

Exercise 14.1

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1. Find the common factors of the given terms.(i) $12x$, 36 (ii) $2y$, $22xy$ (iii) $14pq$, $28p^2q^2$ (iv) $2x$, $3x^2$, 4 (v) $6abc$, $24ab^2$, $12a^2b$ (vi) $16x^3$, $-4x^2$, $32x$ (vii) $10pq$, $20qr$, $30rp$ (viii) $3x^2y^3$, $10x^3y^2$, $6x^2y^2z$ **Solution:**(i) Factors of $12x$ and 36

$$12x = 2 \times 2 \times 3 \times x$$

$$36 = 2 \times 2 \times 3 \times 3$$

Common factors of $12x$ and 36 are $2, 2, 3$ and, $2 \times 2 \times 3 = 12$ (ii) Factors of $2y$ and $22xy$

$$2y = 2 \times y$$

$$22xy = 2 \times 11 \times x \times y$$

Common factors of $2y$ and $22xy$ are $2, y$ and, $2 \times y = 2y$ (iii) Factors of $14pq$ and $28p^2q$

$$14pq = 2 \times 7 \times p \times q$$

$$28p^2q = 2 \times 2 \times 7 \times p \times p \times q$$

Common factors of $14pq$ and $28p^2q$ are $2, 7, p, q$ and, $2 \times 7 \times p \times q = 14pq$ (iv) Factors of $2x$, $3x^2$ and 4

$$2x = 2 \times x$$

$$3x^2 = 3 \times x \times x$$

$$4 = 2 \times 2$$

Common factors of $2x$, $3x^2$ and 4 is 1 .(v) Factors of $6abc$, $24ab^2$ and $12a^2b$

$$6abc = 2 \times 3 \times a \times b \times c$$

$$24ab^2 = 2 \times 2 \times 2 \times 3 \times a \times b \times b$$

$$12a^2b = 2 \times 2 \times 3 \times a \times a \times b$$

Common factors of $6abc$, $24ab^2$ and $12a^2b$ are 2, 3, a, b
and, $2 \times 3 \times a \times b = 6ab$

(vi) Factors of $16x^3$, $-4x^2$ and $32x$

$$16x^3 = 2 \times 2 \times 2 \times 2 \times x \times x \times x$$

$$-4x^2 = -1 \times 2 \times 2 \times x \times x$$

$$32x = 2 \times 2 \times 2 \times 2 \times 2 \times x$$

Common factors of $16x^3$, $-4x^2$ and $32x$ are 2, 2, x
and, $2 \times 2 \times x = 4x$

(vii) Factors of $10pq$, $20qr$ and $30rp$

$$10pq = 2 \times 5 \times p \times q$$

$$20qr = 2 \times 2 \times 5 \times q \times r$$

$$30rp = 2 \times 3 \times 5 \times r \times p$$

Common factors of $10pq$, $20qr$ and $30rp$ are 2, 5
and, $2 \times 5 = 10$

(viii) Factors of $3x^2y^3$, $10x^3y^2$ and $6x^2y^2z$

$$3x^2y^3 = 3 \times x \times x \times y \times y \times y$$

$$10x^3y^2 = 2 \times 5 \times x \times x \times x \times y \times y$$

$$6x^2y^2z = 3 \times 2 \times x \times x \times y \times y \times z$$

Common factors of $3x^2y^3$, $10x^3y^2$ and $6x^2y^2z$ are x^2 , y^2
and, $x^2 \times y^2 = x^2y^2$

2. Factorise the following expressions

(i) $7x - 42$

(ii) $6p - 12q$

(iii) $7a^2 + 14a$

(iv) $-16z + 20z^3$

(v) $20l^2m + 30alm$

(vi) $5x^2y - 15xy^2$

(vii) $10a^2 - 15b^2 + 20c^2$

(viii) $-4a^2 + 4ab - 4ca$

(ix) $x^2yz + xy^2z + xyz^2$

(x) $ax^2y + bxy^2 + cxyz$

Solution:

$$(i) \quad 7x = 7 \times x$$

$$42 = 2 \times 3 \times 7$$

The common factor is 7

$$\therefore 7x - 42 = (7 \times x) - (2 \times 3 \times 7) = 7(x - 6)$$

$$(ii) \quad 6p = 2 \times 3 \times p$$

$$12q = 2 \times 2 \times 3 \times q$$

The common factors are 2 and 3

$$\therefore 6p - 12q = (2 \times 3 \times p) - (2 \times 2 \times 3 \times q)$$

$$= 2 \times 3 [p - (2 \times q)]$$

$$= 6(p - 2q)$$

$$(iii) \quad 7a^2 = 7 \times a \times a$$

$$14a = 2 \times 7 \times a$$

The common factors are 7 and a

$$\therefore 7a^2 + 14a = (7 \times a \times a) + (2 \times 7 \times a)$$

$$= 7 \times a [a + 2] = 7a(a + 2)$$

$$(iv) \quad 16z = 2 \times 2 \times 2 \times 2 \times z$$

$$20z^3 = 2 \times 2 \times 5 \times z \times z \times z$$

The common factors are 2, 2, and z.

