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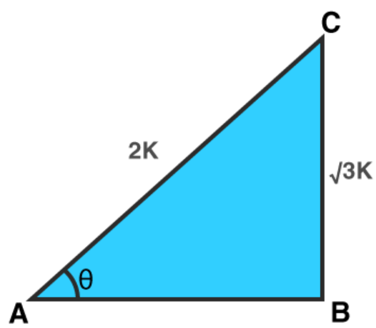
Question 1:
If sin θ = √3/2, find the value of all T-ratios of θ.

Solution:
Given function: sin θ = √3/2
Let us first draw a right ΔABC, ∠B = 90 degrees and ∠A = θ



(where k is a positive)

We know that sin θ = BC/AC = (Perpendicular)/Hypotenuse = √3/2

By Pythagoras Theorem:

AC^2 = AB^2 + BC^2

Or AB^2 = AC^2 - BC^2 = 4k^2 – 3k^2 = k^2
AB = k

Find other T-rations using their definitions:
Cos = AB/AC = 1/2

Tan θ = BC/AB = √3

$cosec\ \theta$ = 1/sinθ = 2/√3

sec θ = 1/cos θ = 2

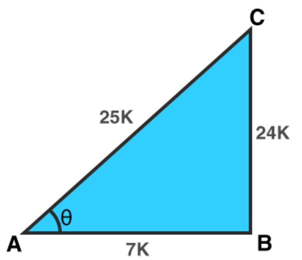
cot θ = 1/tan θ = 1/√3

Question 2:
If cos θ=7/25 , find the values of all T-ratios of θ.

Solution:

Given function: cos θ = 7/25

Draw a right ΔABC, ∠B = 90 degrees and ∠A = θ



(where k is a positive)

We know that cos θ = AB/AC = Base/Hypotenuse = 7/25

By Pythagoras Theorem:

AC^2 = AB^2 + BC^2

Or BC^2 = AC^2 - AB^2 = 625k^2 – 49k^2 = 576k^2

AB = 24k

Find other T-rations using their definitions:
Sin θ = BC/AC = 24/25

Tan θ = BC/AB = 24/7

$cosec\ \theta$ = 1/sinθ = 25/24

sec θ = 1/cos θ = 25/7

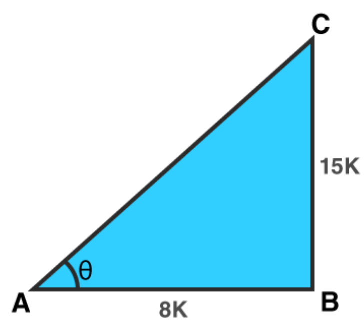
cot θ = 1/tan θ = 7/24

Question 3:
If tan θ=15/8, find the values of all T-ratios of θ.

Solution:

Given function: tan θ=15/8

Draw a right ΔABC, ∠B = 90 degrees and ∠A = θ



We know that tan θ = BC/AB = perpendicular/base = 15/8

(where k is a positive)

By Pythagoras Theorem:

AC^2 = AB^2 + BC^2

= 64k^2 + 225k^2

= 289k^2

AC = 17k

Find other T-rations using their definitions:

Sin θ = BC/AC = 15/17

cos θ = AB/AC = 8/17

$cosec\ \theta$ = 1/sinθ = 17/15

sec θ = 1/cos θ = 17/8

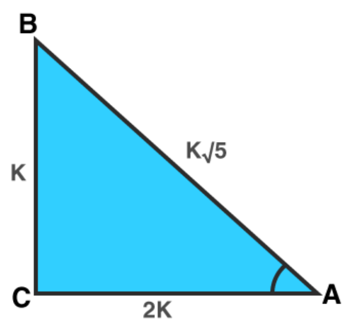
cot θ = 1/tan θ = 8/15

Question 4:
If cot θ = 2, find the value of all T-ratios of θ.

Solution:

Given function: cot θ= 2

Draw a right ΔABC, ∠C = 90 degrees and ∠A = θ



We know that cot θ = AC/BC = base/perpendicular = 2/1

(where k is a positive)

By Pythagoras Theorem:

AB^2 = BC^2 + AC^2

= k^2 + 4k^2

= 5k^2

AB = k√5

Find other T-rations using their definitions:
Sin θ = BC/AB = k/(k√5) = 1/√5

cos θ = AC/AB = (2k)/(k√5) = 2/√5

tan θ = BC/AC = sinθ /cosθ = k/(2k) = 1/2

$cosec\ \theta$ = 1/sinθ = √5

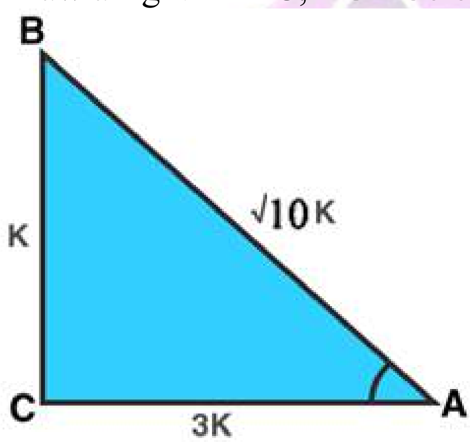
sec θ = 1/cos θ = √5/2

Question 5:
If cosec θ = √10, the find the values of all T-ratios of θ.

