

RBSE Class 12th Maths Question Paper With Solution 2017

QUESTION PAPER CODE 816

SECTION - A

Question 1: Find the value of $\sin [\pi / 3 + \sin^{-1} (-1 / 2)]$.

Solution:

$$\begin{aligned} & \sin [\pi / 3 + \sin^{-1} (-1 / 2)] \\ &= \sin [\pi / 3 - \sin^{-1} (1 / 2)] \\ &= \sin ([\pi / 3] - [\pi / 6]) \\ &= \sin (\pi / 6) \\ &= 1 / 2 \end{aligned}$$

$$A = \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

Question 2: If $A = \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, then find $2A - B$.

Solution:

$$\begin{aligned} A &= \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix} & B &= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \\ 2A - B &= 2 \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 8 \\ -6 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 4-1 & 8-3 \\ -6+2 & 4-5 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 5 \\ -4 & -1 \end{bmatrix} \end{aligned}$$

Question 3: If $A = [2 \quad -4 \quad 3]$ and $B = \begin{bmatrix} 2 \\ -4 \\ 8 \end{bmatrix}$, then find $(AB)'$.

Solution:

$$A = [2 \quad -4 \quad 3] \text{ and } B = \begin{bmatrix} 2 \\ -4 \\ 8 \end{bmatrix}$$
$$AB = [4 + 16 + 24]$$
$$= [44]$$
$$(AB)' = 44$$

Question 4: Find: $\int(\sqrt{x} + (1 / \sqrt{x})^2 dx$.

Solution:

$$\int(\sqrt{x} + (1 / \sqrt{x})^2 dx$$
$$= \int(x + (1 / x) + 2) dx$$
$$= (x^2 / 2) + \log x + 2x + c$$

Question 5: Find the general solution of the differential equation: $(dy / dx) = 2x / y^2$.

Solution:

$$(dy / dx) = 2x / y^2$$
$$\int y^2 dy = \int 2x dx$$
$$(y^3 / 3) = x^2 + c$$

Question 6: If vector $a = 2i - 2j + 2k$ and vector $b = i + j - k$, then find the unit vector along the vector $(a + b)$.

Solution:

$$a = 2i - 2j + 2k$$

$$b = i + j - k$$

$$a + b = 2i - 2j + 2k + i + j - k$$

$$= 3i - j + k$$

The unit vector along the vector $(a + b)$ is given by

$$n = \frac{a + b}{|a + b|}$$

$$= \frac{(3i - j + k)}{\sqrt{3^2 + (-1)^2 + 1^2}}$$

$$= \frac{(3i - j + k)}{\sqrt{11}}$$

Question 7: Find the cartesian form of the equation of the line passing through the points $(1, 0, 2)$ and $(4, 5, 6)$.

Solution:

The cartesian form of the equation passing through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$\frac{(x - x_1)}{(x_2 - x_1)} = \frac{(y - y_1)}{(y_2 - y_1)} = \frac{(z - z_1)}{(z_2 - z_1)}$$

$$\frac{(x - 1)}{(4 - 1)} = \frac{(y - 0)}{(5 - 0)} = \frac{(z - 2)}{(6 - 2)}$$

$$\frac{(x - 1)}{3} = \frac{y}{5} = \frac{(z - 2)}{4}$$

Question 8: If a line makes 120° , 45° and 90° angles with the x, y and z-axis respectively then find its direction-cosines.

