

Exercise 12.1

Page No: 197

1. Evaluate:

(i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $(1/2)^{-5}$

Solution:

$$(i) 3^{-2} = (1/3)^2$$
$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= 1/9$$

$$(ii) (-4)^{-2} = (1/-4)^2$$
$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= 1/16$$

$$(iii) (1/2)^{-5} = (2/1)^5$$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= 2^5$$

$$= 32$$

2. Simplify and express the result in power notation with positive exponent:

(i) $(-4)^4 \div (-4)^8$

(ii) $(1/2^3)^2$

(iii) $-(3)^4 \times (5/3)^4$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$

(v) $2^{-3} \times (-7)^{-3}$

Solution:

(i) $(-4)^5 \div (-4)^8$

$$= (-4)^5 / (-4)^8$$

$[\because a^m \div a^n = a^{m-n}]$

$$= (-4)^{5-8}$$

$$= 1/(-4)^3$$

(ii) $(1/2^3)^2$

$$= 1^2 / (2^3)^2$$

$$\left[\because \left(\frac{a}{b} \right)^m = \frac{a^m}{b^m} \right]$$

$$= 1/2^{3 \times 2} = 1/2^6$$

$$\left[\because (a^m)^n = a^{m \times n} \right]$$

(iii) $-(3)^4 \times (5/3)^4$

$$(-3)^4 \times \left(\frac{5}{3} \right)^4 = (-3)^4 \times \frac{5^4}{3^4} \left[\because \left(\frac{a}{b} \right)^m = \frac{a^m}{b^m} \right]$$

$$= (-1)^4 \times 3^4 \times (5^4/3^4)$$

$$\left[\because (ab)^m = a^m b^m \right]$$

$$= 3^{(4-4)} \times 5^4$$

$$\left[\because a^m \div a^n = a^{m-n} \right]$$

$$= 3^0 \times 5^4 = 5^4$$

$$[\because a^0 = 1]$$

$$(iv) (3^{-7} \div 3^{-10}) \times 3^{-5}$$

$$= (3^{-7}/3^{-10}) \times 3^{-5}$$

$$= 3^{-7 - (-10)} \times 3^{-5}$$

$$[\because a^m \div a^n = a^{m-n}]$$

$$= 3^{(-7+10)} \times 3^{-5}$$

$$= 3^3 \times 3^{-5}$$

$$= 3^{(3+(-5))}$$

$$[\because a^m \times a^n = a^{m+n}]$$

$$= 3^{-2}$$

$$= 1/3^2$$

$$[\because a^{-m} = \frac{1}{a^m}]$$

$$(v) 2^{-3} \times (-7)^{-3}$$

$$= (2 \times -7)^{-3}$$

(Because $a^m \times b^m = (ab)^m$)

$$= 1/(2 \times -7)^3$$

$$[\because a^{-m} = \frac{1}{a^m}]$$

$$= 1/(-14)^3$$

3. Find the value of :

(i) $(3^0+4^{-1})\times 2^2$

(ii) $(2^{-1}\times 4^{-1})\div 2^{-2}$

(iii) $(1/2)^{-2}+(1/3)^{-2}+(1/4)^{-2}$

(iv) $(3^{-1}+4^{-1}+5^{-1})^0$

(v) $\{(-2/3)^{-2}\}^2$

Solution:

(i) $(3^0+4^{-1})\times 2^2 = (1+(1/4))\times 2^2$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= ((4+1)/4)\times 2^2$$

$$= (5/4)\times 2^2$$

$$= (5/2^2)\times 2^2$$
$$= 5\times 2^{(2-2)}$$

$$\left[\because a^m \div a^n = a^{m-n} \right]$$

$$= 5\times 2^0$$

$$= 5\times 1 = 5$$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

(ii) $(2^{-1}\times 4^{-1})\div 2^{-2}$

$$= [(1/2)\times (1/4)] \div (1/4)$$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= (1/2 \times 1/2^2) \div 1/4$$

$$= 1/2^3 \div 1/4$$

$$= (1/8) \times (4)$$

$$= 1/2$$

$$\text{(iii) } (1/2)^{-2} + (1/3)^{-2} + (1/4)^{-2}$$
$$= (2^{-1})^{-2} + (3^{-1})^{-2} + (4^{-1})^{-2}$$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= 2^{(-1 \times -2)} + 3^{(-1 \times -2)} + 4^{(-1 \times -2)}$$