Solution:

Given function: cosec θ = √10

Draw a right ∆ABC, ∠C = 90 degrees and ∠A = θ



We know that, cosecθ = AB/BC = hypotenuse/perpendicular = (k√10)/k

(where k is a positive)

By Pythagoras
Theorem:

$AC^2 = AB^2 + BC^2$

$= 10k^2 + k^2$
$= 9k^2$

$AC = 3k$

Find other T-rations using their definitions:
$\sin\theta = BC/AB = 1/\sqrt{10}$

$\cos\theta = AC/AB = (3k)/(k\sqrt{10}) = 3/\sqrt{10}$

$\tan\theta = BC/AC = \sin\theta/\cos\theta = 1/3$

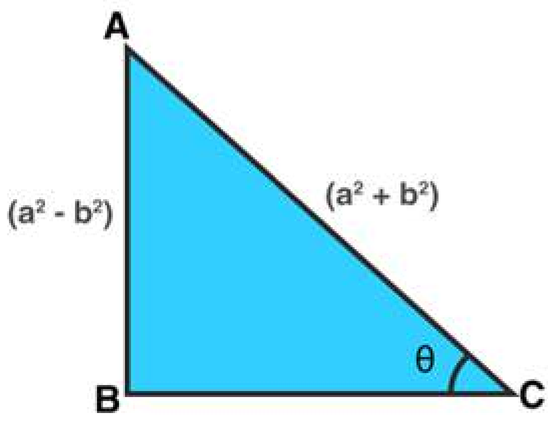
$\sec\theta = AB/AC = 1/\cos\theta = \sqrt{10}/3$

$\cot\theta = AC/BC = 1/\tan\theta = 3$

Question 6:
If $\sin\theta = (a^2-b^2)/(a^2+b^2)$, find the values of all T-ratios of θ.

Solution:

Draw a triangle, ΔABC, Let ∠ACB = θ and ∠B = 90 degrees



Sin θ = (a^2 - b^2) / (a^2 + b^2)

AB = (a^2 - b^2)

AC = (a^2 + b^2)

By Pythagoras theorem:

BC = √[(a^2 + b^2)^2 - (a^2 - b^2)^2]

BC = √(4a^2 b^2)

or BC = 2ab

Find other T-rations using their definitions:

cos θ = base/hypotenuse = 2ab / (a^2 + b^2)

tan θ = perpendicular/base = (a^2 - b^2) / 2ab

cosec θ = 1/sin θ = (a^2 + b^2)/(a^2 - b^2)

sec θ = 1/cos θ = (a^2 + b^2)/2ab

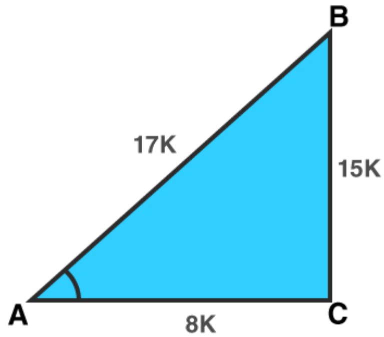
cotθ = 1/tanθ = 2ab/(a^2 - b^2)

Question 7:
If 15cotA=8, find the values of sinA and secA.

Solution:

Given: 15 cot A = 8

cotA = (8k)/(15k) = 1/tanA = AC/BC



Where k is any positive.
By Pythagoras theorem:

AB^2 = BC^2 + AC^2

= (15k)^2 + (8k)^2

= 289k^2

AB = 17k

Find other T-rations using their definitions:

sin A = perpendicular/ hypotenuse = (15k)/(17k) = 15/17

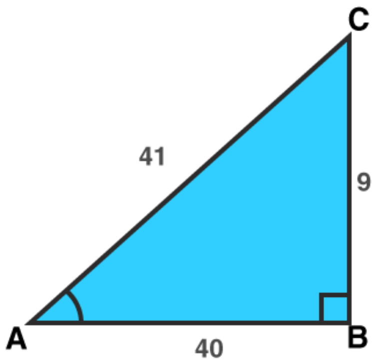
sec A = hypotenuse /base = 17/8

Question 8:
If sinA=9/41, find the values of cosA and tanA.