$$\therefore -16z + 20z^3 = -(2 \times 2 \times 2 \times 2 \times z) + (2 \times 2 \times 5 \times z \times z \times z)$$

$$= (2 \times 2 \times z) [-(2 \times 2) + (5 \times z \times z)]$$

$$= 4z(-4 + 5z^2)$$

$$(v) \quad 20l^2m = 2 \times 2 \times 5 \times l \times l \times m$$

$$30alm = 2 \times 3 \times 5 \times a \times l \times m$$

The common factors are 2, 5, l and m

$$\therefore 20l^2m + 30alm = (2 \times 2 \times 5 \times l \times l \times m) + (2 \times 3 \times 5 \times a \times l \times m)$$

$$= (2 \times 5 \times l \times m) [(2 \times l) + (3 \times a)]$$

$$= 10lm(2l + 3a)$$

$$(vi) \quad 5x^2y = 5 \times x \times x \times y$$

$$15xy^2 = 3 \times 5 \times x \times y \times y$$

The common factors are 5, x , and y

$$\therefore 5x^2y - 15xy^2 = (5 \times x \times x \times y) - (3 \times 5 \times x \times y \times y)$$

$$= 5 \times x \times y [x - (3 \times y)]$$

$$= 5xy(x - 3y)$$

$$(vii) \quad 10a^2 - 15b^2 + 20c^2$$

$$10a^2 = 2 \times 5 \times a \times a$$

$$- 15b^2 = -1 \times 3 \times 5 \times b \times b$$

$$20c^2 = 2 \times 2 \times 5 \times c \times c$$

Common factor of $10a^2$, $15b^2$ and $20c^2$ is 5

$$10a^2 - 15b^2 + 20c^2 = 5(2a^2 - 3b^2 + 4c^2)$$

$$(viii) \quad -4a^2 + 4ab - 4ca$$

$$-4a^2 = -1 \times 2 \times 2 \times a \times a$$

$$4ab = 2 \times 2 \times a \times b$$

$$-4ca = -1 \times 2 \times 2 \times c \times a$$

Common factor of $-4a^2$, $4ab$, $-4ca$ are 2, 2, a i.e. $4a$

So,

$$-4a^2 + 4ab - 4ca = 4a(-a + b - c)$$

$$(ix) \quad x^2yz + xy^2z + xyz^2$$

$$x^2yz = x \times x \times y \times z$$

$$xy^2z = x \times y \times y \times z$$

$$xyz^2 = x \times y \times z \times z$$

Common factor of x^2yz , xy^2z and xyz^2 are x , y , z i.e. xyz

$$\text{Now, } x^2yz + xy^2z + xyz^2 = xyz(x+y+z)$$

$$(x) \ ax^2y + bxy^2 + cxyz$$

$$ax^2y = a \times x \times x \times y$$

$$bxy^2 = b \times x \times y \times y$$

$$cxyz = c \times x \times y \times z$$

Common factors of ax^2y , bxy^2 and $cxyz$ are xy

$$\text{Now, } ax^2y + bxy^2 + cxyz = xy(ax + by + cz)$$

3. Factorise.

$$(i) \ x^2 + xy + 8x + 8y$$

$$(ii) \ 15xy - 6x + 5y - 2$$

$$(iii) \ ax + bx - ay - by$$

$$(iv) \ 15pq + 15 + 9q + 25p$$

$$(v) \ z - 7 + 7xy - xyz$$

Solution:

$$(i) \ x^2 + xy + 8x + 8y = x \times x + x \times y + 8 \times x + 8 \times y$$

$$= x(x + y) + 8(x + y)$$

$$= (x + y)(x + 8)$$

$$(ii) \ 15xy - 6x + 5y - 2 = 3 \times 5 \times x \times y - 3 \times 2 \times x + 5xy - 2$$

$$= 3x(5y - 2) + 1(5y - 2)$$

$$= (5y - 2)(3x + 1)$$

$$(iii) \ ax + bx - ay - by = a \times x + b \times x - a \times y - b \times y$$

$$= x(a + b) - y(a + b)$$

$$= (a + b)(x - y)$$

$$(iv) \ 15pq + 15 + 9q + 25p = 15pq + 9q + 25p + 15$$

$$= 3 \times 5 \times p \times q + 3 \times 3 \times q + 5 \times 5 \times p + 3 \times 5$$

$$= 3q(5p + 3) + 5(5p + 3)$$

$$= (5p + 3)(3q + 5)$$

$$(v) \ z - 7 + 7xy - xyz = z - x \times y \times z - 7 + 7 \times x \times y$$

$$= z(1 - xy) - 7(1 - xy)$$

$$= (1 - xy)(z - 7)$$

Exercise 14.2

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1. Factorise the following expressions.

