RBSE Class 12th Maths Question Paper With Solution 2018

QUESTION PAPER CODE 1050

SECTION - A

Question 1: If f : R → R, f (x) = x^2 - 5x + 7, then find the value of f^{-1} (1).

Solution:

f : R → R

f (x) = x^2 - 5x + 7

Let f^{-1} (1) = x ---- (1)

f (x) = 1

x^2 - 5x + 7 = 1

x^2 - 5x + 7 - 1 = 0

x^2 - 5x + 6 = 0

x^2 - 3x - 2x + 6 = 0

x (x - 3) - 2 (x - 3) = 0

x = 2, 3

f^{-1} (1) = 2, 3 [from (1)]

Question 2: Find the value of $\sin^{-1}$ (1 / 2) + 2 $\cos^{-1}$ (1 / 2).

Solution:

$\sin^{-1}$ (1 / 2) + 2 $\cos^{-1}$ (1 / 2)

= (π / 6) + 2 * (π / 3)

= (π / 6) + (2π / 3)

= (π + 4π) / 6

= 5π / 6

Question 3: Find A, if 2A - $\begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 3 & 2 \\ 0 & 1 \end{bmatrix}$.

Solution:

$$2A - \begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 3 & 2 \\ 0 & 1 \end{bmatrix}$$

$$2A = \begin{bmatrix} 1 & 5 \\ 3 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \\ 0 & 6 \end{bmatrix}$$

$$A = \frac{1}{2}\begin{bmatrix} 4 & 4 \\ 4 & 4 \\ 0 & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix}$$

Question 4: If A = $\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, then find A^{-1}.

Solution:

Write the augmented matrix

	A_1	A_2	B_1	B_2
1	3	4	1	0
2	1	2	0	1

Find the pivot in the 1st column and swap the 2nd and the 1st rows

	A_1	A_2	B_1	B_2
1	1	2	0	1
2	3	4	1	0

Eliminate the 1st column

	A_1	A_2	B_1	B_2
1	1	2	0	1
2	0	-2	1	-3

Make the pivot in the 2nd column by dividing the 2nd row by -2

	A_1	A_2	B_1	B_2
1	1	2	0	1
2	0	1	-1/2	3/2

Eliminate the 2nd column

	A_1	A_2	B_1	B_2
1	1	0	1	-2
2	0	1	-1/2	3/2

There is the inverse matrix on the right

	A_1	A_2	B_1	B_2
1	1	0	1	-2
2	0	1	-1/2	3/2

Question 5: Find $\int xe^x$ dx.

Solution:

$\int u * v \, dx = u \int v \, dx - \int[(d(u) / dx) * \int v \, dx] \, dx$

$I = x \int e^x \, dx - \int[(d(x) / dx) * \int e^x \, dx] \, dx$

$= xe^x - \int 1 . e^x \, dx$

$= xe^x - e^x + c$

$= e^x (x - 1) + c$

Question 6: Find a vector of magnitude 5 units along the vector i - 2j + 2k.

Solution:

a = i - 2j + 2k

$|a| = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$

$a = a_1 i + a_2 j + a_3 k$

$a = \sqrt{a_1^2 + a_2^2 + a_3^2}$

a = a / |a| = [i - 2j + 2k] / 3 ---- (i)

The vector of magnitude 5 units along the vector a = 5 * a

= 5 * {[i - 2j + 2k] / 3}

= (5 / 3) (i - 2j + 2k)

Question 7: Find the projection of the vector i - j on the vector i + j.

Solution:

a = i - j

b = i + j

The projection of vector a on b is given by [a . b] / |b|

$= (i - j) (i + j) / \sqrt{1^2 + 1^2}$

$= (1 - 1) / \sqrt{2}$

$= 0 / \sqrt{2}$

= 0

Question 8: Find the direction cosines of the line (x - 2) / 2 = (y + 1) / -2 = (z - 1) / 1.

Solution:

The line is $(x - 2) / 2 = (y + 1) / -2 = (z - 1) / 1$ --- (i)

