Exercise 14.3   Page No: 227

1. Carry out the following divisions.

(i) $28x^4 \div 56x$
(ii) $-36y^3 \div 9y^2$
(iii) $66pq^2r^3 \div 11qr^2$
(iv) $34x^3y^3z^3 \div 51xy^2z^3$
(v) $12a^8b^8 \div (-6a^6b^4)$

Solution:

(i) $28x^4 = 2\times2\times7\times x\times x\times x\times x$
$56x = 2\times2\times2\times7\times x$

$$28x^4 \div 56x = \frac{2\times2\times7\times x\times x\times x\times x}{2\times2\times2\times7\times x} = \frac{x^3}{2} = \frac{1}{2}x^3$$

$$\text{(ii) } -36y^3 \div 9y^2 = \frac{-2\times2\times3\times3\times y\times y\times y}{3\times3\times y\times y} = -4y$$

$$\text{(iii) } 66pq^2r^3 \div 11qr^2 = \frac{2\times3\times11\times p\times q\times q\times r\times r\times r}{11\times q\times r\times r} = 6pqr$$

$$\text{(iv) } 34x^3y^3z^3 \div 51xy^2z^3 = \frac{2\times17\times x\,x\times x\,x\times x\,x\times y\times y\times y\times z\times z\times z}{3\times17\times x\times xy\times xy\times xz\times xz\times xz} = \frac{2}{3}x^2y$$

$$\text{(v) } 12a^8b^8 \div (-6a^6b^4) = \frac{2\times2\times3\times a^8\times b^8}{-2\times3\times a^6\times b^4} = -2\,a^2\,b^4$$

2. Divide the given polynomial by the given monomial.

(i) $(5x^2-6x) \div 3x$

(ii)$(3y^8 - 4y^6 + 5y^4) \div y^4$

(iii) $8(x^3 y^2 z^2 + x^2 y^3 z^2 + x^2 y^2 z^3) \div 4x^2 y^2 z^2$

(iv)$(x^3 + 2x^2 + 3x) \div 2x$

(v) $(p^3 q^6 - p^6 q^3) \div p^3 q^3$

Solution:

(i) $5x^2 - 6x = x(5x - 6)$

$(5x^2 - 6x) \div 3x = \dfrac{x(5x - 6)}{3x} = \dfrac{1}{3}(5x - 6)$

(ii) $3y^8 - 4y^6 + 5y^4 = y^4(3y^4 - 4y^2 + 5)$

$(3y^8 - 4y^6 + 5y^4) \div y^4 = \dfrac{y^4(3y^4 - 4y^2 + 5)}{y^4} = 3y^4 - 4y^2 + 5$

(iii) $8(x^3 y^2 z^2 + x^2 y^3 z^2 + x^2 y^2 z^3) = 8x^2 y^2 z^2(x + y + z)$

$8(x^3 y^2 z^2 + x^2 y^3 z^2 + x^2 y^2 z^3) \div 4x^2 y^2 z^2 = \dfrac{8x^2 y^2 z^2(x + y + z)}{4x^2 y^2 z^2} = 2(x + y + z)$

(iv) $x^3 + 2x^2 + 3x = x(x^2 + 2x + 3)$

$(x^3 + 2x^2 + 3x) \div 2x = \dfrac{x(x^3 + 2x^2 + 3)}{2x} = \dfrac{1}{2}(x^2 + 2x + 3)$

(v) $p^3 q^6 - p^6 q^3 = p^3 q^3(q^3 - p^3)$

$(p^3 q^6 - p^6 q^3) \div p^3 q^3 = \dfrac{p^3 q^3(q^3 - p^3)}{p^3 q^3} = q^3 - p^3$

3. Work out the following divisions.

(i) $(10x - 25) \div 5$

(ii) $(10x - 25) \div (2x - 5)$

(iii) $10y(6y + 21) \div 5(2y + 7)$

(iv) $9x^2 y^2(3z - 24) \div 27xy(z - 8)$

(v) $96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$

Solution:

(i) $(10x–25) \div 5 = 5(2x-5)/5 = 2x-5$
(ii) $(10x–25) \div (2x–5) = 5(2x-5)/(2x-5) = 5$

(iii) $10y(6y+21) \div 5(2y+7) = 10y×3(2y+7)/5(2y+7) = 6y$
(iv) $9x^2y^2(3z–24) \div 27xy(z–8) = 9x^2y^2×3(z-8)/27xy(z-8) = xy$

(v) $96abc(3a –12)(5b–30) \div 144(a–4)(b–6) = \dfrac{96\ abc\ ×3(a-4)×5(b-6)}{144(a-4)\ (b-6)} = 10abc$

4. Divide as directed.

(i) $5(2x+1)(3x+5) \div (2x+1)$

(ii) $26xy(x+5)(y–4) \div 13x(y–4)$

(iii) $52pqr(p+q)(q+r)(r+p) \div 104pq(q+r)(r+p)$

(iv) $20(y+4)(y^2+5y+3) \div 5(y+4)$

(v) $x(x+1)(x+2)(x+3) \div x(x+1)$

Solution:

(i) $5(2x + 1)(3x + 5) \div (2x + 1) = \dfrac{5(2x + 1)(3x + 5)}{(2x + 1)}$

$= 5(3x + 5)$

(ii) $26\,xy\,(x + 5)(y - 4) \div 13\,x\,(y - 4) = \dfrac{2 \times 13 \times xy\,(x + 5)(y - 4)}{13\,x\,(y - 4)}$

$= 2\,y\,(x + 5)$

(iii) $52\,pqr\,(p + q)(q + r)(r + p) \div 104\,pq\,(q + r)(r + p)$

$= \dfrac{2 \times 2 \times 13 \times p \times q \times r \times (p + q) \times (q + r) \times (r + p)}{2 \times 2 \times 2 \times 13 \times p \times q \times (q + r) \times (r + p)}$

$= \dfrac{1}{2}r(p + q)$

(iv) $20\,(y + 4)(y^2 + 5y + 3) = 2 \times 2 \times 5 \times (y + 4)(y^2 + 5y + 3)$

$20\,(y + 4)(y^2 + 5y + 3) \div 5\,(y + 4) = \dfrac{2 \times 2 \times 5 \times (y + 4) \times (y^2 + 5y + 3)}{5 \times (y + 4)}$

$= 4(y^2 + 5y + 3)$

(v) $x\,(x + 1)(x + 2)(x + 3) \div x\,(x + 1) = \dfrac{x(x + 1)(x + 2)(x + 3)}{x(x + 1)}$

$= (x + 2)(x + 3)$

5. Factorise the expressions and divide them as directed.
(i) $(y^2+7y+10) \div (y+5)$

(ii) $(m^2-14m-32) \div (m+2)$

(iii) $(5p^2-25p+20) \div (p-1)$

(iv) $4yz(z^2+6z-16) \div 2y(z+8)$

(v) $5pq(p^2-q^2) \div 2p(p+q)$

(vi) $12xy(9x^2-16y^2) \div 4xy(3x+4y)$

(vii) $39y^3(50y^2-98) \div 26y^2(5y+7)$

Solution:

(i) $(y^2+7y+10) \div (y+5)$

First solve for equation, $(y^2+7y+10)$

$(y^2+7y+10) = y^2+2y+5y+10 = y(y+2)+5(y+2) = (y+2)(y+5)$

Now, $(y^2+7y+10) \div (y+5) = (y+2)(y+5)/(y+5) = y+2$

(ii) $(m^2–14m–32) \div (m+2)$

Solve for $m^2–14m–32$, we have

$m^2–14m–32 = m^2+2m-16m–32 = m(m+2)–16(m+2) = (m–16)(m+2)$

Now, $(m^2–14m–32) \div (m+2) = (m–16)(m+2)/(m+2) = m-16$

(iii) $(5p^2–25p+20) \div (p–1)$

Step 1: Take 5 common from the equation, $5p^2–25p+20$, we get

$5p^2–25p+20 = 5(p^2–5p+4)$

Step 2: Factorize $p^2–5p+4$

$p^2–5p+4 = p^2–p-4p+4 = (p–1)(p–4)$

Step 3: Solve original equation

$(5p^2–25p+20) \div (p–1) = 5(p–1)(p–4)/(p-1) = 5(p–4)$

(iv) $4yz(z^2 + 6z–16) \div 2y(z+8)$

Factorize $z^2+6z–16$,

$z^2+6z–16 = z^2-2z+8z–16 = (z–2)(z+8)$

Now, $4yz(z^2+6z–16) \div 2y(z+8) = 4yz(z–2)(z+8)/2y(z+8) = 2z(z-2)$

(v) $5pq(p^2–q^2) \div 2p(p+q)$

$p^2–q^2$ can be written as (p–q)(p+q) using identity.

$5pq(p^2–q^2) ÷ 2p(p+q) = 5pq(p–q)(p+q)/2p(p+q) = 5/2q(p–q)$

(vi) $12xy(9x^2–16y^2) ÷ 4xy(3x+4y)$

Factorize $9x^2–16y^2$, we have

$9x^2–16y^2 = (3x)^2–(4y)^2 = (3x+4y)(3x-4y)$ using identity: $p^2–q^2 = (p–q)(p+q)$

Now, $12xy(9x^2–16y^2) ÷ 4xy(3x+4y) = 12xy(3x+4y)(3x-4y) /4xy(3x+4y) = 3(3x-4y)$

(vii) $39y^3(50y^2–98) ÷ 26y^2(5y+7)$

First solve for $50y^2–98$, we have

$50y^2–98 = 2(25y^2–49) = 2((5y)^2–7^2) = 2(5y–7)(5y+7)$

Now, $39y^3(50y^2–98) ÷ 26y^2(5y+7) =$

$$\frac{3 \times 13 \times y^3 \times 2(5y - 7)(5y + 7)}{2 \times 13 \times y^2 (5y+7)} = 3y(5y - 7)$$