(i) $a^2+8a+16$

(ii) $p^2-10p+25$

(iii) $25m^2+30m+9$

(iv) $49y^2+84yz+36z^2$

(v) $4x^2-8x+4$

(vi) $121b^2-88bc+16c^2$

(vii) $(l+m)^2-4lm$ (Hint: Expand $(l+m)^2$ first)

(viii) $a^4+2a^2b^2+b^4$

Solution:

(i) $a^2+8a+16$

$= a^2+2 \times 4 \times a+4^2$

$= (a+4)^2$

Using identity: $(x+y)^2 = x^2+2xy+y^2$

(ii) $p^2-10p+25$

$= p^2-2 \times 5 \times p+5^2$

$= (p-5)^2$

Using identity: $(x-y)^2 = x^2-2xy+y^2$

(iii) $25m^2+30m+9$

$= (5m)^2+2 \times 5m \times 3+3^2$

$= (5m+3)^2$

Using identity: $(x+y)^2 = x^2+2xy+y^2$

(iv) $49y^2+84yz+36z^2$

$= (7y)^2+2 \times 7y \times 6z+(6z)^2$

$= (7y+6z)^2$

Using identity: $(x+y)^2 = x^2+2xy+y^2$

(v) $4x^2-8x+4$

$= (2x)^2-2 \times 2 \times x+2^2$

$= (2x-2)^2$

Using identity: $(x-y)^2 = x^2-2xy+y^2$

(vi) $121b^2-88bc+16c^2$

$= (11b)^2-2 \times 11b \times 4c+(4c)^2$

$= (11b-4c)^2$

Using identity: $(x-y)^2 = x^2-2xy+y^2$

(vii) $(l+m)^2 - 4lm$ (Hint: Expand $(l+m)^2$ first)
 Expand $(l+m)^2$ using identity: $(x+y)^2 = x^2 + 2xy + y^2$
 $(l+m)^2 - 4lm = l^2 + m^2 + 2lm - 4lm$
 $= l^2 + m^2 - 2lm$
 $= (l-m)^2$
 Using identity: $(x-y)^2 = x^2 - 2xy + y^2$

(viii) $a^4 + 2a^2b^2 + b^4$
 $= (a^2)^2 + 2 \times a^2 \times b^2 + (b^2)^2$
 $= (a^2 + b^2)^2$
 Using identity: $(x+y)^2 = x^2 + 2xy + y^2$

2. Factorise.

- (i) $4p^2 - 9q^2$
- (ii) $63a^2 - 112b^2$
- (iii) $49x^2 - 36$
- (iv) $16x^5 - 144x^3$ differ
- (v) $(l+m)^2 - (l-m)^2$
- (vi) $9x^2y^2 - 16$
- (vii) $(x^2 - 2xy + y^2) - z^2$
- (viii) $25a^2 - 4b^2 + 28bc - 49c^2$

Solution:

(i) $4p^2 - 9q^2$
 $= (2p)^2 - (3q)^2$
 $= (2p-3q)(2p+3q)$
 Using identity: $x^2 - y^2 = (x+y)(x-y)$

(ii) $63a^2 - 112b^2$
 $= 7(9a^2 - 16b^2)$
 $= 7((3a)^2 - (4b)^2)$
 $= 7(3a+4b)(3a-4b)$
 Using identity: $x^2 - y^2 = (x+y)(x-y)$