$(a, b, c) = (2, -2, 1)$

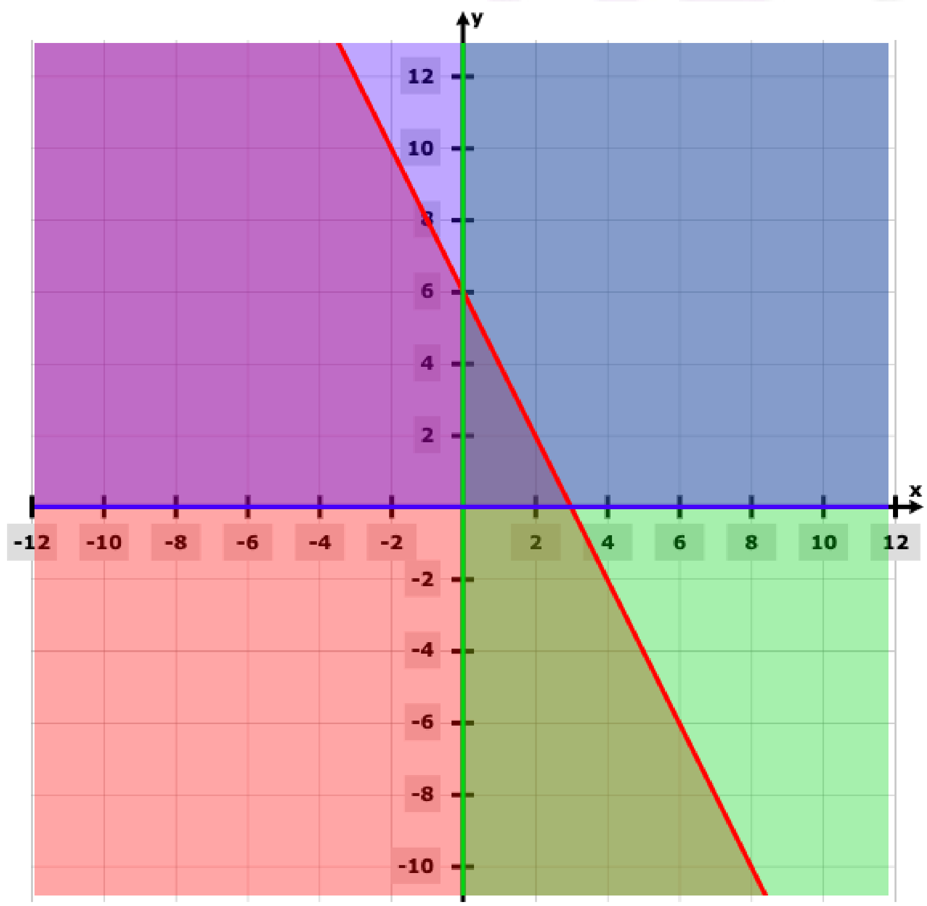
The direction ratios are $\pm a / \sqrt{a^2 + b^2 + c^2}, \pm b / \sqrt{a^2 + b^2 + c^2}, \pm c / \sqrt{a^2 + b^2 + c^2}$

$= \pm 2 / \sqrt{4 + 4 + 1}, \pm (-2) / \sqrt{4 + 4 + 1}, \pm 1 / \sqrt{4 + 4 + 1}$

$= \pm 2 / 3, \pm 2 / 3, \pm 1 / 3$

Question 9: Show the region of the feasible solution under the following constraints $2x + y \leq 6$; $x \geq 0$; $y \geq 0$.

Solution:



Question 10: If A and B are two independent events with P (A) = 0.2 and P (B) = 0.5 then find the value of P (A ∪ B).

Solution:

P (A) = 0.2

P (B) = 0.5

P (A ∩ B) = 0.2 * 0.5 = 0.10

P (A ∪ B) = P (A) + P (B) − P (A ∩ B)

= 0.2 + 0.5 - 0.10

= 0.7 - 0.10

= 0.60

SECTION - B

Question 11: If f : R → R and g : R → R, are defined such that f (x) = x^2 + 3, g (x) = 1 - [1 / (1 - x)] then find g o f (x) and f o g (x).

Solution:

f : R → R

f (x) = $x^2 + 3$

g : R → R

g (x) = 1 - [1 / (1 - x)]

= - x / [1 - x]

= x / (x - 1)

g o f (x) = g [f (x)]

= g [x^2 + 3]

= [x^2 + 3] / [x^2 + 3 - 1]

= [x^2 + 3] / [x^2 + 2]

f o g (x) = f [g (x)]

= $(x / x - 1)^2 + 3$

= [$x^2 / (x - 1)^2$] + 3

= [$x^2 + 3 (x - 1)^2$] / $(x - 1)^2$

= [$4x^2 - 6x + 3$] / [$x^2 - 2x + 1$]

Question 12: If $A=\begin{bmatrix}1 & -2\\ 2 & 3\end{bmatrix}$ and $B=\begin{bmatrix}-5 & -2\\ 1 & 2\end{bmatrix}$**, then find $2A^2$ - 3B.**