Solution:

Draw a triangle, ΔABC, Let ∠ACB = θ and ∠B = 90 degress

Sin A = perpendicular/ hypotenuse = 9/41



By Pythagoras theorem:

AC^2 = AB^2 + BC^2

AB^2 = AC^2 – BC^2

= 41^2 – 9^2

= 1600

AB = 40

Find other T-rations using their definitions:

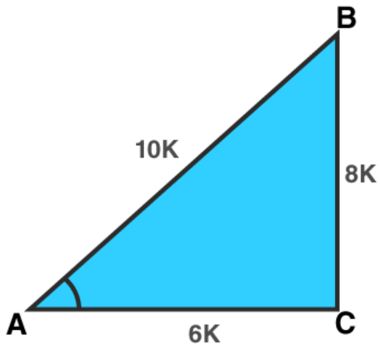
cos A = base/ hypotenuse = 40/41

tan A = perpendicular /base = 9/40

Question 9:
If cos θ = 0.6, show that (5sinθ − 3tanθ) = 0.

Solution:

cos θ = 0.6 = (6k)/(10k) = AC/AB



Where k is any positive.
By Pythagoras theorem:

AB^2 = BC^2 + AC^2

BC^2 = AB^2 - AC^2

= (10k)^2 + (6k)^2

= 64k^2

BC = 8k

Find other T-rations using their definitions:

sin θ = perpendicular/ hypotenuse = 8/10

tan θ = perpendicular/base = 8/6

Now,

LHS = 5sinθ - 3tanθ

= 5(8/10) - 3(8/6)

= 4 - 3(4/3)

= 4(3) - 3(4)
= 12 - 12
= 0
=RHS
Hence proved.

Question 10:
If cosec θ = 2, show that

$$\left(cot\theta + \frac{sin\theta}{1 + cos\theta}\right) = 2$$

Solution:

cosec θ = 2

or 1/sinθ = 2

(cosecθ is reciprocal of sin θ)

sin θ = 1/2

which implies θ = 30 degrees.

Find the values of cos θ and cot θ at θ = 30 degrees.

cos 30^0 = √3/2 and cot 30^0 = √3

Now,

LHS = cot 0 + sin0/(1 +cos0)

= √3 + 1/2 / (1 + √3/2)

$= \frac{2\sqrt{3} + 3 + 1}{2 + \sqrt{3}}$

$= \frac{4 + 2\sqrt{3}}{2 + \sqrt{3}}$

$= \frac{2(2 + \sqrt{3})}{2 + \sqrt{3}}$

=2
=RHS

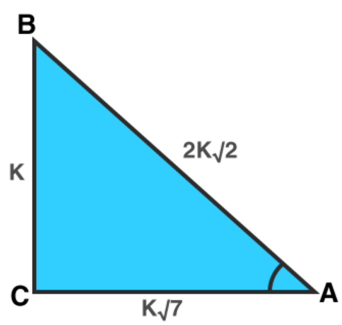
Hence proved.

Question 11:
If tan θ = 1/√7, show that

$$\frac{(cosec^2\theta - sec^2\theta)}{(cosec^2\theta + sec^2\theta)} = \frac{3}{4}$$

Solution:
Given: tan θ = 1/√7

tanθ = k/(k√7) = BC/AC



Where k is any positive.

By Pythagoras theorem:

AB^2 = BC^2 + AC^2

= k^2 + 7k^2

AB = 2k√2

Find cosec θ and sec θ using their definitions:

cosec θ = AB/BC = 2k√2

secθ = AB/AC = 2√2/√7

Now,
LHS =

$$\frac{(cosec^2\theta - sec^2\theta)}{(cosec^2\theta + sec^2\theta)}$$

$$= \frac{(2\sqrt{2})^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{(2\sqrt{2})^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$$

= 48/64

=3/4

= RHS

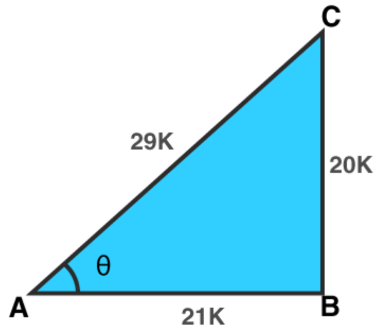
Hence proved

Question 12.
If tan θ = 20/21, show that
$$\left(\frac{1 - sin\theta + cos\theta}{1 + sin\theta + cos\theta}\right) = \frac{3}{7}$$

Solution:

Given: tan θ = 20/21

tanθ = 20k/(21k)



Where k is any positive.