Solution:

If a line makes α , β , γ with the x, y and z-axis respectively, then the direction cosines are given by

$$l = \cos \alpha$$

$$m = \cos \beta$$

$$n = \cos \gamma$$

$$l = \cos 120^\circ = -1 / 2$$

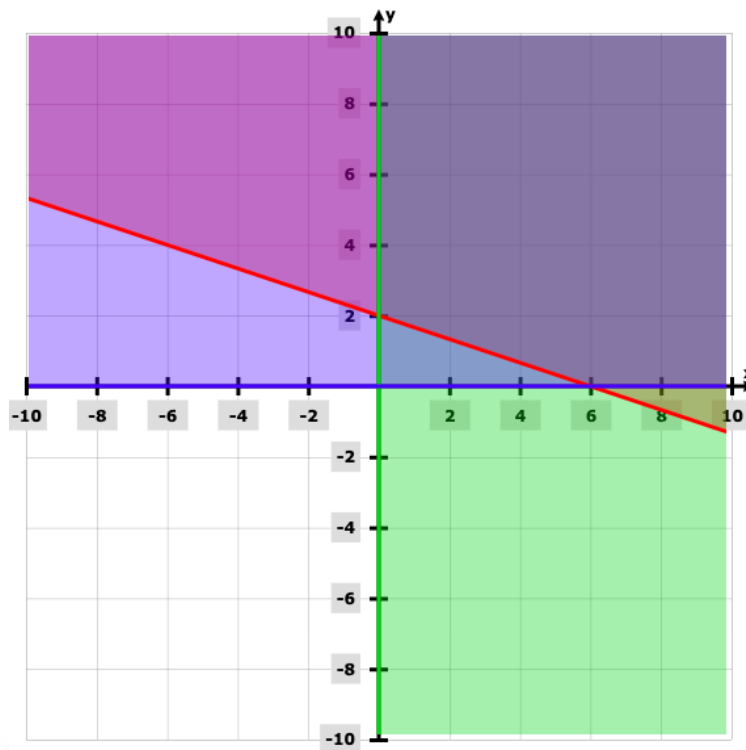
$$m = \cos 45^\circ = 1 / \sqrt{2}$$

$$n = \cos 90^\circ = 0$$

It is conceptually a wrong question because $l^2 + m^2 + n^2 = 1$.

Question 9: Show the region of the feasible solution under the following constraints: $x + 3y \geq 6$, $x \geq 0$, $y \geq 0$ in the answer book.

Solution:



Question 10: If $P(B/A) = 0.2$, $P(A) = 0.8$, then find $P(A \cap B)$.

Solution:

$$P(B/A) = 0.2$$

$$P(A) = 0.8$$

$$P(B/A) = P(A \cap B) / P(A)$$

$$0.2 = P(A \cap B) / 0.8$$

$$0.2 * 0.8 = P(A \cap B)$$

$$P(A \cap B) = 0.16$$

SECTION - B

Question 11: [a] Prove that the relation R defined on the set Z as $aRb \Leftrightarrow a - b$ is divisible by 3 is an equivalence relation.

OR

[b] If function $f, g: \mathbb{R} \rightarrow \mathbb{R}$, are defined as $f(x) = x^2, g(x) = 2x$ then find

[i] $f \circ g(x)$

[ii] $g \circ f(x)$

[iii] $f \circ f(3)$

Solution:

[a] $aRb \Leftrightarrow a - b$ is divisible by 3

(i) Reflexive : $aRa \Rightarrow a - a = 0$ is divisible by 3

$\therefore R$ is reflexive.

(ii) Symmetric: Let $(a, b) \in R$

$\Rightarrow aRb \Leftrightarrow a - b$ is divisible by 3

$\Rightarrow bRa \Leftrightarrow b - a$ is divisible by 3

$\Rightarrow (b, a) \in R$

$\therefore R$ is symmetric

(iii) Transitive: Again $(a, b) \in R$ and $(b, c) \in R$

$aRb \Leftrightarrow a - b$ is divisible by 3 and $bRc \Leftrightarrow b - c$ is divisible by 3

$\Rightarrow a - b$ is divisible by 3 and $b - c$ is divisible by 3

$\Rightarrow (a - b) + (b - c)$ is divisible by 3

$\Rightarrow a - c$ is divisible by 3

$\Rightarrow (a, c) \in R$

$\therefore R$ is transitive, hence R is an equivalence relation.

