RBSE Class 12th Maths Question Paper With Solution 2016

QUESTION PAPER CODE 412

SECTION - A

Question 1: Find x, if $\tan^{-1} 3 + \cot^{-1} x = \pi / 2$.

Solution:

$\tan^{-1} 3 + \cot^{-1} x = \pi / 2$

$\tan^{-1} 3 = \pi / 2 - \cot^{-1} x$

$\tan^{-1} x + \cot^{-1} x = \pi / 2$

$\tan^{-1} x = \pi / 2 - \cot^{-1} x$

$\tan^{-1} 3 = \tan^{-1} x$

$x = 3$

Question 2: Construct a 2 × 2 matrix $A = [a_{ij}]$, whose elements are given by $a_{ij} = |-5i + 2j|$.

Solution:

$a_{ij} = |-5i + 2j|$

The matrix A of order 2 × 2 is given by $\begin{bmatrix} a_{11} & a_{12} \\ a_{13} & a_{14} \end{bmatrix}$.

$a_{11} = |-5 + 2| = 3$

$a_{12} = |-5 + 4| = 1$

$a_{21} = |-10 + 2| = 8$

$a_{22} = |-10 + 4| = 6$

$A = \begin{bmatrix} 3 & 1 \\ 8 & 6 \end{bmatrix}$

Question 3: If $[x \quad -3]\begin{bmatrix} 2x \\ 6 \end{bmatrix} = 0$, then find the value of x.

Solution:

$[x \quad -3]\begin{bmatrix} 2x \\ 6 \end{bmatrix} = 0$

$[2x^2 - 18] = 0$

$2x^2 - 18 = 0$

$2x^2 = 18$

$x^2 = 18 / 2$

$x^2 = 9$

$x = \pm 3$

Question 4: Find $\int[\tan x / \cot x]\, dx$.

Solution:

$\int[\tan x / \cot x]\, dx$

$\tan x = 1 / \cot x$

$= \int \tan^2 x\, dx$

$= \int \sec^2 x\, dx - \int 1\, dx$

$= \tan x - x + c$

Question 5: Find the general solution of the differential equation (dy / dx) - (y / x) = 0.

Solution:

$(dy / dx) - (y / x) = 0$

$(dy / dx) = (y / x)$

$\int dy / y = \int dx / x$

$\ln y = \ln x + c$

$\ln y = \ln xc$

$y = xc$

Question 6: If a = 2i - j + 5k and b = 4i - 2j + λk such that a || b, find the value of λ.

Solution:

a || b

a = kb

(2i - j + 5k) = k (4i - 2j + λk)

2 = 4k

- 1 = -2

5k = λk

k = 1 / 2

5 = (1 / 2)λ

λ = 10

Question 7: Find the direction cosine of the line x / 4 = y / 7 = z / 4.

Solution:

The direction ratios of the line are x / 4 = y / 7 = z / 4.

The direction cosine of the line are x / 4 = y / 7 = z / 4 are $4 / \sqrt{4^2 + 7^2 + 4^2}$, $7 / \sqrt{4^2 + 7^2 + 4^2}$, $4 / \sqrt{4^2 + 7^2 + 4^2}$.

= 4 / 9, 7 / 9, 4 / 9

Question 8: Find the angle between planes r . (i - j + k) = 5 and r . (2i + j - k) = 7.

Solution:

Let the normal of the planes be a = i - j + k and b = 2i + j - k.