By Pythagoras theorem:

AC^2 = AB^2 + BC^2

= 441k^2 + 400k^2

AC = 29k

Find sin θ and cos θ using their definitions:

sin θ = perpendicular/ hypotenuse = 20/29

cos θ = base/hypotenuse = 21/29

Now,
LHS =

$$\left(\frac{1 - sin\theta + cos\theta}{1 + sin\theta + cos\theta}\right)$$

$$= \frac{1 - \frac{20}{29} + \frac{21}{29}}{1 + \frac{20}{29} + \frac{21}{29}}$$

=30/70

= 3/7

=RHS
Hence proved

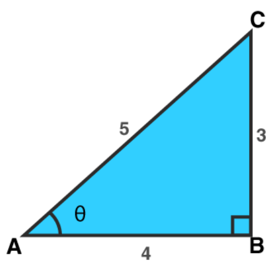
Question 13:
If secθ=5/4, show that

$$\left(\frac{sin\theta - 2cos\theta}{tan\theta - cot\theta}\right) = \frac{12}{7}$$

Solution:

Given: secθ=5/4

sec θ = 5k/(4k) = 5/4 and cos θ = 4/5



Where k is any positive.

By Pythagoras theorem:

AC^2 = AB^2 + BC^2

BC^2 = AC^2 - AB^2

= 25k^2 + 16k^2

BC = 3k

Find sin θ, tan θ and cot θ using their definitions:

sin θ = perpendicular/ hypotenuse = 3/5

tan θ = perpendicular/base = 3/4

cotθ = 1/tanθ = 4/3

Now,
LHS =
$$\left(\frac{sin\theta - 2cos\theta}{tan\theta - cot\theta}\right)$$

$$= \frac{\frac{3}{5} - 2 \times \frac{4}{5}}{\frac{3}{4} - \frac{4}{3}}$$
$$= \frac{\frac{3-8}{5}}{\frac{9-16}{12}}$$

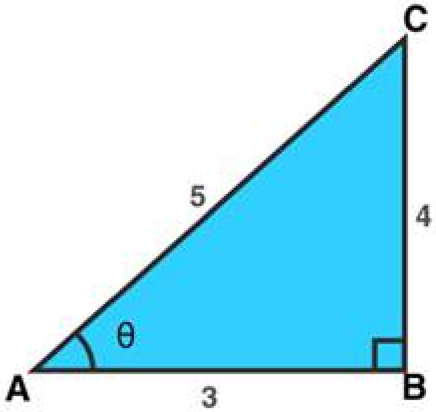
= 12/7

=RHS

Question 14:
If cotθ=3/4, show that
$$\left(\frac{sec\theta - cosc\theta}{sec\theta + cosec\theta}\right) = \frac{1}{\sqrt{7}}$$
Solution:

Given: cotθ=3/4

or cotθ = 3k/4k



Where k is any positive.

By Pythagoras theorem:

AC^2 = AB^2 + BC^2

= 9k^2 +16k^2

= 25k^2

AC = 5k

Find sec θ and cosec θ using their definitions:

sec θ = hypotenuse/base = 5/3

cosec θ = hypotenuse/perpendicular = 5/4

Now,
LHS =

$$\left(\frac{sec\theta - cosc\theta}{sec\theta + cosec\theta}\right)$$

=

$$\sqrt{\frac{5/3-5/4}{5/3+5/4}}$$

$$=\sqrt{\frac{\frac{20-15}{12}}{\frac{20+15}{12}}}$$

$$=\frac{1}{\sqrt{7}}$$

= RHS

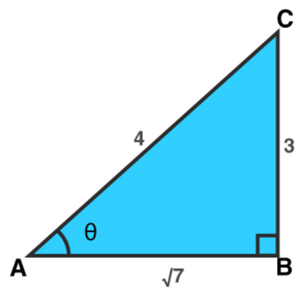
Question 15:
If sinθ=3/4, show that

$$\sqrt{\frac{cosec^2\theta-cot^2\theta}{sec^2\theta-1}}=\frac{\sqrt{7}}{3}$$

Solution:

Given: sinθ=3/4

or sinθ=3k/4k



Where k is any positive.

By Pythagoras theorem:

AC^2 = AB^2 + BC^2

16k^2 = AB^2 + 9k^2

AB = √7

Find sec θ and cot θ using their definitions:

sec θ = hypotenuse/base = 4/√7

cotθ = base/perpendicular = √7/3

Now,
LHS =

$$\sqrt{\frac{cosec^2\theta - cot^2\theta}{sec^2\theta - 1}}$$

=

$$\sqrt{\frac{(\frac{4}{3})^2 - (\frac{\sqrt{7}}{3})^2}{(\frac{4}{\sqrt{7}})^2 - 1}}$$

=

$$\sqrt{\frac{\frac{16/9 - 7/9}{12}}{16/7 - 1}}$$

$$= \frac{\sqrt{7}}{3}$$

= RHS

Question 16:

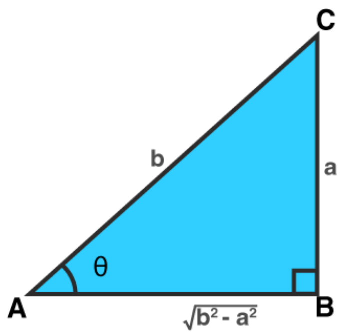
If sinθ=a/b, show that

$$sec\ \theta + tan\ \theta = \sqrt{\frac{b+a}{b-a}}$$

Solution:

Given: sinθ = a/b

or sinθ= ak/bk



Where k is any positive.