OR

[b] $f(x) = x^2, g(x) = 2x$

$$(i) f \circ g(x) = f[g(x)] = f(2x) = (2x)^2 = 4x^2$$

$$(ii) g \circ f(x) = g[f(x)] = g(x^2) = 2x^2$$

$$(iii) f \circ f(x) = f[f(x)] = f(x^2) = (x^2)^2 = x^4$$

$$\therefore f \circ f(3) = (3)^4 = 81$$

Question 12: [a] Express the function $\tan^{-1}[(\cos x - \sin x) / (\cos x + \sin x)]$; $\pi / 4 < x < 3\pi / 4$ in the simplest form.

OR

[b] Prove that $\sin^{-1}(8/17) + \sin^{-1}(3/5) = \tan^{-1}(77/36)$.

Solution:

$$\begin{aligned} [a] & \tan^{-1}[(\cos x - \sin x) / (\cos x + \sin x)] \\ &= \tan^{-1}[(1 - \{\sin x / \cos x\}) / (1 + \{\sin x / \cos x\})] \\ &= \tan^{-1}[(1 - \tan x) / (1 + \tan x)] \\ &= \tan^{-1}[(\tan \pi / 4 - \tan x) / [1 + \tan(\pi / 4) \tan x]] \\ &= \tan^{-1}(\tan(\pi / 4 - x)) \\ &= \pi / 4 - x \end{aligned}$$

OR

$$\begin{aligned} [b] & \sin^{-1}(8/17) + \sin^{-1}(3/5) \\ &= \sin^{-1}[(8/17) \sqrt{1 - (3/5)^2} + (3/5) \sqrt{1 - (8/17)^2}] \\ &= \sin^{-1}[(8/17)(4/5) + (3/5)(15/17)] \\ &= \sin^{-1}[(32/85) + (45/85)] \\ &= \sin^{-1}[77/85] \\ &= \tan^{-1}[(77/85) / \sqrt{1 - (77/85)^2}] \\ &= \tan^{-1}(77/36) \end{aligned}$$

Question 13: If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then prove that $A^2 - 5A + 7I_2 = 0$, where I_2 is the identity matrix of order 2.

Solution:

$$\begin{aligned}
 A &= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \\
 A^2 &= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \\
 A^2 - 5A + 7I_2 &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
 \end{aligned}$$

Question 14: Examine the continuity of function $f(x) = \begin{cases} x+5 & x \leq 1 \\ x-5 & x > 1 \end{cases}$ at point $x = 1$.

Solution:

$$f(x) = \begin{cases} x+5 & x \leq 1 \\ x-5 & x > 1 \end{cases}$$

$$\text{Left hand limit} = \lim_{x \rightarrow 1^-} (x + 5) = 6$$

$$\text{Right hand limit} = \lim_{x \rightarrow 1^+} (x - 5) = -4$$

Since $\text{LHL} \neq \text{RHL}$, $f(x)$ is discontinuous at $x = 1$.

Question 15: [a] Find the equation of the tangent to the curve $y = x^3 - x + 1$ at the point whose x coordinate is 1.

OR

[b] The length x of a rectangle is decreasing at the rate 3 cm/minute and the width y is increasing at the rate 5cm / minute. When $x = 10$ cm and $y = 6$ cm, find the area of the rectangle.

Solution:

$$[a] y = x^3 - x + 1 \dots(i)$$

$$dy / dx = 3x^2 - 1$$

$$\text{At } x = 1 \Rightarrow dy / dx = 2$$

$$\text{At } x = 1 \text{ from the equation of curve } y = 1$$

∴ The equation of tangent at (1, 1) is

$$y - 1 = (dy / dx)_{(1,1)} (x - 1)$$

$$\Rightarrow y - 1 = 2 (x - 1)$$

$$\Rightarrow 2x - y - 1 = 0$$

OR

[b] Let at any instant of time t, length be x, breadth y and the area A, then given that dt

$$dx / dt = -3 \text{ cm / min}$$

$$dy / dt = 5 \text{ cm / min}$$

$$\text{Area } A = xy$$

On differentiating with respect to t,

$$dA / dt = x (dy / dt) + y (dx / dt)$$

$$= 10 (5) + 6 (-3)$$

$$dA / dt = 32 \text{ cm}^2 / \text{min}$$

Question 16: Find the maximum profit that a company can make, if the profit function is given by $P(x) = 51 - 72x - 18x^2$.