a . b = |a| . |b| cos θ

cos θ = (a . b) / |a| . |b|

cos θ = [i - j + k] [2i + j - k] / |i - j + k| |2i + j - k|

= [2 − 1 − 1] / $\sqrt{1 + 1 + 1}$ $\sqrt{4 + 1 + 1}$

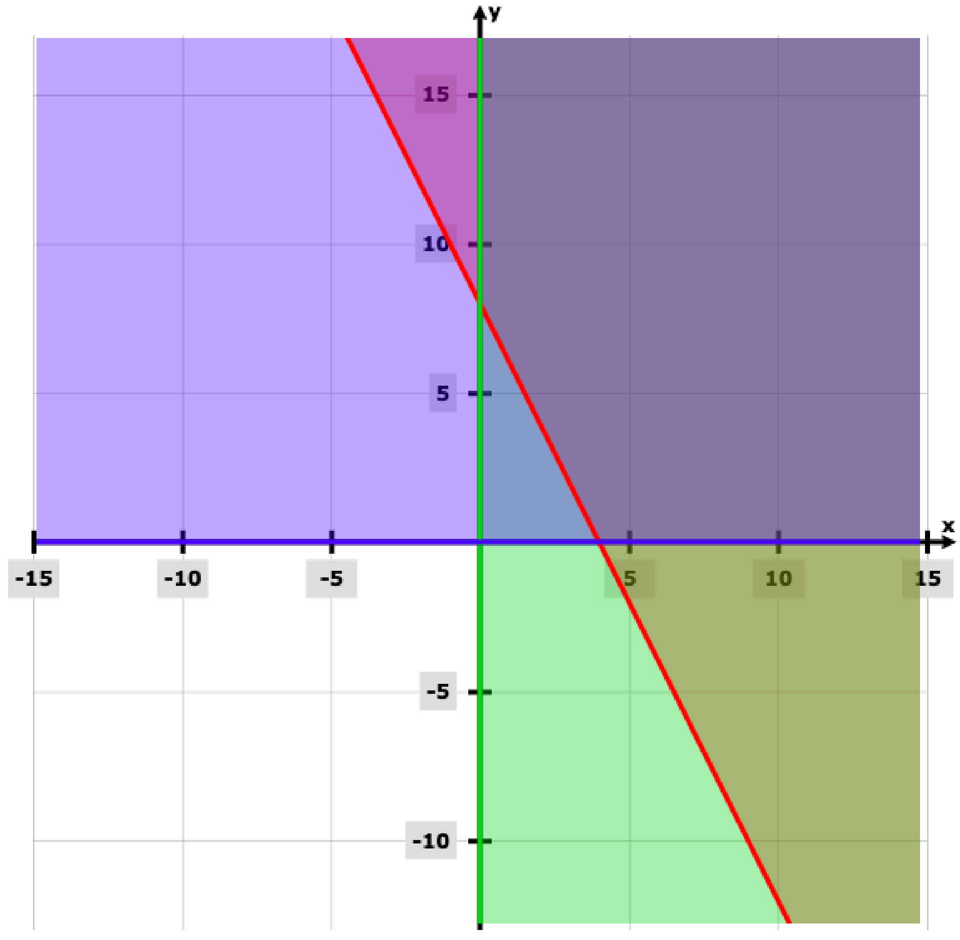
cos θ = 0

θ = $\cos^{-1} 0$

$\theta = \pi / 2$

Question 9: Show the region of the feasible solution under the following constraints 2x + y ≥ 8, x ≥ 0, y ≥ 0 in the answer book.

Solution:



Question 10: If A and B are independent events with P (A) = 0.2 and P (B) = 0.5, then find the value of P (A ∪ B).

Solution:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$

$= (0.2) + (0.5) - (0.2)(0.5)$

= 0.7 - 0.10
= 0.6

SECTION - B

Question 11: [a] Prove that the relation R in a set of real numbers R defined as R = {(a, b) : a ≥ b} is reflexive and transitive but not symmetric.

OR

[b] Consider f : R → R given by f (x) = 2x + 3. Show that f is invertible. Find also the inverse of function f.

Solution:

[a] R = {(a, b) : a ≥ b}

Reflexive
R = {(a, a) : a > a}
Hence, R is reflexive.

Symmetric
Let (a, b) ∈ R
aRb ⇔ a ≥ b
bRa ⇔ b ≥ a not true
(b, a) ∉ R
Hence, R is not symmetric.

Transitive
Let (a, b) ∈ R and (b, c) ∈ R
aRb ⇔ a ≥ b
bRc ⇔ b ≥ c
a ≥ c
(a, c) ∈ R

Hence, R is transitive.
So, R is reflexive and transitive but not symmetric.

