5}$

Solution:

(i) $3^{-2} = (1/3)^2$

$\left[\because \ a^{-m} = \frac{1}{a^m}\right]$

= 1/9

(ii) $\mathbf{(-4)^{-2} = (1/-4)^2}$

$\left[\because \ a^{-m} = \frac{1}{a^m}\right]$

= 1/16

(iii) $\mathbf{(1/2)^{-5} = (2/1)^5}$

$\left[\because \ a^{-m} = \frac{1}{a^m}\right]$

$= 2^5$

= 32

2. Simplify and express the result in power notation with positive exponent:

(i) $(-4)^4 \div (-4)^8$

(ii) $(1/2^3)^2$

(iii) $-(3)^4 \times (5/3)^4$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$

(v) $2^{-3}\times(-7)^{-3}$

Solution:

(i) $(-4)^5 \div (-4)^8$

$= (-4)^5/(-4)^8$

$\left[\because \ a^m \div a^n = a^{m-n}\right]$

$= (-4)^{5-8}$

$= 1/(-4)^3$

(ii) **$(1/2^3)^2$**

$= 1^2/(2^3)^2$

$\left[\because \ \left(\frac{a}{b}\right)^m = \frac{a^m}{a^n}\right]$

$= 1/2^{3\times2} = 1/2^6$

$\left[\because \ \left(a^m\right)^n = a^{m\times n}\right]$

(iii) **$-(3)^4\times(5/3)^4$**

$$(-3)^4 \times \left(\frac{5}{3}\right)^4 = (-3)^4 \times \frac{5^4}{3^4} \quad \left[\because \ \left(\frac{a}{b}\right)^m = \frac{a^m}{a^n}\right]$$

$= (-1)^4\times3^4\times(5^4/3^4)$

$\left[\because \ (ab)^m = a^m b^m\right]$

$= 3^{(4-4)}\times5^4$

$\left[\because \ a^m \div a^n = a^{m-n}\right]$

$= 3^0 \times 5^4 = 5^4$

$\left[\because \ a^0 = 1\right]$

(iv) $\left(3^{-7} \div 3^{-10}\right) \times 3^{-5}$

$= (3^{-7}/3^{-10}) \times 3^{-5}$

$= 3^{-7-(-10)} \times 3^{-5}$

$\left[\because \ a^m \div a^n = a^{m-n}\right]$

$= 3^{(-7+10)} \times 3^{-5}$

$= 3^3 \times 3^{-5}$

$= 3^{(3+-5)}$

$\left[\because \ a^m \times a^n = a^{m+n}\right]$

$= 3^{-2}$

$= 1/3^2$

$\left[\because \ a^{-m} = \frac{1}{a^m}\right]$

(v) $2^{-3} \times (-7)^{-3}$

$= (2 \times -7)^{-3}$

(Because $a^m \times b^m = (ab)^m$)

$= 1/(2 \times -7)^3$

$\left[\because \ a^{-m} = \frac{1}{a^m}\right]$

$= 1/(-14)^3$

3. Find the value of :

(i) $(3^0+4^{-1})\times 2^2$

(ii) $(2^{-1}\times 4^{-1})\div 2^{-2}$

(iii) $(1/2)^{-2}+(1/3)^{-2}+(1/4)^{-2}$

(iv) $(3^{-1}+4^{-1}+5^{-1})^0$

(v) $\{(-2/3)^{-2}\}^2$

Solution:

(i)$(3^0+4^{-1})\times 2^2 = (1+(1/4))\times 2^2$

$$\left[\because\ a^{-m}=\frac{1}{a^m}\right]$$

$= ((4+1)/4\ )\times 2^2$

$= (5/4)\times 2^2$

$= (5/2^2)\times 2^2$
$= 5\times 2^{(2-2)}$

$$\left[\because\ a^m\div a^n=a^{m-n}\right]$$

$= 5\times 2^0$

$= 5\times 1 = 5$

$$\left[\because\ a^{-m}=\frac{1}{a^m}\right]$$

(ii)$(2^{-1}\times 4^{-1})\div 2^{-2}$

$= [(1/2)\times(1/4)]\div(1/4)$

$\left[\because \ a^{-m}=\frac{1}{a^m}\right]$

$= (1/2\times 1/2^2)\div 1/4$

$= 1/2^3 \div 1/4$

$= (1/8)\times(4)$
$= 1/2$

(iii) $\mathbf{(1/2)^{-2}+(1/3)^{-2}+(1/4)^{-2}}$
$= (2^{-1})^{-2}+(3^{-1})^{-2}+(4^{-1})^{-2}$