Solution:

$$P(x) = 51 - 72x - 18x^2$$

$$P'(x) = -72 - 36x$$

Equate $P'(x)$ to 0 and solve for x.

$$P'(x) = 0$$

$$-72 - 36x = 0$$

$$x = -2$$

Since, $P'(x) < 0$ at $x = -2$, $P(x)$ is maximum at $x = -2$ which is given by

$$P(-2) = 51 - 72 * (-2) - 18 * (-2)^2$$

$$= 51 + 144 - 72$$

$$= 123$$

Question 17: [a] Find: $\int (dx / x (x^5 + 1))$.

OR

[b] Find: $\int (x \sin^{-1} x / \sqrt{1 - x^2}) dx$.

Solution:

$$[a] \int (dx / x (x^5 + 1))$$

$$I = \int (dx / x (x^5 + 1))$$

$$= \int (x^4 dx / x^5 (x^5 + 1))$$

$$\text{Put } x^5 + 1 = t$$

$$5x^4 dx = dt$$

$$x^4 dx = dt / 5$$

$$\int (x^4 dx / x^5 (x^5 + 1)) = (1 / 5) \int dt / t (t - 1)$$

$$= (1 / 5) \int [(1 / t - 1) - (1 / t)] dt$$

$$= (1 / 5) \log |t - 1| - \log t + c$$

$$= (1 / 5) \log |(t - 1) / t| + c$$

$$= (1 / 5) \log (x^5 / x^5 + 1) + c$$

OR

$$[b] I = \int (x \sin^{-1} x / \sqrt{1 - x^2}) dx$$

$$\text{Put } t = \sin^{-1} x$$

$$dt = 1 / \sqrt{1 - x^2} dx$$

$$\int (x \sin^{-1} x / \sqrt{1 - x^2}) dx = \int t \sin t dt$$

$$= t \int \sin t dt - \int [(dt / dt) \int \sin t dt] dt$$

$$= -t \cos t + \sin t + c$$

$$= x - \sin^{-1} x \sqrt{1 - x^2} + c$$

Question 18: Find: $\int \sec^2 x dx / \sqrt{\tan^2 x + 4}$.

Solution:

$$I = \int \sec^2 x dx / \sqrt{\tan^2 x + 4}$$

$$\text{Put } t = \tan x$$

$$dt = \sec^2 x dx$$

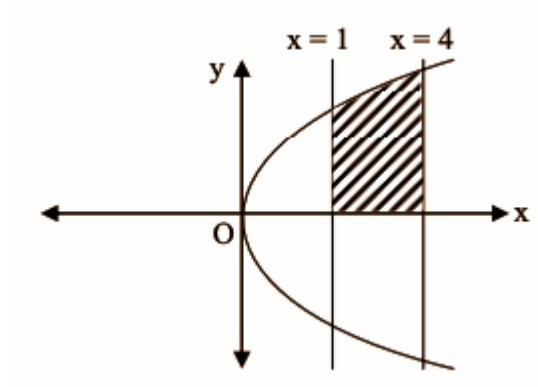
$$\int \sec^2 x dx / \sqrt{\tan^2 x + 4} = \int dt / \sqrt{t^2 + 4}$$

$$= \log |t + \sqrt{t^2 + 4}| + c$$

$$= \log |\tan x + \sqrt{\tan^2 x + 4}| + c$$

Question 19: Find the area of the region bounded by parabola $y^2 = 16x$ and the lines $x = 1$, $x = 4$ and x -axis in the first quadrant.