OR

[b] f (x) = 2x + 3
f (x_1) = f (x_2)
2x_1 + 3 = 2x_2 + 3
$x_1 = x_2$
Therefore, f (x) is a one-one function.
In f (x) = 2x + 3, the domain of x and y are R, hence the domain and co-domain are the same.
Therefore, f (x) is onto function.
So, f (x) is a one-one and onto function.
Hence, the inverse of f (x) exists.
Let f (x) = y
y = 2x + 3
2x = y - 3
x = (y - 3) / 2
Replacing x by y and y by x,
y = (x - 3) / 2
f^{-1} (x) = (x - 3) / 2

Question 12: [a] Prove that $\tan^{-1}$ (2 / 9) + $\tan^{-1}$ (1 / 4) = (1 / 2) $\sin^{-1}$ (4 / 5)

OR

[b] Solve 2 $\tan^{-1}$ (sin x) = $\tan^{-1}$ (2 sec x), 0 < x < π / 2.

Solution:

[a] LHS = $\tan^{-1}$ (2 / 9) + $\tan^{-1}$ (1 / 4)
$\tan^{-1}$ x + $\tan^{-1}$ y = $\tan^{-1}$ [(x + y) / (1 - xy)]
= $\tan^{-1}$ [(2 / 9) + (1 / 4) / (1 - (2 / 9) (1 / 4)]
= $\tan^{-1}$ {[(8 + 9) / 36] / (1 - 2 / 36)}
= $\tan^{-1}$ [(17 / 36) / (34 / 36)]
= $\tan^{-1}$ (1 / 2)
= (1 / 2) $\sin^{-1}$ (2 * (1 / 2)) / (1 + (1 / 4))

$= (1 / 2) \sin^{-1} (4 / 5)$

OR

[b] $2 \tan^{-1} (\sin x) = \tan^{-1} (2 \sec x)$

$2 \tan^{-1} x = \tan^{-1} (2x / (1 - x))$

$\tan^{-1} [(2 \sin x) / (1 - \sin^2 x)] = \tan^{-1} [2 \sec x]$

$(2 \sin x) / (1 - \sin^2 x) = 2 \sec x$

$\sin x / \cos^2 x = \sec x$

$\sin x \sec x = 1$

$\sin x / \cos x = 1$

$\tan x = 1$

$x = \pi / 4$

Question 13: Express the matrix A = $\begin{bmatrix} 2 & -4 & -2 \\ -1 & 4 & 3 \\ 1 & -3 & 2 \end{bmatrix}$ **as the sum of a symmetric and a skew-symmetric matrix.**

Solution:

$A = [(A + A^T) / 2] + [(A - A^T) / 2]$

$$= (\frac{1}{2})(\begin{bmatrix} 2 & -4 & -2 \\ -1 & 4 & 3 \\ 1 & -3 & 2 \end{bmatrix}) + (\begin{bmatrix} 2 & -1 & 1 \\ -4 & 4 & -3 \\ -2 & 3 & 2 \end{bmatrix}) + (\frac{1}{2})(\begin{bmatrix} 2 & -4 & -2 \\ -1 & 4 & 3 \\ 1 & -3 & 2 \end{bmatrix}) - (\begin{bmatrix} 2 & -1 & 1 \\ -4 & 4 & -3 \\ -2 & 3 & 2 \end{bmatrix})$$

$$= (\frac{1}{2}) \begin{bmatrix} 4 & -5 & -1 \\ -5 & 8 & 0 \\ -1 & 0 & 4 \end{bmatrix} + (\frac{1}{2}) \begin{bmatrix} 0 & -3 & -3 \\ 3 & 0 & 6 \\ 3 & -6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -5/2 & -1/2 \\ -5/2 & 4 & 0 \\ -1/2 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -3/2 & -3/2 \\ 3/2 & 0 & 3 \\ 3/2 & -3 & 0 \end{bmatrix}$$

Question 14: Find the value of K so that the function is continuous at the point x = π / 2.

$$f(x)=\begin{cases}\dfrac{K\cos x}{\pi-2x} & ; \quad x\neq\dfrac{\pi}{2}\\ 5 & ; \quad x=\dfrac{\pi}{2}\end{cases}$$