By Pythagoras theorem:

AC^2 = AB^2 + BC^2

AB^2 = AC^2 - BC^2

= b^2 - a^2

$$AB = \sqrt{b^2 - a^2}$$

Find sec θ and tan θ using their definitions:

sec θ = hypotenuse/base = b/√(b^2 - a^2)

tan θ = perpendicular/base = a/√(b^2 - a^2)

Now,

LHS = sec θ + tan θ

$$= \frac{b}{\sqrt{b^2 - a^2}} + \frac{a}{\sqrt{b^2 - a^2}}$$

$$= \frac{b + a}{\sqrt{b^2 - a^2}}$$

$$= \frac{b + a}{\sqrt{(b + a)(b - a)}}$$

$$= \frac{\sqrt{b + a} \times \sqrt{b + a}}{\sqrt{(b + a)} \times \sqrt{b - a}}$$

$$= \sqrt{\frac{b + a}{b - a}}$$

= RHS

Question 17:

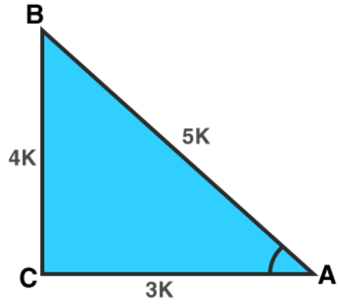
If cosθ=3/5, show that

$$\frac{(sin\theta - cot\theta)}{2tan\theta} = \frac{3}{160}$$

Solution:

Given: cosθ=3/5

cosθ = (3k)/(5k) = AC/AB



Where k is any positive.

By Pythagoras theorem:

AB^2 = BC^2 + AC^2

BC = 4k

Find sin θ, tan θ and cot θ using their definitions:

sin θ = perpendicular/ hypotenuse = 4/5

tan θ = perpendicular/base = 4/3

cot θ = 1/tan θ = 3/4

Now,
LHS =

$$\frac{(sin\theta - cot\theta)}{2tan\theta}$$

=

$$\frac{4/5 - 3/4}{2(4/3)}$$

= 3/160

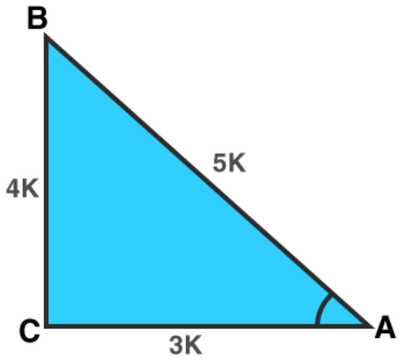
= RHS

Question 18:
If tan θ = 4/3, show that (sin θ + cos θ) = 7/5.

Solution:

Given: tan θ = 4/3

tanθ = (4k)/(3k) = BC/AC



Where k is any positive.

By Pythagoras theorem:

AB^2 = BC^2 + AC^2

AB^2 = 16k^2 + 9k^2

AB = 5k

Find sin θ and cos θ using their definitions:

sin θ = perpendicular/ hypotenuse = 4/5

cos θ = base/hypotenuse = 3/5

Now,
LHS = sin θ + cos θ

= 4/5 + 3/5

= 7/5

= RHS

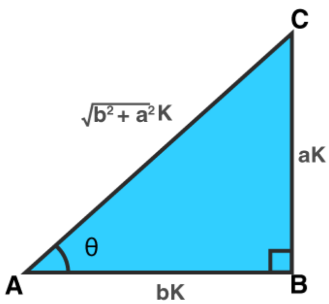
Question 19:
If tan θ = a/b, show that

$$\left(\frac{asin\theta - bcos\theta}{asin\theta + bcos\theta}\right) = \frac{(a^2 - b^2)}{(a^2 + b^2)}$$

Solution:

Given: tan θ = a/b

tanθ = (ak)/(bk) = BC/AB



Where k is any positive.