$$\left[\because (a^m)^n = a^{m \times n} \right]$$

$$= 2^2 + 3^2 + 4^2$$

$$= 4 + 9 + 16$$

$$= 29$$

$$\text{(iv) } (3^{-1} + 4^{-1} + 5^{-1})^0$$

$$= 1$$

$$\left[\because a^0 = 1 \right]$$

$$\text{(v) } \{(-2/3)^{-2}\}^2 = (-2/3)^{-2 \times 2}$$

$$\left[\because (a^m)^n = a^{m \times n} \right]$$

$$= (-2/3)^{-4}$$

$$= (-3/2)^4$$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= 81/16$$

4. Evaluate

(i) $(8^{-1} \times 5^3) / 2^{-4}$

(ii) $(5^{-1} \times 2^{-2}) \times 6^{-1}$

Solution:

(i) $(8^{-1} \times 5^3) / 2^{-4}$

$$\frac{8^{-1} \times 5^3}{2^{-4}} = \frac{(2^3)^{-1} \times 5^3}{2^{-4}} = \frac{2^{-3} \times 5^3}{2^{-4}} \quad \left[\because (a^m)^n = a^{m \times n} \right]$$

$$= 2^{-3-(-4)} \times 5^3 = 2^{-3+4} \times 5^3 \quad \left[\because a^m \div a^n = a^{m-n} \right]$$

$$= 2 \times 125 = 250$$

(ii) $(5^{-1} \times 2^{-2}) \times 6^{-1}$

$$(5^{-1} \times 2^{-1}) \times 6^{-1} = \left(\frac{1}{5} \times \frac{1}{2} \right) \times \frac{1}{6} \quad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= (1/10) \times 1/6$$

$$= 1/60$$

5. Find the value of m for which $5^m \div 5^{-3} = 5^5$

Solution:

$$5^m \div 5^{-3} = 5^5$$

$$5^{(m-(-3))} = 5^5$$

$$\left[\because a^m \div a^n = a^{m-n} \right]$$

$$5^{m+3} = 5^5$$

Comparing exponents both sides, we get

$$m+3 = 5$$

$$m = 5-3$$

$$m = 2$$

6. Evaluate

(i)

$$\left\{ \left(\frac{1}{3} \right)^{-1} - \left(\frac{1}{4} \right)^{-1} \right\}^{-1}$$

(ii)

$$\left(\frac{5}{8} \right)^{-7} \times \left(\frac{8}{5} \right)^{-4}$$

Solution:

(i)

$$\left\{ \left(\frac{1}{3} \right)^{-1} - \left(\frac{1}{4} \right)^{-1} \right\}^{-1} = \left\{ \left(\frac{3}{1} \right)^1 - \left(\frac{4}{1} \right)^1 \right\}^{-1} \quad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= 3-4$$

$$= -1$$

(ii)

$$\left(\frac{5}{8} \right)^{-7} \times \left(\frac{8}{5} \right)^{-4} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-4}}{5^{-4}} \quad \left[\because \left(\frac{a}{b} \right)^m = \frac{a^m}{b^m} \right]$$

$$= 5^{-7-(-4)} \times 8^{-4-(-7)} \quad \left[\because a^m \div a^n = a^{m-n} \right]$$

$$= 5^{-7+4} \times 8^{-4+7}$$

$$= 5^{-3} \times 8^3 = \frac{8^3}{5^3} \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= 512/125$$

7. Simplify.

(i)

$$\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} \quad (t \neq 0)$$

(ii)

$$\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$$

Solution 7:

(i)

$$\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$$

$$= \frac{5^2 \times t^{-4}}{5^{-3} \times 5 \times 2 \times t^{-8}}$$

$$= \frac{5^{2-(-3)-1} \times t^{-4-(-8)}}{2}$$

$$\left[\because a^m \div a^n = a^{m-n} \right]$$

$$= \frac{5^{2+3-1} \times t^{-4+8}}{2} = \frac{5^4 \times t^4}{2} = \frac{625}{2} t^4$$

(ii)

$$\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$$

$$= \frac{3^{-5} \times (2 \times 5)^{-5} \times 5^3}{5^{-7} \times (2 \times 3)^{-5}}$$