Solution:



$$A = \int_1^4 y \, dx$$

$$= \int_1^4 4\sqrt{x} \, dx$$

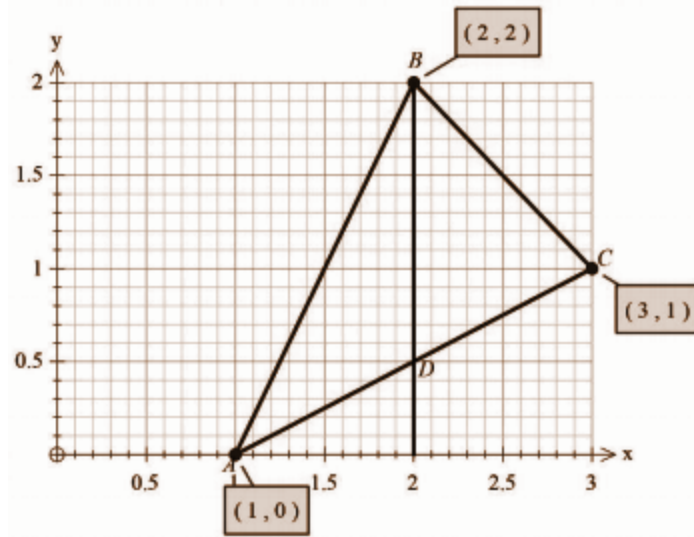
$$= 4 \left(\frac{x^{3/2}}{3/2} \right) \Big|_1^4$$

$$= \frac{8}{3} (8 - 1)$$

$$= \frac{56}{3} \text{ square units}$$

Question 20: Using integration, find the area of the region bounded by the triangle ABC whose vertices are A (1, 0), B (2, 2) and C (3, 1).

Solution:



Equation of AB

$$\Rightarrow y - 2 = (2 - 0 / 2 - 1) (x - 2)$$

$$\Rightarrow y - 2 = 2x - 4$$

$$\Rightarrow 2x = y + 2$$

$$\Rightarrow x = (1 / 2) (y + 2)$$

Equation of AC

$$\Rightarrow y - 0 = (1 - 0 / 3 - 1) (x - 1)$$

$$\Rightarrow 2y = x - 1$$

$$\Rightarrow x = 2y + 1$$

Equation of BC

$$\Rightarrow y - 2 = (2 - 1 / 2 - 3) (x - 2)$$

$$\Rightarrow y - 2 = -x + 2$$

$$\Rightarrow x = 4 - y$$

Required area = Area of ABD + Area of DBCE – Area of ACE

$$= \int_1^2 (2x - 2) dx + \int_2^3 (4 - x) dx + \int_1^3 (x - 1) / 2 dx$$

$$= (x^2 - 2x)_1^2 + (4x - (x^2 / 2))_2^3 - (1 / 2) ((x^2 / 2) - x)_1^3$$

$$= |(4 - 4) - (1 - 2)| + |(12 - (9 / 2)) - (8 - 2)| - (1 / 2) |((9 / 2) - 3) - ((1 / 2) - 1)|$$

$$= 1 + (3 / 2) - (1 / 2) (2)$$

$$= 3 / 2$$

Question 21: If $\mathbf{a} = 5\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$, then find the angle between the vectors $(\mathbf{a} + \mathbf{b})$ and $(\mathbf{a} - \mathbf{b})$.

Solution:

$$a + b = 6i + 2j - 8k$$

$$a - b = 4i - 4j + 2k$$

Angle between $a + b$ and $a - b$ is given by

$$\cos \theta = \frac{(a + b) \cdot (a - b)}{|a + b| |a - b|}$$

$$\frac{(24 - 8 - 16)}{\sqrt{104} \sqrt{36}} = 0$$

$$\theta = 90^\circ$$

Question 22: Find the area of a parallelogram whose adjacent sides are vectors $a = i - j + 3k$ and $b = 2i - 7j + k$.

Solution:

$$a = i - j + 3k$$

$$b = 2i - 7j + k$$

$$A = |a \times b|$$

$$(a \times b) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

$$= i(-1 + 21) - j(1 - 6) + k(-7 + 2)$$

$$= 20i + 5j - 5k$$

$$A = |a \times b| = \sqrt{400 + 25 + 25} = \sqrt{450}$$

Question 23: By graphical method solve the following linear programming problem for minimize.