By Pythagoras theorem:

AC^2 = AB^2 + BC^2

$AC = \sqrt{a^2 + b^2}\ k$

Find sin θ and cos θ using their definitions:

sin θ = perpendicular/ hypotenuse = a/(√(a^2+b^2)

cos θ = base/hypotenuse = b/(√(a^2+b^2)

Now,

LHS:

$$\left(\frac{asin\theta - bcos\theta}{asin\theta + bcos\theta}\right)$$

$$= \frac{a.\frac{a}{\sqrt{a^2+b^2}} - b.\frac{b}{\sqrt{a^2+b^2}}}{a.\frac{a}{\sqrt{a^2+b^2}} + b.\frac{b}{\sqrt{a^2+b^2}}}$$

$$= \frac{\frac{a^2-b^2}{\sqrt{a^2+b^2}}}{\frac{a^2+b^2}{\sqrt{a^2+b^2}}}$$

$$= \frac{(a^2-b^2)}{(a^2+b^2)}$$

= RHS

Question 20:

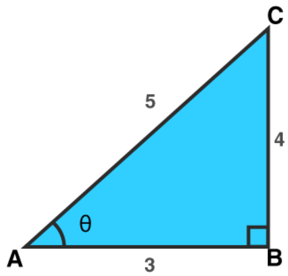
If 3 tan θ = 4, show that

$$\left(\frac{4cos\theta - sin\theta}{2cos\theta + sin\theta}\right) = \frac{4}{5}$$

Solution:

Given: 3 tan θ = 4

or tan θ = 4/3



Where k is any positive.

By Pythagoras theorem:

AC^2 = AB^2 + BC^2

AC = 5

Find sin θ and cos θ using their definitions:

sin θ = perpendicular/ hypotenuse = 4/5

cos θ = base/hypotenuse = 3/5

LHS:

$$\left(\frac{4cos\theta - sin\theta}{2cos\theta + sin\theta}\right)$$

$$= \frac{4 \times \frac{3}{5} - \frac{4}{5}}{2 \times \frac{3}{5} + \frac{4}{5}}$$

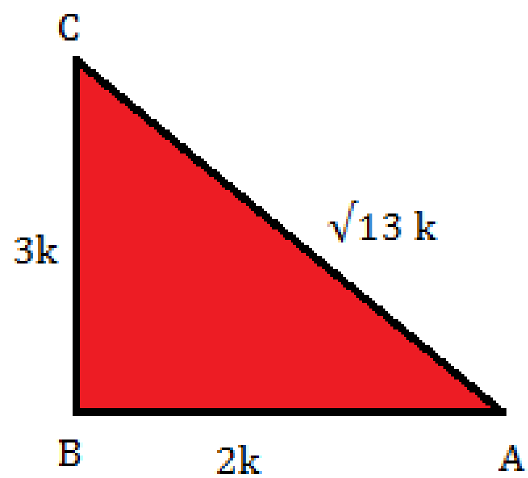
=(8/5)/(10/5)

= 4/5

= RHS

Question 21:
If 3 cot θ = 2, show that

$$\left(\frac{4sin\theta - 3cos\theta}{2sin\theta + 6cos\theta}\right) = \frac{1}{3}$$



Solution:

Given: 3 cot θ = 2

or cot θ = 2/3
Where k is any positive.

By Pythagoras theorem:

AC^2 = AB^2 + BC^2

AC = √13 k

Find sin θ and cos θ using their definitions:

sin θ = perpendicular/ hypotenuse = 3/√13

cos θ = base/hypotenuse =2/√13

Now,

LHS:

$$\left(\frac{4sin\theta - 3cos\theta}{2sin\theta + 6cos\theta}\right)$$

$$=\frac{4\times\frac{3}{\sqrt{13}}-3\times\frac{2}{\sqrt{13}}}{2\times\frac{3}{\sqrt{13}}+6\times\frac{2}{\sqrt{13}}}$$

$$=\frac{\frac{12-6}{\sqrt{13}}}{\frac{6+12}{\sqrt{13}}}$$

=1/3

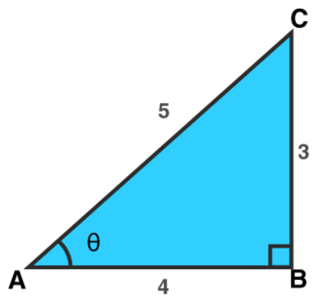
=RHS

Question 22:
If 3cotθ=4, show that (1-tan^2 θ)/(1+tan^2 θ) = (cos^2 θ – sin^2 θ).

Solution:

Given: 3cotθ=4

or cot θ = 4/3



Where k is any positive.

By Pythagoras theorem:

AC^2 = AB^2 + BC^2

= 16 + 9

AC = 5 k

Find sin θ and cos θ using their definitions:

sin θ = perpendicular/ hypotenuse = 3/5

cos θ = base/hypotenuse = 4/5

Now,

LHS = (1-tan^2 θ)/(1+tan^2 θ)

=

$$\frac{1-(3/4)^2}{1+(3/4)^2}$$

=

$$\frac{1-9/16}{1+9/16} = \frac{8}{16} \times \frac{16}{25}$$

= 8/25

Again,

RHS= (cos^2 θ – sin^2 θ)

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

= 16/12 – 9/25

= 8/25

Therefore, RHS = LHS

Hence Proved.