(iii) $49x^2 - 36$
 $= (7a)^2 - 6^2$
 $= (7a+6)(7a-6)$
 Using identity: $x^2 - y^2 = (x+y)(x-y)$

$$\begin{aligned} \text{(iv)} \quad & 16x^5 - 144x^3 \\ &= 16x^3(x^2 - 9) \\ &= 16x^3(x^2 - 9) \\ &= 16x^3(x-3)(x+3) \\ \text{Using identity: } x^2 - y^2 &= (x+y)(x-y) \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad & (l+m)^2 - (l-m)^2 \\ &= \{(l+m) - (l-m)\} \{(l+m) + (l-m)\} \\ \text{Using Identity: } x^2 - y^2 &= (x+y)(x-y) \\ &= (l+m-l+m)(l+m+l-m) \\ &= (2m)(2l) \\ &= 4ml \end{aligned}$$

$$\begin{aligned} \text{(vi)} \quad & 9x^2y^2 - 16 \\ &= (3xy)^2 - 4^2 \\ &= (3xy-4)(3xy+4) \\ \text{Using Identity: } x^2 - y^2 &= (x+y)(x-y) \end{aligned}$$

$$\begin{aligned} \text{(vii)} \quad & (x^2 - 2xy + y^2) - z^2 \\ &= (x-y)^2 - z^2 \\ \text{Using Identity: } (x-y)^2 &= x^2 - 2xy + y^2 \\ &= \{(x-y) - z\} \{(x-y) + z\} \\ &= (x-y-z)(x-y+z) \\ \text{Using Identity: } x^2 - y^2 &= (x+y)(x-y) \end{aligned}$$

$$\begin{aligned} \text{(viii)} \quad & 25a^2 - 4b^2 + 28bc - 49c^2 \\ &= 25a^2 - (4b^2 - 28bc + 49c^2) \\ &= (5a)^2 - \{(2b)^2 - 2(2b)(7c) + (7c)^2\} \\ &= (5a)^2 - (2b-7c)^2 \end{aligned}$$

Using Identity: $x^2 - y^2 = (x+y)(x-y)$, we have
 $= (5a+2b-7c)(5a-2b-7c)$

3. Factorise the expressions.

- (i) $ax^2 + bx$
- (ii) $7p^2 + 21q^2$
- (iii) $2x^3 + 2xy^2 + 2xz^2$
- (iv) $am^2 + bm^2 + bn^2 + an^2$
- (v) $(lm+l)+m+1$
- (vi) $y(y+z)+9(y+z)$
- (vii) $5y^2 - 20y - 8z + 2yz$
- (viii) $10ab + 4a + 5b + 2$
- (ix) $6xy - 4y + 6 - 9x$

Solution:

- (i) $ax^2 + bx = x(ax+b)$
- (ii) $7p^2 + 21q^2 = 7(p^2 + 3q^2)$
- (iii) $2x^3 + 2xy^2 + 2xz^2 = 2x(x^2 + y^2 + z^2)$
- (iv) $am^2 + bm^2 + bn^2 + an^2 = m^2(a+b) + n^2(a+b) = (a+b)(m^2 + n^2)$
- (v) $(lm+l)+m+1 = lm+m+l+1 = m(l+1)+(l+1) = (m+1)(l+1)$
- (vi) $y(y+z)+9(y+z) = (y+9)(y+z)$
- (vii) $5y^2 - 20y - 8z + 2yz = 5y(y-4) + 2z(y-4) = (y-4)(5y+2z)$
- (viii) $10ab + 4a + 5b + 2 = 5b(2a+1) + 2(2a+1) = (2a+1)(5b+2)$
- (ix) $6xy - 4y + 6 - 9x = 6xy - 9x - 4y + 6 = 3x(2y-3) - 2(2y-3) = (2y-3)(3x-2)$

4. Factorise.

- (i) $a^4 - b^4$
- (ii) $p^4 - 81$
- (iii) $x^4 - (y+z)^4$
- (iv) $x^4 - (x-z)^4$
- (v) $a^4 - 2a^2b^2 + b^4$

Solution:

- (i) $a^4 - b^4$

$$\begin{aligned} &= (a^2)^2 - (b^2)^2 \\ &= (a^2 - b^2)(a^2 + b^2) \\ &= (a - b)(a + b)(a^2 + b^2) \end{aligned}$$

(ii) $p^4 - 81$

$$\begin{aligned} &= (p^2)^2 - (9)^2 \\ &= (p^2 - 9)(p^2 + 9) \\ &= (p^2 - 3^2)(p^2 + 9) \\ &= (p - 3)(p + 3)(p^2 + 9) \end{aligned}$$

(iii) $x^4 - (y + z)^4 = (x^2)^2 - [(y + z)^2]^2$

$$\begin{aligned} &= \{x^2 - (y + z)^2\} \{x^2 + (y + z)^2\} \\ &= \{x - (y + z)\} \{x + (y + z)\} \{x^2 + (y + z)^2\} \\ &= (x - y - z)(x + y + z) \{x^2 + (y + z)^2\} \end{aligned}$$