Solution:

$$A=\begin{bmatrix}1 & -2\\ 2 & 3\end{bmatrix} B=\begin{bmatrix}-5 & -2\\ 1 & 2\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & -2\\ 2 & 3\end{bmatrix}\begin{bmatrix}1 & -2\\ 2 & 3\end{bmatrix}=\begin{bmatrix}1-4 & -2-6\\ 2+6 & -4+9\end{bmatrix}=\begin{bmatrix}-3 & -8\\ 8 & 5\end{bmatrix}$$

$$3B=3\begin{bmatrix}-5 & -2\\ 3 & 6\end{bmatrix}=\begin{bmatrix}-15 & -6\\ 3 & 6\end{bmatrix}$$

$$2A^2-3B=2\begin{bmatrix}-3 & -8\\ 8 & 5\end{bmatrix}=\begin{bmatrix}-6 & -16\\ 16 & 10\end{bmatrix}-\begin{bmatrix}-15 & -6\\ 3 & 6\end{bmatrix}=\begin{bmatrix}9 & -10\\ 13 & 4\end{bmatrix}$$

Question 13: Examine the continuity of function f defined by

$$f(x)=\begin{cases}\dfrac{e^{1/x}}{1+e^{1/x}}; & x\neq 0\\ 0 \quad ; & x=0\end{cases} \quad \text{at } x=0.$$

Solution:

f (0) = 0

f (0 - 0) = $\lim_{h\to 0}$ f (0 - h)

= $\lim_{h\to 0} e^{([1/0] - h)} / [1 + e^{([1/0] - h)}]$

= $\lim_{h\to 0} e^{-1/h} / [1 + e^{-1/h}]$

= $e^{-\infty} / [1 + e^{-\infty}]$

$e^{-\infty} = 1 / [e^{\infty}] = 1 / \infty = 1 / (1 / 0) = 0 / 1 = 0$

f (0 - 0) = 0 / [1 + 0] = 0 / 1 = 0

f (0 + 0) = $\lim_{h\to 0}$ f (0 + h)

= $\lim_{h\to 0} e^{1/h} / [1 + e^{1/h)}]$

= $\lim_{h\to 0} e^{1/h} / e^{1/h} [e^{-1/h} + 1]$

= $\lim_{h\to 0} 1 / [e^{-1/h} + 1]$

= $1 / [1 + e^{-\infty}]$
= 1 / 0 + 1
= 1
f (0) = f (0 − 0) ≠ f (0 + 0)
So, f (x) is not continuous at x = 0.

Question 14: Find ∫dx / √1 + x − √x.

Solution:
I = ∫dx / √1 + x − √x
= ∫(1 / (√1 + x − √x)) * [(√1 + x + √x) / (√1 + x + √x)] dx
= ∫[(√1 + x + √x) / (1 + x − x)] dx
= ∫[(√1 + x + √x) dx
= ∫(√1 + x) dx + ∫√ x dx
= $[(1 + x)^{(½ + 1)} / (1 / 2 + 1)] + (x)^{(½ + 1)} / (1 / 2 + 1) + c$
= $(2 / 3) (1 + x)^{3/2} + (2 / 3) x^{3/2} + c$

Question 15: Find the vector product of the vectors 2i - j + k and 3i + j - 2k.

Solution:
a = 2i - j + k
b = 3i + j - 2k

a x b = $\begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 3 & 1 & -2 \end{vmatrix}$
= [(-1) * (-2) - (1 * 1)] i - [2 * (-2) - 1 * 3] j + [2 * 1 - (-1 * 3)] k
= (2 - i) i - (- 4 - 3) j + (2 - (- 3)) k
= i + 7j + 5k

SECTION - C

Question 16: [a] Solve the equation $\cos^{-1} x + \cos^{-1} 2x = 2\pi / 3$.

OR

[b] Solve the equation $\sec^{-1}(x / a) - \sec^{-1}(x / b) = \sec^{-1} b - \sec^{-1} a$.

Solution:

[a] $\cos^{-1} x + \cos^{-1} 2x = 2\pi / 3$

$\cos^{-1} x + \cos^{-1} y = \cos^{-1} [xy - \sqrt{1 - x^2}\sqrt{1 - y^2}]$

$\cos^{-1} (x * 2x - \sqrt{1 - x^2}\sqrt{1 - (2x)^2}) = 2\pi / 3$

$\cos^{-1} (2x^2 - \sqrt{1 - x^2}\sqrt{1 - (2x)^2}) = 2\pi / 3$

$2x^2 - \sqrt{1 - x^2}\sqrt{1 - (2x)^2} = 2\pi / 3$

$2x^2 - \sqrt{1 - x^2}\sqrt{1 - 4x^2} = 2\pi / 3$

$2x^2 + (1 / 2) = \sqrt{1 - x^2}\sqrt{1 - 4x^2}$

On squaring both sides,

$(2x^2 + (1 / 2))^2 = (1 - x^2)(1 - 4x^2)$

$4x^4 + 2x^2 + (1 / 4) = 1 - 4x^2 - x^2 + 4x^4$

$7x^2 = 1 - (1 / 4)$

$7x^2 = 3 / 4$

$x^2 = 3 / 28$

$x = \pm \sqrt{3 / 28}$

The solution of $\cos^{-1} x + \cos^{-1} 2x = 2\pi / 3$ is $\pm \sqrt{3 / 28}$.

OR

[b] $\sec^{-1}(x / a) - \sec^{-1}(x / b) = \sec^{-1} b - \sec^{-1} a$

$\cos^{-1}(a / x) - \cos^{-1}(b / x) = \cos^{-1}(1 / b) - \cos^{-1}(1 / a)$

$\cos^{-1}(a / x) + \cos^{-1}(1 / a) = \cos^{-1}(1 / b) + \cos^{-1}(b / x)$

$\cos^{-1} [(a / x) * (1 / a) - \sqrt{1 - (a^2 / x^2)}\sqrt{1 - (1 / a)^2}] = \cos^{-1} [(b / x) * (1 / b) - \sqrt{1 - (1 / b^2)}\sqrt{1 - (b^2 / x^2)}]$

$[(1 / x) - \sqrt{1 - (a^2 / x^2)}\sqrt{1 - (1 / a)^2}] = [(1 / x) - \sqrt{1 - (1 / b^2)}\sqrt{1 - (b^2 / x^2)}]$

On squaring both sides,

$[1 - (a^2 / x^2)][1 - (1 / a)^2] = [1 - (b^2 / x^2)][1 - (1 / b)^2]$

$1 - (a^2 / x^2) - (1 / a^2) + (a^2 / x^2a^2) = 1 - (1 / b)^2 - (b^2 / x^2) + (b^2 / x^2b^2)$

$(b^2 / x^2) + (1 / b)^2 = (a^2 / x^2) + (1 / a^2)$

$(b^2 / x^2) - (a^2 / x^2) = (1 / a^2) - (1 / b)^2$

$[(b^2 - a^2) / x^2] = (1 / a^2) - (1 / b^2)$
$[(b^2 - a^2) / x^2] = [b^2 - a^2 / b^2 a^2]$
$x^2 = b^2 a^2$
$x = \pm \sqrt{ab}$
The solution of $\sec^{-1} (x / a) - \sec^{-1} (x / b) = \sec^{-1} b - \sec^{-1} a$ is $x = \pm \sqrt{ab}$.

Question 17: Prove that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$ **= (5x + 4) (x - 4)².**

Solution:

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$
$$R_1 \to R_1 + R_2 + R_3$$
$$\begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$
$$(5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$
$$C_1 \to C_1 - C_2; C_2 \to C_2 - C_3$$
$$(5x+4)\begin{vmatrix} 0 & 0 & 1 \\ x-4 & 4-x & 2x \\ 0 & x-4 & x+4 \end{vmatrix}$$

On taking the value of the determinant,
$= (5x + 4) (x - 4)^2$

Question 18: Solve the following system of equations by using Cramer's rule.
5x – 4y = 7
x + 3y = 9

Solution:

5x – 4y = 7 ---- (1)
x + 3y = 9 ---- (2)

$$\Delta = \begin{vmatrix} 5 & -4 \\ 1 & 3 \end{vmatrix} = (5 * 3 - (1)(-4)) = 15 + 4 = 19$$

$$\Delta_1 = \begin{vmatrix} 7 & -4 \\ 9 & 3 \end{vmatrix} = (7 * 3 - (9)(-4)) = 21 + 36 = 57$$

$$\Delta_2 = \begin{vmatrix} 5 & 7 \\ 1 & 9 \end{vmatrix} = (5 * 9 - (7)(1)) = 45 - 7 = 38$$

$$x = \Delta_1/\Delta = 57/19 = 3$$

$$y = \Delta_2/\Delta = 38/19 = 2$$

Question 19: Find the intervals in which the function f given by f (x) = sin x + cos x; 0 ≤ x ≤ 2π is
a) Strictly increasing
b) Strictly decreasing

Solution:
f (x) = sinx + cosx
f ' (x) = cosx - sinx
Put f' (x) = 0
cosx - sinx = 0
cosx = sinx
Divide throughout by cosx
1 = sinx / cosx
1 = tanx
x = $\tan^{-1}$ (1)
x = π / 4
f' (x) > 0
Therefore, we can divide the given interval into two parts (0, (π / 4)) and ((π / 4), 2π).
Take any value in the first interval (0, (π / 4)),
f '(x) = cos (π / 6) - sin (π / 6) > 0
Therefore, the given function f (x) = sinx + cosx is strictly increasing in the interval (0, (π / 4)).