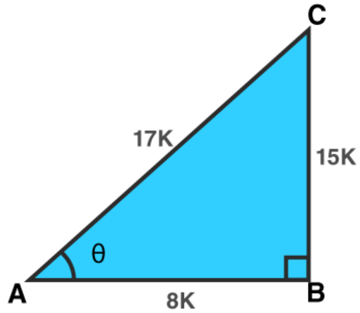
Question 23:
If sec θ = 17/8, verify that

$$\frac{3-4sin^2\theta}{4cos^2\theta-3}=\frac{3-tan^2\theta}{1-3tan^2\theta}$$

Solution:

Given: sec θ = 17/8

or sec θ = 17k/8k



Where k is any positive.

By Pythagoras theorem:

AC^2 = AB^2 + BC^2

BC^2 = AC^2 - AB^2

= 289k^2 - 64k^2

= 225 k^2

BC = 15 k

Find sin θ and cos θ using their definitions:

sin θ = perpendicular/ hypotenuse = 15/17
cos θ = base/hypotenuse = 15/8
Now,
LHS

$$\frac{3-4sin^2\theta}{4cos^2\theta-3}$$

$$=\frac{3-4\times\left(\frac{15}{17}\right)^2}{4\times\left(\frac{8}{17}\right)^2-3}=\frac{3-\frac{4\times225}{289}}{4\times\frac{64}{289}-3}$$

= (867-900) / (256-867)

= 33/611

RHS:

$$\frac{3-tan^2\theta}{1-3tan^2\theta}$$

$$=\frac{3-\frac{225}{64}}{1-3\times\frac{225}{64}}=\frac{\frac{3\times64-225}{64}}{\frac{64-3\times225}{64}}$$

= (192-225) /(64-675)

= 33/611

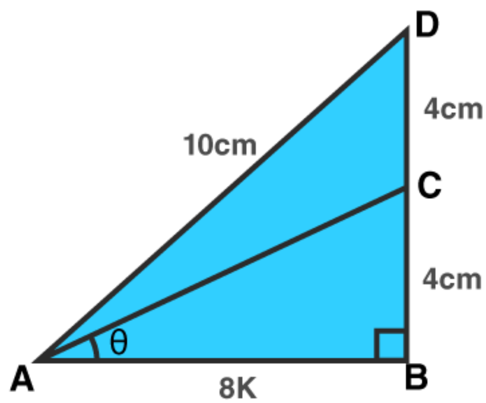
LHS = RHS

Verified.

Question 24:
In the adjoining figure, ∠B=90°, ∠BAC=θ°, BC=CD=4 cm and AD=10 cm. Find (i) sinθ and (ii) cosθ.

Solution:

Draw a triangle using given instructions:



From figure: Δ ABC and Δ ABD are right angled triangles

where AD = 10cm BC = CD = 4cm

BD = BC + CD = 8cm

By Pythagoras theorem:

AD^2 = BD^2 + AB^2

(10)^2 = (8)^2 + AB^2

100 = 64 + AB^2

AB^2 = 36 = (6)2

or AB = 6cm

Again,

AC^2 = BC^2 + AB^2

AC^2 = (4)^2 + (6)^2

AC^2 = 16 + 36 = 52

or AC = √52 = 2√13 cm

(i) Find sin θ

sin θ = BC/AC = 4/2√13 = 2√13/13

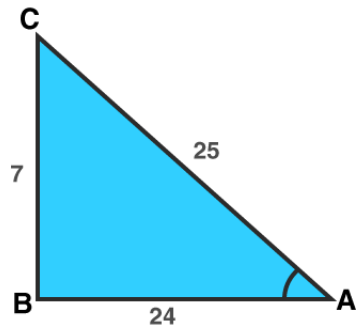
(ii) Find cos θ

cosθ = AB/AC = 6/2√13 = 3/√13 = 3√13/13

Question 25:
In a ∆ABC, it is given that ∠B = 90°, AB = 24 cm and BC = 7 cm.
Find the value of
(i) sin A
(ii) cos A
(iii) sin C
(iv) cos C

Solution:

Draw a triangle using given instructions:



From figure: Δ ABC is a right angled triangle

By Pythagoras theorem:

AC^2 = BC^2 + AB^2

AC = 25

(i) Find sin A

sin A = BC/AC = 7/25

(ii) Find cos A

cos A = AB/AC = 24/25

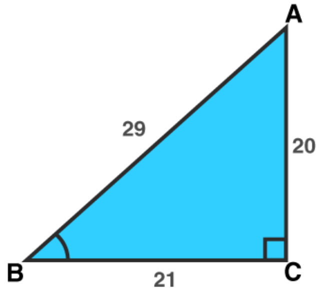
(iii) sin C = AB/AC = 24/25

(iv) cosC = BC/AC = 7/25

Question 26:
In a ΔABC, in which ∠C = 90°, ∠ABC = θ°, BC = 21 units, AB = 29 units. Show that (cos^2θ – sin^2 θ) = 41/841.