(iv) $x^4 - (x - z)^4 = (x^2)^2 - \{(x - z)^2\}^2$

$$\begin{aligned} &= \{x^2 - (x - z)^2\} \{x^2 + (x - z)^2\} \\ &= \{x - (x - z)\} \{x + (x - z)\} \{x^2 + (x - z)^2\} \\ &= z(2x - z)(x^2 + x^2 - 2xz + z^2) \\ &= z(2x - z)(2x^2 - 2xz + z^2) \end{aligned}$$

(v) $a^4 - 2a^2b^2 + b^4 = (a^2)^2 - 2a^2b^2 + (b^2)^2$

$$\begin{aligned} &= (a^2 - b^2)^2 \\ &= ((a - b)(a + b))^2 \end{aligned}$$

5. Factorise the following expressions.

(i) $p^2 + 6p + 8$

(ii) $q^2 - 10q + 21$

(iii) $p^2+6p-16$

Solution:

(i) p^2+6p+8

We observed that, $8 = 4 \times 2$ and $4+2 = 6$

p^2+6p+8 can be written as $p^2+2p+4p+8$
Taking Common terms, we get

$$p^2+6p+8 = p^2+2p+4p+8 = p(p+2)+4(p+2)$$

Again $p+2$ is common in both the terms.

$$= (p+2)(p+4)$$

This implies: $p^2+6p+8 = (p+2)(p+4)$

(ii) $q^2-10q+21$

Observed that, $21 = -7 \times -3$ and $-7+(-3) = -10$

$$q^2-10q+21 = q^2-3q-7q+21$$

$$= q(q-3)-7(q-3)$$

$$= (q-7)(q-3)$$

This implies $q^2-10q+21 = (q-7)(q-3)$

(iii) $p^2+6p-16$

We observed that, $-16 = -2 \times 8$ and $8+(-2) = 6$

$$p^2+6p-16 = p^2-2p+8p-16$$

$$= p(p-2)+8(p-2)$$

$$= (p+8)(p-2)$$

So, $p^2+6p-16 = (p+8)(p-2)$

Exercise 14.3

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1. Carry out the following divisions.

(i) $28x^4 \div 56x$

(ii) $-36y^3 \div 9y^2$

(iii) $66pq^2r^3 \div 11qr^2$

(iv) $34x^3y^3z^3 \div 51xy^2z^3$

(v) $12a^8b^8 \div (-6a^6b^4)$

Solution:

(i) $28x^4 = 2 \times 2 \times 7 \times x \times x \times x \times x$

$56x = 2 \times 2 \times 2 \times 7 \times x$

$$28x^4 \div 56x = \frac{2 \times 2 \times 7 \times x \times x \times x \times x}{2 \times 2 \times 2 \times 7 \times x} = \frac{x^3}{2} = \frac{1}{2}x^3$$

$$(ii) -36y^3 \div 9y^2 = \frac{-2 \times 2 \times 3 \times 3 \times y \times y \times y}{3 \times 3 \times y \times y} = -4y$$

$$(iii) 66pq^2r^3 \div 11qr^2 = \frac{2 \times 3 \times 11 \times p \times q \times q \times r \times r \times r}{11 \times q \times r \times r} = 6pqr$$

$$(iv) 34x^3y^3z^3 \div 51xy^2z^3 = \frac{2 \times 17 \times x \times x \times x \times y \times y \times y \times z \times z \times z}{3 \times 17 \times x \times y \times y \times z \times z \times z} = \frac{2}{3}x^2y$$

$$(v) 12a^8b^8 \div (-6a^6b^4) = \frac{2 \times 2 \times 3 \times a^8 \times b^8}{-2 \times 3 \times a^6 \times b^4} = -2a^2b^4$$

2. Divide the given polynomial by the given monomial.

(i) $(5x^2 - 6x) \div 3x$

(ii) $(3y^8 - 4y^6 + 5y^4) \div y^4$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$

(iv) $(x^3 + 2x^2 + 3x) \div 2x$

(v) $(p^3q^6 - p^6q^3) \div p^3q^3$

Solution:

(i) $5x^2 - 6x = x(5x - 6)$

$$(5x^2 - 6x) \div 3x = \frac{x(5x - 6)}{3x} = \frac{1}{3}(5x - 6)$$

(ii) $3y^8 - 4y^6 + 5y^4 = y^4(3y^4 - 4y^2 + 5)$

$$(3y^8 - 4y^6 + 5y^4) \div y^4 = \frac{y^4(3y^4 - 4y^2 + 5)}{y^4} = 3y^4 - 4y^2 + 5$$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) = 8x^2y^2z^2(x + y + z)$

$$8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2 = \frac{8x^2y^2z^2(x + y + z)}{4x^2y^2z^2} = 2(x + y + z)$$