Objective function $Z = 5x + 7y$

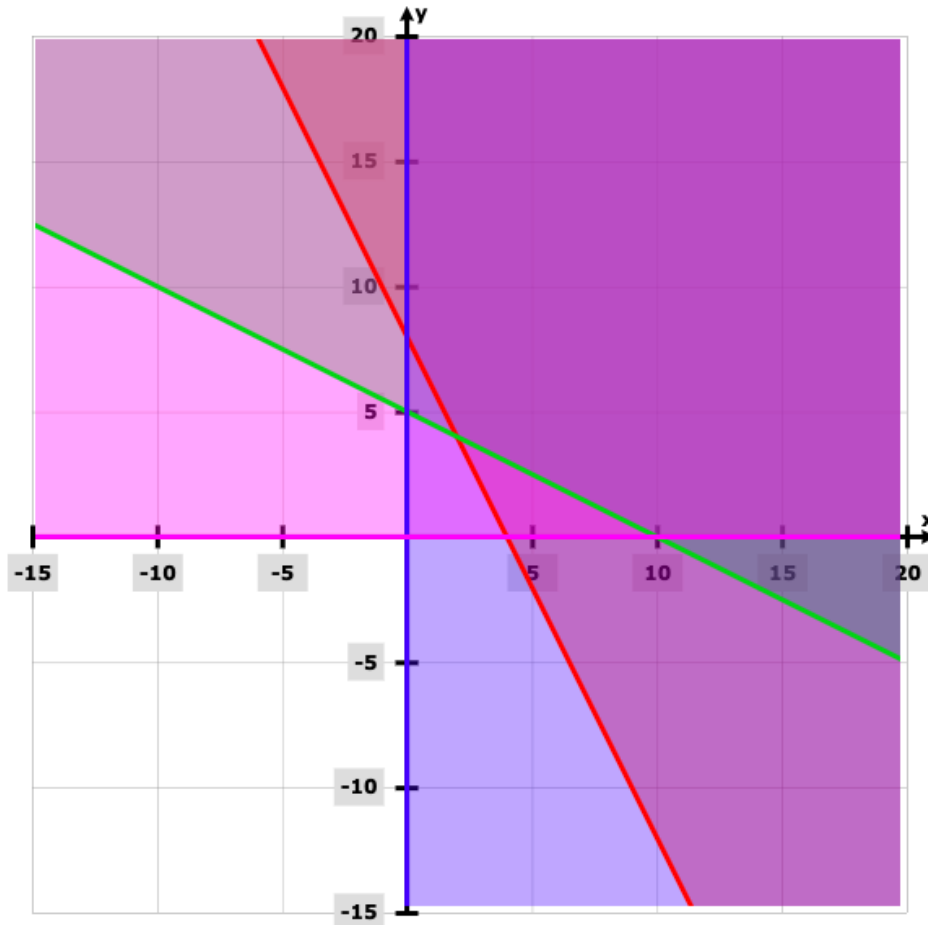
Constraints

$$2x + y \geq 8$$

$$x + 2y \geq 10$$

$$x \geq 0, y \geq 0$$

Solution:



Points	Z
(10, 0)	50
(0, 8)	56
(2, 4)	38

The minimum value of Z is 38 at the point (2, 4).

Question 24: Given three identical boxes I, II and III each containing two coins. In the box I both coins are gold coins in box II both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of silver what is the probability that the other coin in the box is also of silver.