Take any value in the first interval ((π / 4), 2π),

f '(x) = cos (π / 2) - sin (π / 2) < 0

Therefore, the given function f (x) = sinx + cosx is strictly decreasing in the interval ((π / 4), 2π).

Question 20: Prove that the value of function x / [1 + x tanx] is maximum at x = cos x.

Solution:

f (x) = x / [1 + x tanx]

f ' (x) = [1 (1 + x tanx) - x (tanx + sec^2 x)] / $[1 + x tanx]^2$

= [1 + x tanx - x tanx - x^2 sec^2 x] / $[1 + x tanx]^2$

= [1 - x^2 sec^2 x] / $[1 + x tanx]^2$

Equate f' (x) = 0

0 = [1 - x^2 sec^2 x] / $[1 + x tanx]^2$

[1 - x^2 sec^2 x] = 0

1 = x^2 sec^2 x

x^2 = 1 / sec^2 x

x = ± cos x

Therefore, the given function x / [1 + x tanx] is maximum at x = cos x.

Question 21: [a] Find ∫dx / √(5x − 6 − x^2)

OR

[b] Find ∫dx / x [6 $(log x)^2$ + 7 log x+ 2]

Solution:

[a] ∫dx / √(5x − 6 − x^2)

5x - 6 - x^2 = - (x^2 - 5x + 6)

= - (x^2 - 5x + (25 / 4) + 6 - (25 /4)]

= - [(x - $(5 / 2)^2$ - (1 / 4)]

= (1 / 4) - (x - $(5 / 2)^2$)

∫dx / √(5x − 6 − x^2) = ∫dx / √$(1 / 2)^2$ + (x - $(5 / 2)^2$)

= sin^{-1} {(x - [5 / 2]) / (1 / 2)} + c

= sin^{-1} {(2x - 5) / (1 / 2)} + c

$= \sin^{-1}(2x - 5) + c$

OR

[b] $\int dx / x\, [6 (\log x)^2 + 7 \log x + 2]$

Let $u = \log x$

$du / dx = 1 / x$

$\int dx / x\, [6 (\log x)^2 + 7 \log x + 2] = \int 1\, du / 6u^2 + 7u + 2$

$= \int du / [u^2 + (7 / 6) u + (1 / 3)]$

$= \int du / [u^2 + (7 / 6) u + (49 / 144) + (1 / 3) - (49 / 144)]$

$= \int du / \{[u + (7 / 12)^2] - (1 / 144)\}$

$= \int du / \{[u + (7 / 12)^2] - (1 / 12)^2\}$

$= [1 / 2 (1 / 12)] \log \{(u + (7 / 12) - (1 / 12)) / (u + (7 / 12) + (1 / 12))\} + c$

$= 6 \log [(12u + 6) / (12u + 8)] + c$

$= 6 \log [(6u + 3) / (6u + 4)] + c$

Substitute $u = \log x$

$= 6 \log [(6 \log x + 3) / (6 \log x + 4)] + c$

Question 22: Find the area bounded by curves $x^2 + y^2 = 1$ and $y = |x|$.

Solution:

$x^2 + y^2 = 1$ and $y = |x|$

Area $= 2 \int_0^{(1 / \sqrt{2})} (\sqrt{(1 - x^2)} - x)\, dx$

$= 2 [(x / 2) (\sqrt{(1 - x^2)}) + (1 / 2) \sin^{-1}(x) - (x^2 / 2)]_0^{(1 / \sqrt{2})}$

$= 2 [(1 / 4) + (\pi / 8) - (1 / 4)]$

$= \pi / 4$ square units

Question 23: Find the area of the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.

Solution:

$y^2 = 4x$ and $x^2 = 4y$

Put $x = y^2 / 4$ in $x^2 = 4y$

$y^4 / 16 = 4y$

$y^4 - 64y = 0$

$y (y^3 - 64) = 0$

$y = 0, 4$

Area = $\int_0^4 (2\sqrt{4x} - (x^2 / 4))dx$

$= [(4 / 3) (x)^{3/2} - (x^3 / 12)]_0^4$

$= [(4 / 3) (4)^{3/2} - (4^3 / 12)]$

$= (32 / 3) - (16 / 3)$

$= 16 / 3$ square units

Question 24: [a] For any vector a, prove that $|a \times i|^2 + |a \times j|^2 + |a \times k|^2 = 2|a|^2$.