(iv) $x^3 + 2x^2 + 3x = x(x^2 + 2x + 3)$

$$(x^3 + 2x^2 + 3x) \div 2x = \frac{x(x^2 + 2x + 3)}{2x} = \frac{1}{2}(x^2 + 2x + 3)$$

(v) $p^3q^6 - p^6q^3 = p^3q^3(q^3 - p^3)$

$$(p^3q^6 - p^6q^3) \div p^3q^3 = \frac{p^3q^3(q^3 - p^3)}{p^3q^3} = q^3 - p^3$$

3. Work out the following divisions.

(i) $(10x - 25) \div 5$

(ii) $(10x - 25) \div (2x - 5)$

(iii) $10y(6y + 21) \div 5(2y + 7)$

(iv) $9x^2y^2(3z - 24) \div 27xy(z - 8)$

(v) $96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$

Solution:

$$(i) (10x-25) \div 5 = 5(2x-5)/5 = 2x-5$$

$$(ii) (10x-25) \div (2x-5) = 5(2x-5)/(2x-5) = 5$$

$$(iii) 10y(6y+21) \div 5(2y+7) = 10y \times 3(2y+7)/5(2y+7) = 6y$$

$$(iv) 9x^2y^2(3z-24) \div 27xy(z-8) = 9x^2y^2 \times 3(z-8)/27xy(z-8) = xy$$

$$(v) \underline{96abc(3a-12)(5b-30)} \div 144(a-4)(b-6) = \frac{96abc \times 3(a-4) \times 5(b-6)}{144(a-4)(b-6)} = 10abc$$

4. Divide as directed.

$$(i) 5(2x+1)(3x+5) \div (2x+1)$$

$$(ii) 26xy(x+5)(y-4) \div 13x(y-4)$$

$$(iii) 52pqr(p+q)(q+r)(r+p) \div 104pq(q+r)(r+p)$$

$$(iv) 20(y+4)(y^2+5y+3) \div 5(y+4)$$

$$(v) x(x+1)(x+2)(x+3) \div x(x+1)$$

Solution:

$$(i) \ 5(2x + 1)(3x + 5) \div (2x + 1) = \frac{5(2x + 1)(3x + 5)}{(2x + 1)}$$

$$= 5(3x + 5)$$

$$(ii) \ 26xy(x + 5)(y - 4) \div 13x(y - 4) = \frac{2 \times 13 \times xy(x + 5)(y - 4)}{13x(y - 4)}$$

$$= 2y(x + 5)$$

$$(iii) \ 52pqr(p + q)(q + r)(r + p) \div 104pq(q + r)(r + p)$$

$$= \frac{2 \times 2 \times 13 \times p \times q \times r \times (p + q) \times (q + r) \times (r + p)}{2 \times 2 \times 2 \times 13 \times p \times q \times (q + r) \times (r + p)}$$

$$= \frac{1}{2}r(p + q)$$

$$(iv) \ 20(y + 4)(y^2 + 5y + 3) = 2 \times 2 \times 5 \times (y + 4)(y^2 + 5y + 3)$$

$$20(y + 4)(y^2 + 5y + 3) \div 5(y + 4) = \frac{2 \times 2 \times 5 \times (y + 4) \times (y^2 + 5y + 3)}{5 \times (y + 4)}$$

$$= 4(y^2 + 5y + 3)$$

$$(v) \ x(x + 1)(x + 2)(x + 3) \div x(x + 1) = \frac{x(x + 1)(x + 2)(x + 3)}{x(x + 1)}$$

$$= (x + 2)(x + 3)$$

5. Factorise the expressions and divide them as directed.

(i) $(y^2 + 7y + 10) \div (y + 5)$

(ii) $(m^2 - 14m - 32) \div (m + 2)$

(iii) $(5p^2 - 25p + 20) \div (p - 1)$

(iv) $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

(v) $5pq(p^2 - q^2) \div 2p(p + q)$

(vi) $12xy(9x^2 - 16y^2) \div 4xy(3x + 4y)$

(vii) $39y^3(50y^2 - 98) \div 26y^2(5y + 7)$

Solution:

(i) $(y^2 + 7y + 10) \div (y + 5)$

First solve for equation, $(y^2+7y+10)$
 $(y^2+7y+10) = y^2+2y+5y+10 = y(y+2)+5(y+2) = (y+2)(y+5)$