Solution:

Let E_1, E_2 and E_3 be three boxes.

$$P(E_1) = P(E_2) = P(E_3) = 1/3$$

$A \rightarrow$ A coin of silver being drawn

$$P(A/E_1) = 0/2 = 0$$

$$P(A/E_2) = 1$$

$$P(A/E_3) = 1/2$$

$$P(E_2/A) = [P(E_2) * P(A/E_2)] / [P(E_1) * P(A/E_1) + P(E_2) * P(A/E_2) + P(E_3) * P(A/E_3)]$$

$$= [(1/3) * 1] / [(1/3) * 0 + (1/3) * 1 + (1/3) * 1/2]$$

$$= 1 / (3/2)$$

$$= 2/3$$

Question 25: Find the variance of the number obtained on a throw of an unbiased die.

Solution:

Sample space $S = \{1, 2, 3, 4, 5, 6\}$

Probability distribution table

x	1	2	3	4	5	6
P(x)	1/6	1/6	1/6	1/6	1/6	1/6

$$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6$$

$$E(x) = \sum x * P(x)$$

$$= (1 * (1/6)) + (2 * (1/6)) + (3 * (1/6)) + (4 * (1/6)) + (5 * (1/6)) + (6 * (1/6))$$

$$= 21/6$$

$$E(x^2) = 1^2 (1/6) + 2^2 (1/6) + 3^2 (1/6) + 4^2 (1/6) + 5^2 (1/6)$$

$$= 91/6$$

$$\text{Var}(x) = E(x^2) - (E(x))^2$$

$$= (91/6) - (21/6)^2$$

= 35 / 12

SECTION - C

Question 26: Show that
$$\begin{vmatrix} a & a^2 & 1+pa^3 \\ b & b^2 & 1+pb^3 \\ c & c^2 & 1+pc^3 \end{vmatrix} = (1 + pabc) (a - b) (b - c) (c - a).$$

Solution:

$$\begin{aligned} & \begin{vmatrix} a & a^2 & pa^3 \\ b & b^2 & pb^3 \\ c & c^2 & pc^3 \end{vmatrix} + \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \\ &= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + abcp \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \\ &= (1 + pabc) \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \\ & R_1 \rightarrow R_1 - R_2, R_2 \rightarrow R_2 - R_3 \\ &= (1 + abcp) \begin{vmatrix} a-b & a^2-b^2 & 0 \\ b-c & b^2-c^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \\ &= (1 + abcp) [(a-b)(b^2-c^2) - (b-c)(a^2-b^2)] \\ &= (1 + abcp) (a-b)(b-c) \{(b+c) - (a+b)\} \\ &= (a-b)(b-c)(c-a)(1 + abcp) \end{aligned}$$

Question 27: If $y = x^x + x^p + p^x + p^p$, $p > 0$ and $x > 0$, find dy / dx .

Solution:

$$y = x^x + x^p + p^x + p^p$$

$$\text{Consider } z = x^x$$

$$\log z = x \log x$$

$$(1/z)(dz/dx) = \log x + 1$$

$$dz/dx = x^x (1 + \log x)$$

$$dy / dx = x^x (1 + \log x) + px^{p-1} + p^x \log p$$

Question 28: Show that $\int_0^\pi [x dx / (a^2 \cos^2 x + b^2 \sin^2 x)] = \pi^2 / 2ab$.

Solution:

$$I = \int_0^\pi [x dx / (a^2 \cos^2 x + b^2 \sin^2 x)] \text{ ---- (1)}$$

$$I = \int_0^\pi (\pi - x) dx / (a^2 \cos^2 x + b^2 \sin^2 x) \text{ ---- (2)}$$

Adding (1) and (2)

$$2I = \int_0^\pi (\pi) dx / (a^2 \cos^2 x + b^2 \sin^2 x)$$

$$2I = 2\pi \int_0^{\pi/2} dx / (a^2 \cos^2 x + b^2 \sin^2 x)$$

$$I = \pi \int_0^{\pi/2} \sec^2 x dx / a^2 + b^2 \tan^2 x$$

Let $\tan x = t$

$$\sec^2 x dx = dt$$

$$I = \pi \int_0^\infty dt / a^2 + b^2 t^2$$

$$I = \pi / b^2 \int_0^\infty dt / [(a^2 / b^2) + t^2]$$

$$I = (\pi / b^2) * (1 / (a / b)) [\tan^{-1} t / (a / b)]_0^\infty$$

$$I = (\pi / ab) * [(\pi / 2) - 0]$$

$$I = \pi^2 / 2ab$$

Question 29: [a] Find the solution of the differential equation $(x - y) dy - (x + y) dx = 0$.