Solution:

$$f(x)=\begin{cases}\dfrac{K\cos x}{\pi-2x} & ; \quad x\neq\dfrac{\pi}{2}\\ 5 & ; \quad x=\dfrac{\pi}{2}\end{cases}$$

Right hand limit at x = π / 2

$\lim_{x\to(\pi/2)^+} f(x) = \lim_{h\to 0} f(\pi/2 + h)$

$= \lim_{h\to 0} [K \cos(\pi/2 + h)] / [(\pi - 2)(\pi/2 + h)]$

$= \lim_{h\to 0} - K \sin h / - 2h$

$= (K/2) \lim_{h\to 0} \sin h / h$

$= K / 2$

For f (x) to be continuous, $f(\pi/2)^+ = f(\pi/2)$

$K / 2 = 5$

$K = 10$

Question 15: [a] Find the intervals in which the function f given by f (x) = x^2 – 6x + 5 is

i) Strictly increasing

ii) Strictly decreasing

OR

[b] Find the equation of the tangent to the curve $x^{2/3} + y^{2/3} = 1$ at the point (1, 1).

Solution:

[a] $f(x) = x^2 - 6x + 5$

$f'(x) = 2x - 6$

Put $f'(x) = 0$

$2x - 6 = 0$
$x = 3$
Then divide the given interval into two parts as $(-\infty, 3)$ and $(3, \infty)$.

[i] Take the interval $(3, \infty)$
$f'(x) = 2x - 6 > 0$
The given function is strictly increasing in the interval $(3, \infty)$.

[ii] Take the interval $(-\infty, 3)$
$f'(x) = 2x - 6 < 0$
The given function is strictly decreasing in the interval $(-\infty, 3)$.

OR

[b] $x^{2/3} + y^{2/3} = 1$
$(2 / 3) x^{-1/3} + (2 / 3) y^{-1/3} (dy / dx) = 0$
$(1 / y^{1/3}) dy / dx = (- 1 / x^{1/3})$
$dy / dx = - (y / x)^{1/3}$
At (1, 1) $dy / dx = -1$
$y – 1 = – 1 (x – 1)$
$y – 1 + x – 1 = 0$
$x + y – 2 = 0$

Question 16: The radius of a circle is increasing uniformly at the rate of 5 cm/sec. Find the rate at which the area of the circle is increasing when the radius is 6 cm.

Solution:
Let the radius and area of the circle be r and A.
$dr / dt = 5$ cm/sec
$A = \pi r^2$

$dA / dt = 2\pi r (dr / dt)$

$= 2\pi * 6 * 5$

$= 60\pi \text{ cm}^2/\text{sec}$

Question 17: [a] Find $\int [(x - 1)(x - \log x)^3 / x] \, dx$

OR

[b] Find $\int \log (x^2 + 1) \, dx$.

Solution:

[a] $\int [(x - 1)(x - \log x)^3 / x] \, dx$

Put $x - \log x = t$

$(1 - (1 / x)) \, dx = dt$

$([x - 1] / x) \, dx = dt$

$\int [(x - 1)(x - \log x)^3 / x] \, dx = \int t^3 \, dt$

$= t^4 / 4 + c$

$= (x - \log x)^4 / 4 + c$

OR

[b] $\int \log (x^2 + 1) \, dx$

$= \int 1 \log (x^2 + 1) \, dx$

$= \log (x^2 + 1) \int dx - \int [(d / dx)(\log (x^2 + 1)) \int dx] \, dx$

$= x \log (x^2 + 1) - \int 2x^2 / (x^2 + 1) \, dx$

$= x \log (x^2 + 1) - 2 [\int dx - \int 1 / x^2 + 1 \, dx]$

$= x \log (x^2 + 1) - 2 [x - \tan^{-1} x] + c$

$= x \log (x^2 + 1) - 2x + \tan^{-1} x + c$

Question 18: Find $\int 1 / [3x^2 + 6x + 2]$.