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}}$$

$$[\because (ab)^m = a^m b^m]$$

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5+3}}{5^{-7} \times 2^{-5} \times 3^{-5}} = \frac{3^{-5} \times 2^{-5} \times 5^{-2}}{5^{-7} \times 2^{-5} \times 3^{-5}}$$

$$[\because a^m \times a^n = a^{m+n}]$$

$$= 3^{-5-(-5)} \times 2^{-5-(-5)} \times 5^{-2-(-7)} \quad [\because a^m \div a^n = a^{m-n}]$$

$$= 3^{-5+5} \times 2^{-5+5} \times 5^{-2+7} = 3^0 \times 2^0 \times 5^5$$

$$= 1 \times 1 \times 3125 \quad [\because a^0 = 1]$$

$$= 3125$$

Exercise 12.2

Page No: 200

1. Express the following numbers in standard form.

- (i) 0.0000000000085
- (ii) 0.00000000000942
- (iii) 6020000000000000
- (iv) 0.00000000837
- (v) 31860000000

Solution:

- (i) $0.0000000000085 = 0.0000000000085 \times (10^{12}/10^{12}) = 8.5 \times 10^{-12}$
- (ii) $0.00000000000942 = 0.00000000000942 \times (10^{12}/10^{12}) = 9.42 \times 10^{-12}$
- (iii) $6020000000000000 = 6020000000000000 \times (10^{15}/10^{15}) = 6.02 \times 10^{15}$
- (iv) $0.00000000837 = 0.00000000837 \times (10^9/10^9) = 8.37 \times 10^{-9}$
- (v) $31860000000 = 31860000000 \times (10^{10}/10^{10}) = 3.186 \times 10^{10}$

2. Express the following numbers in usual form.

- (i) 3.02×10^{-6}
- (ii) 4.5×10^4
- (iii) 3×10^{-8}
- (iv) 1.0001×10^9
- (v) 5.8×10^{12}
- (vi) 3.61492×10^6

Solution:

- (i) $3.02 \times 10^{-6} = 3.02/10^6 = 0.00000302$
- (ii) $4.5 \times 10^4 = 4.5 \times 10000 = 45000$
- (iii) $3 \times 10^{-8} = 3/10^8 = 0.00000003$
- (iv) $1.0001 \times 10^9 = 1000100000$
- (v) $5.8 \times 10^{12} = 5.8 \times 1000000000000 = 5800000000000$
- (vi) $3.61492 \times 10^6 = 3.61492 \times 1000000 = 3614920$

3. Express the number appearing in the following statements in standard form.

- (i) 1 micron is equal to $1/1000000$ m.
- (ii) Charge of an electron is 0.000, 000, 000, 000, 000, 16 coulomb.
- (iii) Size of bacteria is 0.0000005 m
- (iv) Size of a plant cell is 0.00001275 m
- (v) Thickness of a thick paper is 0.07 mm

Solution :

(i) 1 micron = $1/1000000$

$$= 1/10^6$$

$$= 1 \times 10^{-6}$$

(ii) Charge of an electron is 0.00000000000000000016 coulombs.

$$= 0.00000000000000000016 \times 10^{19}/10^{19}$$

$$= 1.6 \times 10^{-19} \text{ coulomb}$$

(iii) Size of bacteria = 0.0000005

$$= 5/10000000 = 5/10^7 = 5 \times 10^{-7} \text{ m}$$

(iv) Size of a plant cell is 0.00001275 m

$$= 0.00001275 \times 10^5/10^5$$

$$= 1.275 \times 10^{-5} \text{ m}$$

(v) Thickness of a thick paper = 0.07 mm

$$0.07 \text{ mm} = 7/100 \text{ mm} = 7/10^2 = 7 \times 10^{-2} \text{ mm}$$

4. In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Solution:

Thickness of one book
= 20 mm

Thickness of 5 books = $20 \times 5 = 100$ mm

Thickness of one paper = 0.016 mm

Thickness of 5 papers = $0.016 \times 5 = 0.08$ mm

Total thickness of a stack = $100 + 0.08 = 100.08$ mm

= $100.08 \times 10^2 / 10^2$ mm

= 1.0008×10^2 mm