OR

**[b] Find the solution of the differential equation $\cos^2 x * (dy / dx) + y = \tan x$
[$0 \leq x \leq \pi / 2$]**

Solution:

$$[a] (x - y) dy - (x + y) dx = 0$$

$$dy / dx = (x + y) / (x - y)$$

Let $y = vx$

$$dy / dx = v + x (dv / dx)$$

$$v + x (dv / dx) = (x + vx) / (x - vx)$$

$$x (dv / dx) = (1 + v) / (1 - v) - v$$

$$x (dv / dx) = (1 + v - v + v^2) / (1 - v)$$

$$\int (1 - v) / (1 + v^2) dv = \int dx / x$$

$$\int \frac{1}{(1+v^2)} dv - \int \frac{v}{(1+v^2)} dv = \int \frac{dx}{x}$$

$$\tan^{-1} v - (1/2) \log(1+v^2) = \log x + c$$

$$\tan^{-1}(y/x) - (1/2) \log(1+[y^2/x^2]) = \log x + c$$

$$\tan^{-1}(y/x) - (1/2) \log[y^2+x^2] = c$$

OR

[b] $\cos^2 x \cdot (dy/dx) + y = \tan x$
 $dy/dx + y/\cos^2 x = \tan x/\cos^2 x$ which is linear equation in 'y'
 Here, $P = 1/\cos^2 x = \sec^2 x$; $Q = \sec^2 x \cdot \tan x$
 $IF = e^{\int P dx} = e^{\tan x}$
 $ye^{\tan x} = \int e^{\tan x} \tan x \sec^2 x dx + c$
 Put $\tan x = z$ in R.H.S.
 $\sec^2 x dx = dz$
 $\int e^{\tan x} \tan x \sec^2 x dx + c$
 $= \int e^z z dz$
 $= ze^z - \int 1 e^z dz$
 $= ze^z - e^z$
 $= (z-1)e^z$
 $= (\tan x - 1) e^{\tan x}$
 $ye^{\tan x} = (\tan x - 1) e^{\tan x} + c$
 $y = (\tan x - 1) + ce^{-\tan x}$

Question 30: [a] Find the shortest distance between the lines $r = (i - 2j + 3k) + \lambda(-i + j - 2k)$ and $r = (i - j - k) + \mu(i + 2j - 2k)$.

OR

[b] Find the equation of the plane that contains the point (2, -1, 3) and is perpendicular to each of the planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$.

Solution:

[a] $a_1 = i - 2j + 3k$
 $a_2 = i - j - k$
 $b_1 = (-i + j - 2k)$
 $b_2 = (i + 2j - 2k)$

$$\text{Shortest distance} = |(a_2 - a_1) \cdot (b_1 * b_2)| / |(b_1 * b_2)|$$

$$(b_1 * b_2) = i(-2 + 4) - j(2 + 2) + k(-2 - 1)$$

$$= 2i - 4j - 3k$$

$$(a_2 - a_1) = j - 4k$$

$$SD = |(-4 + 12) / \sqrt{4 + 16 + 9}| = 8 / \sqrt{29}$$

OR

[b] Let the equation of plane

$$a(x - 2) + b(y + 1) + c(z - 3) = 0 \dots(1)$$

The plane (1) is perpendicular to the planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$

$$\therefore 2a + 3b - 2c = 0 \dots (2)$$

$$a + 2b - 3c = 0 \dots (3)$$

Solving (2) & (3)

From (1)

$$-5(x - 2) + 4(y + 1) + (z - 3) = 0$$

$$-5x + 4y + z + 11 = 0$$

$$5x - 4y - z - 11 = 0$$