Solution:

$\int 1 / [3x^2 + 6x + 2]$

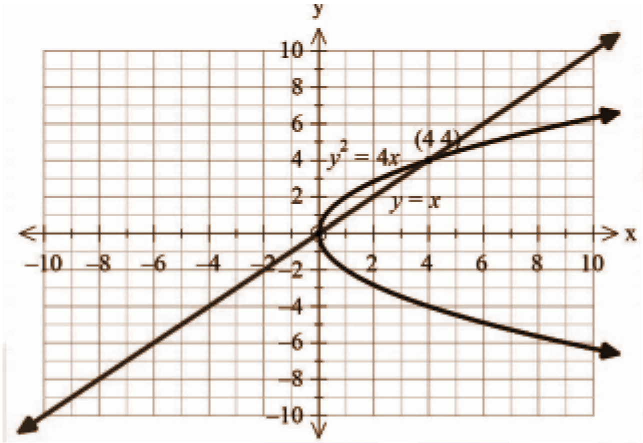
$= (1 / 3) \int [1 / x^2 + 2x + 1 + (2 / 3) - 1] \, dx$

$= (1 / 3) \int [1 / x^2 + 2x + 1 - (1 / 3)]$

$= (1 / 3) \int 1 / (x + 1)^2 - (1 / \sqrt{3})^2$
$= (1 / 3) / 2 (1 / \sqrt{3}) \log |(x + 1 - (1 / \sqrt{3})) / (x + 1 - (1 / \sqrt{3}))| + c$
$= (1 / 2\sqrt{3}) \log |(x + 1 - (1 / \sqrt{3})) / (x + 1 - (1 / \sqrt{3}))| + c$

Question 19: Find the area bounded by the parabola $y^2 = 4x$ and the straight line y = x. (Draw the figure in answer book)

Solution:

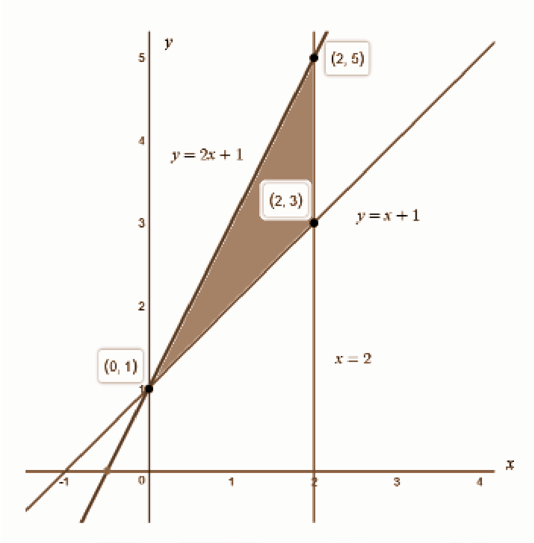


$y^2 = 4x$ and $y = x$
$y^2 = 4y$
$y^2 - 4y = 0$
$y (y - 4) = 0$
$y = 0, 4$
$\text{Area} = \int_0^4 (2\sqrt{x} - x)\, dx$
$= [(4 / 3) x^{3/2} - (x^2 / 2)]_0^4$
$= [(4 / 3) (4)^{3/2} - 8 - 0]$
$= (32 / 3) - 8$
$= 8 / 3$ square units

Question 20: Using integration, find the area of a triangular region whose sides have the equations y = x + 1, y = 2x + 1 and x = 2. (Draw the figure in answer book)

Solution:
Consider $y = x + 1$ and $y = 2x + 1$

Equate both the equations and solve for x
x + 1 = 2x + 1
x = 0
Substitute x = 0 in y = x + 1,
y = 1
Substitute x = 2 in y = 2x + 1
y = 5
Graph of y = x + 1, y = 2x + 1 and x = 2 is given by



Area = $\int_0^2 [2x + 1 - x - 1]\, dx$
$= \int_0^2 x\, dx$
$= [x^2 / 2]_0^2$
= 2 square units

Question 21: If a, b, c are unit vectors such that a + b + c = 0, find the value of a . b + b . c + c . a.

Solution:
a + b + c = 0
$(a + b + c)^2 = 0$
$|a|^2 + |b|^2 + |c|^2 + 2 (a . b + b . c + c . a) = 0$
1 + 1 + 1 + 2 (a . b + b . c + c . a) = 0
2 (a . b + b . c + c . a) = - 3
(a . b + b . c + c . a) = -3 / 2

Question 22: Find a unit vector perpendicular to each of the vectors 2a + b and a - 2b, where a = i + 2j - k, b = i + j + k.