Now, $(y^2+7y+10) \div (y+5) = (y+2)(y+5)/(y+5) = y+2$

(ii) **$(m^2-14m-32) \div (m+2)$**

Solve for $m^2-14m-32$, we have

$$m^2-14m-32 = m^2+2m-16m-32 = m(m+2)-16(m+2) = (m-16)(m+2)$$

Now, $(m^2-14m-32) \div (m+2) = (m-16)(m+2)/(m+2) = m-16$

(iii) **$(5p^2-25p+20) \div (p-1)$**

Step 1: Take 5 common from the equation, $5p^2-25p+20$, we get

$$5p^2-25p+20 = 5(p^2-5p+4)$$

Step 2: Factorize p^2-5p+4

$$p^2-5p+4 = p^2-p-4p+4 = (p-1)(p-4)$$

Step 3: Solve original equation

$$(5p^2-25p+20) \div (p-1) = 5(p-1)(p-4)/(p-1) = 5(p-4)$$

(iv) **$4yz(z^2 + 6z-16) \div 2y(z+8)$**

Factorize $z^2+6z-16$,

$$z^2+6z-16 = z^2-2z+8z-16 = (z-2)(z+8)$$

Now, $4yz(z^2+6z-16) \div 2y(z+8) = 4yz(z-2)(z+8)/2y(z+8) = 2z(z-2)$

(v) **$5pq(p^2-q^2) \div 2p(p+q)$**

$p^2 - q^2$ can be written as $(p-q)(p+q)$ using identity.

$$5pq(p^2 - q^2) \div 2p(p+q) = 5pq(p-q)(p+q) / 2p(p+q) = 5/2q(p-q)$$

(vi) $12xy(9x^2 - 16y^2) \div 4xy(3x+4y)$

Factorize $9x^2 - 16y^2$, we have

$$9x^2 - 16y^2 = (3x)^2 - (4y)^2 = (3x+4y)(3x-4y) \text{ using identity: } p^2 - q^2 = (p-q)(p+q)$$

$$\text{Now, } 12xy(9x^2 - 16y^2) \div 4xy(3x+4y) = 12xy(3x+4y)(3x-4y) / 4xy(3x+4y) = 3(3x-4y)$$

(vii) $39y^3(50y^2 - 98) \div 26y^2(5y+7)$

First solve for $50y^2 - 98$, we have

$$50y^2 - 98 = 2(25y^2 - 49) = 2((5y)^2 - 7^2) = 2(5y-7)(5y+7)$$

$$\text{Now, } 39y^3(50y^2 - 98) \div 26y^2(5y+7) =$$

$$\frac{3 \times 13 \times y^3 \times 2(5y-7)(5y+7)}{2 \times 13 \times y^2(5y+7)} = 3y(5y-7)$$

Exercise 14.4

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Find and correct the errors in the following mathematical statements.

1. $4(x-5) = 4x-5$

Solution:

$$4(x-5) = 4x - 20 \neq 4x - 5 = \text{RHS}$$

The correct statement is $4(x-5) = 4x-20$

2. $x(3x+2) = 3x^2+2$

Solution:

$$\text{LHS} = x(3x+2) = 3x^2+2x \neq 3x^2+2 = \text{RHS}$$

The correct solution is $x(3x+2) = 3x^2+2x$

3. $2x+3y = 5xy$

Solution:

$$\text{LHS} = 2x+3y \neq \text{R. H. S}$$

The correct statement is $2x+3y = 2x+3y$

4. $x+2x+3x = 5x$

Solution:

$$\text{LHS} = x+2x+3x = 6x \neq \text{RHS}$$

The correct statement is $x+2x+3x = 6x$

5. $5y+2y+y-7y = 0$

Solution:

$$\text{LHS} = 5y+2y+y-7y = y \neq \text{RHS}$$

The correct statement is $5y+2y+y-7y = y$

6. $3x+2x = 5x^2$

Solution:

$$\text{LHS} = 3x+2x = 5x \neq \text{RHS}$$

The correct statement is $3x+2x = 5x$

7. $(2x)^2+4(2x)+7 = 2x^2+8x+7$

Solution:

$$\text{LHS} = (2x)^2+4(2x)+7 = 4x^2+8x+7 \neq \text{RHS}$$

The correct statement is $(2x)^2+4(2x)+7 = 4x^2+8x+7$

8. $(2x)^2+5x = 4x+5x = 9x$

Solution:

$$\text{LHS} = (2x)^2+5x = 4x^2+5x \neq 9x = \text{RHS}$$

The correct statement is $(2x)^2+5x = 4x^2+5x$

9. $(3x + 2)^2 = 3x^2+6x+4$

Solution:

$$\text{LHS} = (3x+2)^2 = (3x)^2+2^2+2 \times 2 \times 3x = 9x^2+4+12x \neq \text{RHS}$$