$\left[\because \ a^{-m}=\frac{1}{a^m}\right]$

$= 2^{(-1\times -2)}+3^{(-1\times -2)}+4^{(-1\times -2)}$

$\left[\because \ (a^m)^n = a^{m\times n}\right]$

$= 2^2+3^2+4^2$

$= 4+9+16$

$=29$

(iv) $(3^{-1}+4^{-1}+5^{-1})^0$
$= 1$

$\left[\because \ a^0 = 1\right]$

(v) $\mathbf{\{(-2/3)^{-2}\}^2 = (-2/3)^{-2\times 2}}$

$\left[\because \ (a^m)^n = a^{m\times n}\right]$

$= (-2/3)^{-4}$
$= (-3/2)^4$

$\left[\because \ a^{-m}=\frac{1}{a^m}\right]$

$= 81/16$

4. Evaluate

(i) $(8^{-1}\times5^3)/2^{-4}$

(ii) $(5^{-1}\times2^{-2})\times6^{-1}$

Solution:

(i) $(8^{-1}\times5^3)/2^{-4}$

$$\frac{8^{-1}\times5^3}{2^{-4}}=\frac{\left(2^3\right)^{-1}\times5^3}{2^{-4}}=\frac{2^{-3}\times5^3}{2^{-4}} \qquad \left[\because \ \left(a^m\right)^n=a^{m\times n}\right]$$

$$= \ 2^{-3-(-4)}\times5^3=2^{-3+4}\times5^3 \qquad \left[\because \ a^m\div a^n=a^{m-n}\right]$$

$= 2\times125 = 250$

(ii) $(5^{-1}\times2^{-2})\times6^{-1}$

$$\left(5^{-1}\times2^{-1}\right)\times6^{-1}=\left(\frac{1}{5}\times\frac{1}{2}\right)\times\frac{1}{6} \quad \left[\because \ a^{-m}=\frac{1}{a^m}\right]$$

$= (1/10)\times1/6$

$= 1/60$

5. Find the value of *m* for which $5^m \div 5^{-3} = 5^5$

Solution:

$5^m \div 5^{-3} = 5^5$

$5^{(m-(-3))} = 5^5$

$\left[\because \ a^m\div a^n=a^{m-n}\right]$

$5^{m+3} = 5^5$

Comparing exponents both sides, we get

m+3 = 5

m = 5-3
m = 2

6. Evaluate

(i)

$$\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$$

(ii)

$$\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}$$

Solution:

(i)

$$\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}=\left\{\left(\frac{3}{1}\right)^{1}-\left(\frac{4}{1}\right)^{1}\right\} \quad \left[\because\ a^{-m}=\frac{1}{a^{m}}\right]$$

= 3-4

= -1

(ii)

$$\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}=\frac{5^{-7}}{8^{-7}}\times\frac{8^{-4}}{5^{-4}} \quad \left[\because\ \left(\frac{a}{b}\right)^{m}=\frac{a^{m}}{b^{m}}\right]$$

$$= 5^{-7-(-4)}\times 8^{-4-(-7)} \quad \left[\because\ a^{m}\div a^{n}=a^{m-n}\right]$$

$= 5^{-7+4} \times 8^{-4+7}$

$= 5^{-3} \times 8^{3} = \frac{8^3}{5^3} \quad \left[\because \ a^{-m} = \frac{1}{a^m}\right]$

= 512/125

7. Simplify.

(i)

$\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} \quad (t \neq 0)$

(ii)

$\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

Solution 7:

(i)

$\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$

$= \frac{5^2 \times t^{-4}}{5^{-3} \times 5 \times 2 \times t^{-8}}$

$= \frac{5^{2-(-3)-1} \times t^{-4-(-8)}}{2}$

$\left[\because \ a^m \div a^n = a^{m-n}\right]$

$= \frac{5^{2+3-1} \times t^{-4+8}}{2} = \frac{5^4 \times t^4}{2} = \frac{625}{2} t^4$

(ii)

$$\frac{3^{-5}\times 10^{-5}\times 125}{5^{-7}\times 6^{-5}}$$

$$=\frac{3^{-5}\times(2\times 5)^{-5}\times 5^{3}}{5^{-7}\times(2\times 3)^{-5}}$$

$$=\frac{3^{-5}\times 2^{-5}\times 5^{-5}\times 5^{3}}{5^{-7}\times 2^{-5}\times 3^{-5}}$$

$$\left[\because\ (ab)^{m}=a^{m}b^{m}\right]$$

$$=\frac{3^{-5}\times 2^{-5}\times 5^{-5+3}}{5^{-7}\times 2^{-5}\times 3^{-5}}=\frac{3^{-5}\times 2^{-5}\times 5^{-2}}{5^{-7}\times 2^{-5}\times 3^{-5}}$$

$$\left[\because\ a^{m}\times a^{n}=a^{m+n}\right]$$

$$=3^{-5-(-5)}\times 2^{-5-(-5)}\times 5^{-2-(-7)}\quad\left[\because\ a^{m}\div a^{n}=a^{m-n}\right]$$

$$=3^{-5+5}\times 2^{-5+5}\times 5^{-2+7}=3^{0}\times 2^{0}\times 5^{5}$$

$$=1\times 1\times 3125\quad\left[\because\ a^{0}=1\right]$$

$$=3125$$