Solution:

a = i + 2j - k

b = i + j + k

2a + b = 2 (i + 2j - k) + i + j + k

= 3i + 5j - k

a - 2b = (i + 2j - k) - 2 (i + j + k)

= - i - 3k

The vector perpendicular to both 2a + b and a - 2b is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & -1 \\ -1 & 0 & -3 \end{vmatrix} = -15i + 10j + 5k$$

Unit vector is given by

= [−15i + 10j + 5k] / √225 + 100 + 25

= [−15i + 10j + 5k] / √350

= [−15i + 10j + 5k] / 5√14

= [−3i + 2j + k] / √14

Question 23: By graphical method solve the following linear programming problem for maximization.

Objective function Z = 1000x + 600y

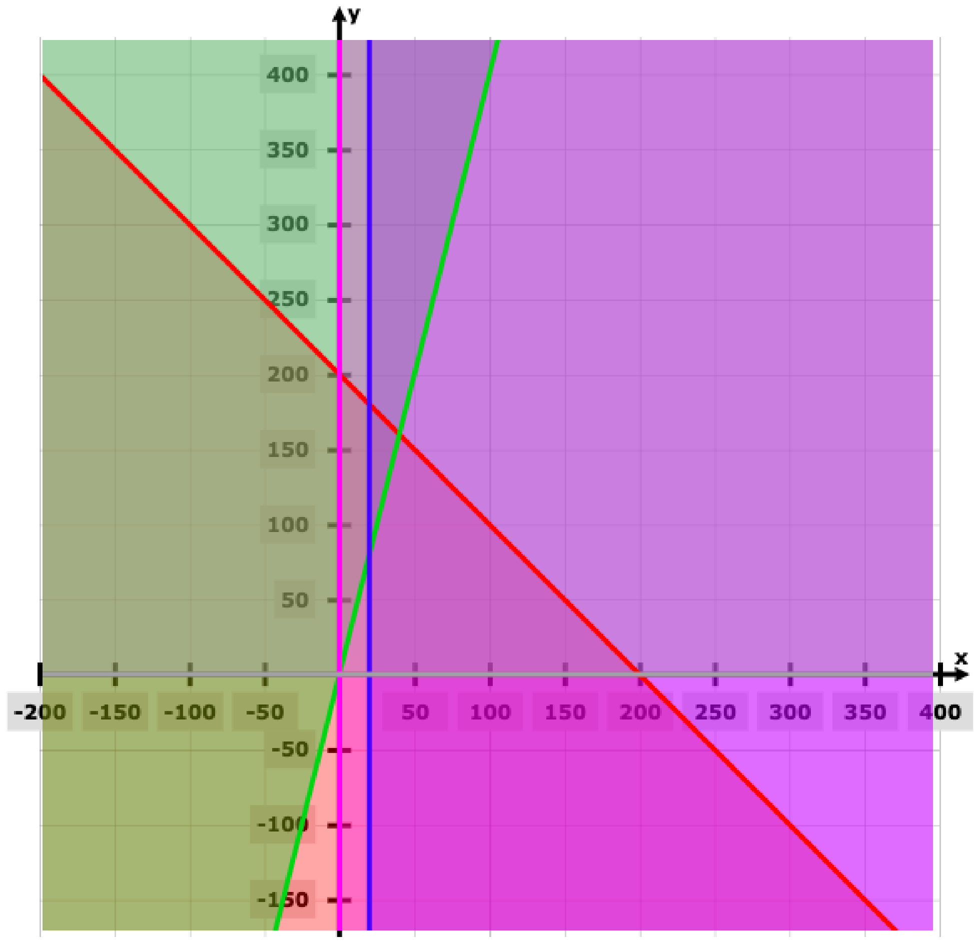
Constraints

$x + y \leq 200$

$4x - y \leq 0$

$x \geq 20,\ x \geq 0,\ y \geq 0.$

Solution:



Points	Z
(20, 80)	68000
(20, 180)	128000
(40, 160)	136000

The maximum value of Z = 1000x+ 600y is 136000 at x = 40 and y =160.