OR

[b] For any vector a, prove that a = (a . i) i + (a . j) j + (a . k) k.

Solution:

[a] $a = xi + yj + zk$

$|a| = \sqrt{x^2 + y^2 + z^2}$

$|a \times i|^2 + |a \times j|^2 + |a \times k|^2$

$= |(xi + yj + zk) i|^2 + |(xi + yj + zk) j|^2 + |(xi + yj + zk) k|^2$

$= |(zj - yk)|^2 + |(-zi + xk)|^2 + |(yi - xj)|^2$

$= (\sqrt{z^2 + (-y)^2})^2 + (\sqrt{(-z)^2 + x^2})^2 + (\sqrt{y + (-x)^2})^2$

$= z^2 + y^2 + z^2 + x^2 + y^2 + x^2$

$= 2 (x^2 + y^2 + z^2)$

$= 2 (\sqrt{(x^2 + y^2 + z^2)})^2$

$= 2 |a|^2$

OR

[b] $a = xi + yj + zk$

$|a| = \sqrt{x^2 + y^2 + z^2}$

$= [(xi + yj + zk) i] i + [(xi + yj + zk) j] j + [(xi + yj + zk) k] k$

$= xi + yj + zk$

$= a$

Question 25: By graphical method solve the following linear programming problem for

Minimum z = 8000x + 12000y

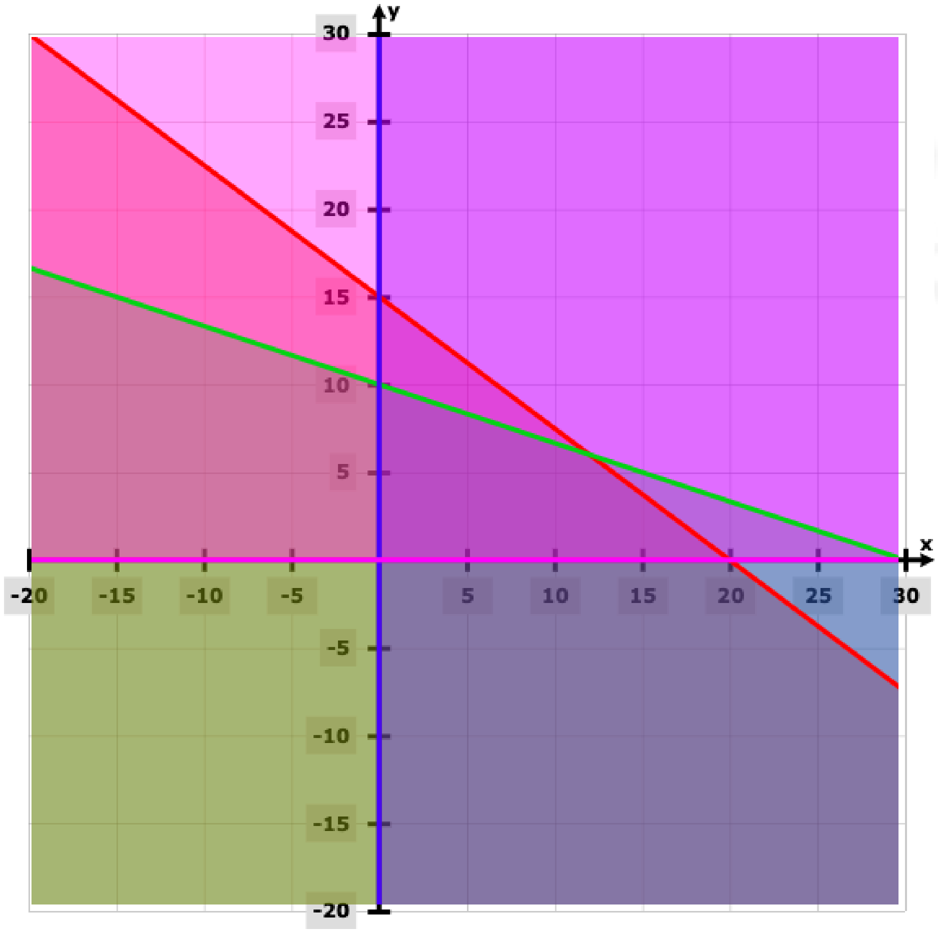
Constraints

3x + 4y ≤ 60

x + 3y ≤ 30

x ≥ 0, y ≥ 0.