The correct statement is $(3x + 2)^2 = 9x^2+4+12x$

10. Substituting $x = -3$ in

(a) $x^2 + 5x + 4$ gives $(-3)^2+5(-3)+4 = 9+2+4 = 15$

(b) $x^2 - 5x + 4$ gives $(-3)^2-5(-3)+4 = 9-15+4 = -2$

(c) $x^2 + 5x$ gives $(-3)^2+5(-3) = -9-15 = -24$

Solution:

(a) Substituting $x = -3$ in x^2+5x+4 , we have

$$x^2+5x+4 = (-3)^2+5(-3)+4 = 9-15+4 = -2. \text{ This is the correct answer.}$$

(b) Substituting $x = -3$ in x^2-5x+4

$$x^2-5x+4 = (-3)^2-5(-3)+4 = 9+15+4 = 28. \text{ This is the correct answer}$$

(c) Substituting $x = -3$ in x^2+5x

$$x^2+5x = (-3)^2+5(-3) = 9-15 = -6. \text{ This is the correct answer}$$

11. $(y-3)^2 = y^2-9$

Solution:

LHS = $(y-3)^2$, which is similar to $(a-b)^2$ identity, where $(a-b)^2 = a^2+b^2-2ab$.

$$(y-3)^2 = y^2+(3)^2-2y \times 3 = y^2+9-6y \neq y^2-9 = \text{RHS}$$

The correct statement is $(y-3)^2 = y^2+9-6y$

12. $(z+5)^2 = z^2+25$

Solution:

LHS = $(z+5)^2$, which is similar to $(a+b)^2$ identity, where $(a+b)^2 = a^2+b^2+2ab$.

$$(z+5)^2 = z^2+5^2+2 \times 5 \times z = z^2+25+10z \neq z^2+25 = \text{RHS}$$

The correct statement is $(z+5)^2 = z^2+25+10z$

13. $(2a+3b)(a-b) = 2a^2-3b^2$

Solution:

$$\begin{aligned} \text{LHS} &= (2a+3b)(a-b) = 2a(a-b)+3b(a-b) \\ &= 2a^2-2ab+3ab-3b^2 \\ &= 2a^2+ab-3b^2 \\ &\neq 2a^2-3b^2 = \text{RHS} \end{aligned}$$

The correct statement is $(2a+3b)(a-b) = 2a^2+ab-3b^2$

14. $(a+4)(a+2) = a^2+8$

Solution:

$$\begin{aligned} \text{LHS} &= (a+4)(a+2) = a(a+2)+4(a+2) \\ &= a^2+2a+4a+8 \\ &= a^2+6a+8 \\ &\neq a^2+8 = \text{RHS} \end{aligned}$$

The correct statement is $(a+4)(a+2) = a^2+6a+8$

15. $(a-4)(a-2) = a^2-8$

Solution:

$$\begin{aligned}\text{LHS} &= (a-4)(a-2) = a(a-2)-4(a-2) \\ &= a^2-2a-4a+8 \\ &= a^2-6a+8 \\ &\neq a^2-8 = \text{RHS}\end{aligned}$$

The correct statement is $(a-4)(a-2) = a^2-6a+8$

16. $3x^2/3x^2 = 0$

Solution:

$$\text{LHS} = 3x^2/3x^2 = 1 \neq 0 = \text{RHS}$$

The correct statement is $3x^2/3x^2 = 1$

17. $(3x^2+1)/3x^2 = 1 + 1 = 2$

Solution:

$$\text{LHS} = (3x^2+1)/3x^2 = (3x^2/3x^2)+(1/3x^2) = 1+(1/3x^2) \neq 2 = \text{RHS}$$

The correct statement is $(3x^2+1)/3x^2 = 1+(1/3x^2)$

18. $3x/(3x+2) = 1/2$

Solution:

$$\text{LHS} = 3x/(3x+2) \neq 1/2 = \text{RHS}$$

The correct statement is $3x/(3x+2) = 3x/(3x+2)$

19. $3/(4x+3) = 1/4x$

Solution:

$$\text{LHS} = 3/(4x+3) \neq 1/4x$$

The correct statement is $3/(4x+3) = 3/(4x+3)$

20. $(4x+5)/4x = 5$

Solution:

$$\text{LHS} = (4x+5)/4x = 4x/4x + 5/4x = 1 + 5/4x \neq 5 = \text{RHS}$$

The correct statement is $(4x+5)/4x = 1 + (5/4x)$

$$21. \frac{7x+5}{5} = 7x$$

Solution:

$$\text{LHS} = (7x+5)/5 = (7x/5) + 5/5 = (7x/5) + 1 \neq 7x = \text{RHS}$$

The correct statement is $(7x+5)/5 = (7x/5) + 1$