Question 24: Bag A contains 2 red and 3 black balls while another bag B contains 3 red and 4 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from bag B.

Solution:

Consider the events E and F as follows:

E: Bag A is selected

F: Bag B is selected

A: Ball drawn is red

Since there are only two bags.

Bag A contains 2 red and 3 black balls

P (A / E) = 2 / 5

Bag B contains 3 red and 4 black balls

P (A / F) = 3 / 7

The required probability is

P (F / A) = [P (F) P (A / F)] / [P (E) P (A / E) + P (F) P (A / F)]

= (1 / 2) (3 / 7) / [(1 / 2) * (2 / 5) + (1 / 2) * (3 / 7)]

= (8 / 35) / [(3 / 14) + (8 / 35)]

= (3 / 14) / (29 / 70)

= 15 / 29

Question 25: From a lot of 30 bulbs which include 6 defectives, a sample of 2 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Solution:

Let X be a random variable.

X = number of defective bulbs.

Clearly, X can assume the value 0, 1 or 2.

P (X = 0) = ${}^{24}C_2 / {}^{30}C_2$ = 92 / 145

P (X = 1) = ${}^{6}C_1 * {}^{24}C_1 / {}^{30}C_2$ = 48 / 145

P (X = 2) = ${}^{6}C_2 / {}^{30}C_2$ = 1 / 129

SECTION - C

Question 26: Prove that $\begin{vmatrix} a & a^2 & b+c \\ b & b^2 & c+a \\ c & c^2 & a+b \end{vmatrix}$ **= (a + b + c) (a - b) (b - c) (c - a).**

Solution:

$$\begin{vmatrix} a & a^2 & b+c \\ b & b^2 & c+a \\ c & c^2 & a+b \end{vmatrix}$$

$$C_1 \to C_1 + C_3$$

$$= \begin{vmatrix} a+b+c & a^2 & b+c \\ b+c+a & b^2 & c+a \\ c+a+b & c^2 & a+b \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & a^2 & b+c \\ 1 & b^2 & c+a \\ 1 & c^2 & a+b \end{vmatrix}$$

$$R_1 \to R_1 - R_3$$

$$= (a+b+c) \begin{vmatrix} 0 & a^2-c^2 & c-a \\ 0 & b^2-c^2 & c-b \\ 1 & c^2 & a+b \end{vmatrix}$$

$= (a + b + c) [(a^2 – c^2) (c – b) – (c – a) (b^2 – c^2)]$

$= (a + b + c) [(a – c) (a + c) (c – b) – (c – a) (b – c) (b + c)]$

$= (a + b + c) (b – c) (c – a) [(a + c) – (b + c)]$

$= (a + b + c) (a – b) (b – c) (c – a)$

Question 27: If $y = [\sin^{-1} x]^2$ **then show that** $(1 - x^2) (d^2y / dx^2) - x (dy / dx) - 2 = 0$**.**

Solution:

$y = [\sin^{-1} x]^2$

$dy / dx = 2 \sin^{-1} x / \sqrt{1 - x^2}$ ------ (1)

$d^2y / dx^2 = \{(2 \sqrt{1 - x^2}) * (1 / \sqrt{1 - x^2}) - (2 \sin^{-1} x) * (1 / 2) * (-2x / \sqrt{1 - x^2})\} / (1 - x^2)$

$(1 - x^2) (d^2y / dx^2) = 2 + x (2 \sin^{-1} x) / \sqrt{1 - x^2})$

$(1 - x^2) (d^2y / dx^2) = 2 + x (dy / dx)$

$(1 - x^2)(d^2y / dx^2) - x(dy / dx) - 2 = 0$

Question 28: Evaluate $\int_0^{\pi} [x \sin x / 1 + \cos^2 x]\, dx$.