Solution:



Points	Z
(0, 0)	0
(20, 0)	160000

(12, 6)	168000
(0, 10)	120000

Therefore, the minimum value of z = 8000 x + 12000 y is 0 at x = 0 and y = 0.

SECTION - D

Question 26: Differentiate $(\log x)^x + x^{\log x}$ with respect to x.

Solution:

$w = (\log x)^x + x^{\log x}$, $y = (\log x)^x$, $z = x^{\log x}$

w = y + z

dw / dx = dy / dx + dz / dx

Take logarithm on both sides to $y = (\log x)^x$, $z = x^{\log x}$

log y = x log (log x) and log z = log x log x

log y = x log (log x) and log z = $[\log x]^2$

(1 / y) (dy / dx) = log (log x) + x (1 / log x) and (1 / z) log z = 2 log x / x

(1 / y) (dy / dx) = log (log x) + (1 / log x) and (1 / z) (dz / dx) = 2 log x / x

dy / dx = $(\log x)^x$ [log (log x) + (1 / log x)] and dz / dx = $x^{\log x}$ 2 log x / x in

dw / dx = dy / dx + dz / dx

dw / dx = $(\log x)^x$ [log (log x) + (1 / log x)] + $x^{\log x}$ 2 log x / x

Question 27: Prove that $\int_0^\pi \log_e (1 + \cos x)\, dx = \pi \log_e (1 / 2)$.

Solution:

$I = \int_0^\pi \log_e (1 + \cos x)\, dx$ ---- (1)

$= \int_0^\pi \log_e (1 + \cos (\pi - x))\, dx$

$= \int_0^\pi \log_e (1 - \cos x)\, dx$ ---- (2)

Adding (1) and (2),

$2I = \int_0^\pi \log_e (1 + \cos x)\, dx + \int_0^\pi \log_e (1 - \cos x)\, dx$

$= \int_0^\pi \log_e (1 + \cos x) + \log_e (1 - \cos x)\, dx$

$= \int_0^\pi \log_e (1 - \cos^2 x)\, dx$

$= \int_0^\pi \log_e (\sin^2 x)\, dx$

$= 2 \int_o^{\pi} \log_e (\sin x)\, dx$
$= 4 \int_o^{\pi/2} \log_e (\sin x)\, dx$
$I = \int_o^{\pi/2} \log_e (\sin x)\, dx$ ---- (3)
$I_1 = \int_o^{\pi/2} \log_e (\sin (\pi / 2 - x))\, dx$
$= \int_o^{\pi/2} \log_e \cos x\, dx$ ---- (4)
Adding (3) and (4),
$2I_1 = \int_o^{\pi/2} \log_e \sin x \cos x\, dx$
$= \int_o^{\pi/2} \log_e \sin 2x\, dx - \int_o^{\pi/2} \log_e 2x\, dx$
$= \int_o^{\pi/2} \log_e \sin 2x\, dx - (\pi / 2) \log_e 2$
Let $2x = t$
$2\, dx = dt$
$dx = dt / 2$
$2I_1 = (1 / 2) \int_o^{\pi/2} \log_e \sin t\, dt - (\pi / 2) \log_e 2$
$2I_1 = I_1 - (\pi / 2) \log_e 2$
$I_1 = (\pi / 2) \log_e (1 / 2)$
$I = 2 * (\pi / 2) \log_e (1 / 2)$
$I = \pi \log_e (1 / 2)$

Question 28: [a] Solve the differential equation dy / dx = [x + y + 1] / [2x + 2y + 3].

OR

[b] Find the particular solution of the differential equation $(\tan^{-1} y - x)\, dy = (1 + y^2)\, dx$ if x = 0 and y = 0.

Solution:
[a] dy / dx = [x + y + 1] / [2x + 2y + 3]
Put x + y = t
1 + (dy / dx) = dt / dx
dy / dx = (dt /dx) - 1
(dt /dx) - 1 = (t + 1) / (2t + 3)
dt / dx = [(t + 1) / (2t + 3)] + 1
= [3t + 4] / [2t + 3]
[2t + 3] / [3t + 4] dt = dx

[(2 / 3) + 1 / [3 (3t + 4)]] dt = dx

$\int (2 / 3)\, dt + (1 / 3) \int 1 / (3t + 4)\, dt = \int dx$

(2 / 3) t + (1 / 9) ln (3t + 4) = x + c

(2 / 3) (x + y) + (1 / 9) ln (3x + 3y + 4) = x + c

OR

[b] $(\tan^{-1} y - x)\, dy = (1 + y^2)\, dx$

$dx / dy = (\tan^{-1} y - x) / (1 + y^2)$

$= (\tan^{-1} y / 1 + y^2) - [x / (1 + y^2)]$

$dx / dy + [x / (1 + y^2)] = (\tan^{-1} y / 1 + y^2)$

This is linear differential equation of the form (dx / dy) + Px = Q

$P = [x / (1 + y^2)]$; $Q = (\tan^{-1} y / 1 + y^2)$

$IF = e^{\int P\, dy} = e^{[x / (1 + y^2)]\, dy} = e^{(\tan^{-1} y)}$

The solution of the differential equation is given by

$x * (IF) = \int Q * (IF)\, dy + c$

$x * e^{(\tan^{-1} y)} = \int (\tan^{-1} y / 1 + y^2) * (e^{(\tan^{-1} y)})\, dy + c$

Let $t = \tan^{-1} y$

$dt = (1 / [1 + y^2])\, dy$

$xe^t = \int te^t\, dt + c$

$xe^t = te^t - \int [\int e^t\, dt]\, dt + c$

$xe^t = te^t - e^t + c$

$x * e^{(\tan^{-1} y)} = e^{(\tan^{-1} y)} (\tan^{-1} y - 1) + c$

c = 1

Therefore, the particular solution is $x * e^{(\tan^{-1} y)} = e^{(\tan^{-1} y)} (\tan^{-1} y - 1) + 1$.

Question 29: Prove that the lines r = i + j - k + λ (3i - j) and r = 4i - k + μ (2i + 3k) are intersecting, also find the point of intersection.

Solution:

The position vectors of arbitrary points on the given lines are

(i + j - k) + λ (3i - j) = (3λ + 1)i + (1 - λ) j - k

If the lines intersect, then they have a common point. So, for some values of λ and μ,

$(3\lambda + 1)$ i + $(1 - \lambda)$ j - k = $(2\mu + 4)$i + 0j + $(3\mu - 1)$k
$(3\lambda + 1) = (2\mu + 4)$; $1 - \lambda = 0$; $-1 = (3\mu - 1)$
Solving the last two of these three equations, $\lambda = 1$ and $\mu = 0$.
These values of λ and m satisfy the first equation.
So, the given lines intersect.
Putting $\lambda = 1$ in first line,
r = (i + j - k) + (3i - j) = 4i + 0j - k which is the position vector of the point of intersection.
Thus, the coordinates of the point of intersection are (4, 0, −1).

Question 30: [a] Bag A contains 3 red and 4 black balls and bag B contains 4 red and 5 black balls. One ball transferred from bag A to bag B and then a ball is drawn from bag B. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

OR

[b] Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution and mean of the number of aces.

Solution:
[a] Total number of balls in bag 1 = 7
Total number of balls in bag 2 = 9
Let E_1 denote the event that a red ball is transferred from bag 1 to bag 2 and E_2 denote the event that a black ball is transferred from bag 1 to bag 2.
$P(E_1) = 3 / 7$
$P(E_2) = 4 / 7$

Let A denote the event that the ball drawn is red.
When a red ball is transferred from bag 1 to bag 2,
$P(A / E_1) = 5 / 10 = 1 / 2$
When a black ball is transferred from bag 1 to bag 2,
$P(A / E_2) = 4 / 10 = 2 / 5$
The probability that the transferred ball is black is denoted as $P(E_2 / A)$

=> $P(E_2 / A) = \{P(E_2) * P(A / E_2)\} / \{P(E_1) * P(A / E_1) + P(E_2) * P(A / E_2)\}$
= {4 / 7 * 2 / 5} / {3 / 7 * 1 / 2 + 4 / 7 * 2 / 5}
= (8 / 5) / (3 / 2 + 8 / 5)
= (8 / 5) / {(15 + 16) / 10}
= (8 / 5) / (31 / 10)
= (8 * 10) / (5 * 31)
= (8 * 2) / 31
= 16 / 31
The probability that the transferred ball is black is 16 / 31.

OR

[b] Let X be the random variable.
X = number of aces obtained in 2 draws.
X = 0, 1, 2
Probability of drawing an ace card = 4 / 52 = 1 / 13
Probability of drawing a non-ace card = 48 / 52 = 12 / 13
P (X = 0) = (1 / 13) (12 / 13) = (12 / 169)
P (X = 1) = (1 / 13) (12 / 13) + (1 / 13) (12 / 13) = 24 / 169
P (X = 2) = (1 / 13) (1 / 13) = 1 / 169
Now, the mean of the number of aces is given by np
= 2 * (1 / 13)
= 2 / 13