Solution:

$I = \int_0^{\pi} [x \sin x / 1 + \cos^2 x]\, dx$ ---- (1)

$= \int_0^{\pi} [(\pi - x) \sin(\pi - x) / 1 + \cos^2(\pi - x)]\, dx$

$= \int_0^{\pi} [(\pi - x) \sin x / 1 + \cos^2 x]\, dx$ ---- (2)

Adding (1) and (2)

$2I = \pi \int_0^{\pi} \sin x / (1 + \cos^2 x)\, dx$

$2I = 2\pi \int_0^{\pi/2} (\sin x\, dx) / (1 + \cos^2 x)$

$I = \pi \int_0^{\pi/2} (\sin x\, dx) / (1 + \cos^2 x)$

Let $\cos x = t$

$- \sin x\, dx = dt$

$I = \pi \int_1^0 - dt / 1 + t^2$

$I = - \pi [\tan^{-1} t]_1^0$

$I = - \pi [\tan^{-1} 0 - \tan^{-1} 1]$

$= - \pi [- \pi / 4]$

$= \pi^2 / 4$

Question 29: [a] Solve the differential equation $2xy + y^2 - 2x^2 (dy / dx) = 0$

OR

[b] Solve the differential equation $dy / dx + y \cot x = 2x + x^2 \cot x$.

Solution:

[a] $2xy + y^2 - 2x^2 (dy / dx) = 0$

$2x^2 (dy / dx) = 2xy + y^2$

$dy / dx = (y / x) + (1 / 2)(y / x)^2$ ----- (1)

Ley $y = vx$

$dy / dx = v + x (dv / dx)$

Put in (1)

$v + x (dv / dx) = v + (1 / 2)v^2$

$x (dv / dx) = v^2 / 2$

$dv / v^2 = dx / 2x$

On integrating both sides,

$\int dv / v^2 = \int dx / 2x$

(- 1 / v) = (1 / 2) log x + c

(- x / y) = (1 / 2) log x + c

OR

[b] dy / dx + y cot x = 2x + x^2 cot x

This is a linear differential equation of the form (dy / dx) + Py = Q

P = cot x, Q = 2x + x^2 cot x

IF = $e^{\int P\, dx} = e^{\int \cot x\, dx} = e^{\log \sin x}$ = sin x

Multiply both sides by I.F. & then integrate

$y \sin x = \int 2x \sin x\, dx + \int x^2 \cot x \sin x\, dx$

$y \sin x = 2 \int x \sin x\, dx + \int x^2 \cos x\, dx$

$= 2 \int x \sin x\, dx + x^2 \sin x - \int 2x \sin x\, dx + c$

y sin x = x^2 sin x + c

Question 30: [a] Find the shortest distance between the lines (x - 1) / 1 = (y - 2) / -1 = (z - 1) / 1 and (x - 2) / 2 = (y + 1) / 1 = (z + 1) / 2.

OR

[b] Prove that if a plane has the intercepts a, b, c and is at distance p units from the origin, then prove that 1 / a^2 + 1 / b^2 + 1 / c^2 = 1 / p^2.

Solution:

[a] (x - 1) / 1 = (y - 2) / -1 = (z - 1) / 1

⇒ (r - [i + 2j + k]) . (i - j + k) = 0

(x - 2) / 2 = (y + 1) / 1 = (z + 1) / 2

⇒ (r - [2i - j - k]) . (2i + j + 2k) = 0

Shortest distance = |[(b - a) . (p x q)] / [p x q]|

[p x q] = $\begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$

= i (- 2 - 1) - j (2 - 2) + k (1 + 2)

$= -3i + 3k$

$(b - a) = (2i - j - k) - (i + 2j + k)$

$= i - 3j - 2k$

Shortest distance $= |(i - 3j - 2k)(-3i + 3k) / 3\sqrt{2}|$

$= |-3 + 0 - 6| / 3\sqrt{2}$

$= 9 / 3\sqrt{2}$

$= 3 / \sqrt{2}$ units

OR

[b] The plane is $(x / a) + (y / b) + (z / c) = 1$

Perpendicular distance from origin (0, 0, 0)

$P = |0 + 0 + 0 - 1| / \sqrt{(1 / a^2 + 1 / b^2 + 1 / c^2)}$

$= 1 / \sqrt{(1 / a^2 + 1 / b^2 + 1 / c^2)}$

$(1 / a^2 + 1 / b^2 + 1 / c^2) = 1 / p^2$