Solution:

Draw a triangle using given instructions:



From figure: Δ ABC is a right angled triangle

AB^2 = AC^2 + BC^2

(29)^2 = AC^2 + (21)^2

841 = AC^2 + 441

AC^2 = 400

or AC = 20

Find sin θ and cos θ:

sinθ = AC/AB = 20/29

cosθ = BC/AB = 21/29

Now:
LHS = cos^2θ – sin^2θ

= (21/29)^2 - (20/29)^2

= 41/841

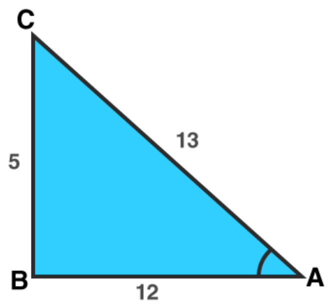
= RHS

Hence proved.

Question 27:

In a ∆ABC, angle B = 90 degrees, AB = 12 cm and BC = 5 cm.
Find (i) cos A (ii) cosec A (iii) cos C (iv) coses C

Solution:



From figure: ∆ ABC is a right angled triangle

AC^2 = BC^2 + AB^2

AC^2 = (5)^2 + (12)^2

AC^2 = 25 + 144

AC^2 = 169 = (13)^2

or AC = 13

Now from figure,

i. cos A = AB/AC = 12/13

ii. cosec A = AC/BC = 13/5

iii. cos C = BC/AC = 5/13

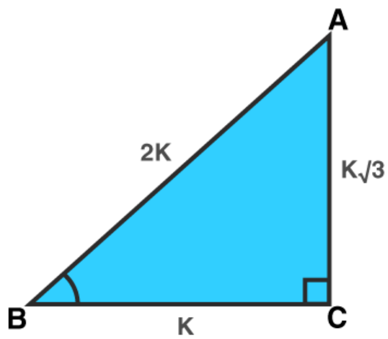
iv. cosec C = AC/AB = 13/12

Hence proved.

Question 28:
If sinα=1/2, prove that (3cosα - 4cos^3α)=0.

Solution:

Given: sinα = k/(2k) = BC/AB



Where k is any positive.

By Pythagoras theorem:

AB^2 = BC^2 + AC^2

AC^2 = AB^2 - BC^2

= (2k)^2 - (k)^2

= 3k^2

or AC= k√3

Find cos α:

cos α = base/hypotenuse = √3/2

Now,

LHS = 3cosα - 4cos^3 α

=

$$3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3$$

$$= 3\left(\frac{\sqrt{3}}{2}\right) - 3\left(\frac{\sqrt{3}}{2}\right)$$

= 0

= RHS

Hence proved.

Question 29:

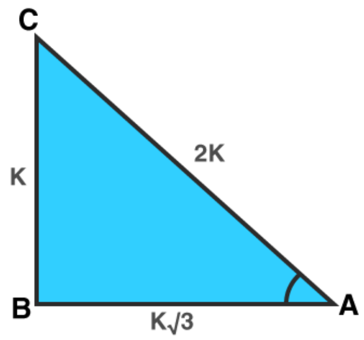
In a △ABC, ∠B = 90° and tan A =1/√3. Prove that

(i) sinA·cosC + cosA·sinC = 1

(ii) cosA·cosC - sinA·sinC = 0

Solution:

Given: tan A = BC/AB = k/(k√3)



Where k is any positive.

By Pythagoras theorem:

AC^2 = BC^2 + AB^2

= (k)^2 - (√3 k)^2

= k^2 + 3k^2

= 4k^2

or AC= 2k

Find sin A, cos A, sin C and cos C

sin A = BC/AC = k/(2k) = 1/2

cos A = AB/AC = (k√3)/(2k) = √3/2

sin C = AB/AC = (k√3)/(2k) = √3/2

cos C = BC/AC = k/(2k) = 1/2

(i) sinA cosC + cosA sinC = 1

LHS = sinA cosC + cosA sinC = (1/2)(1/2) + (√3/2)(√3/2)

= 1/4 + 3/4

= 4/4
= 1
RHS = LHS

(ii) cosA cosC - sinA sinC = 0

LHS = cosA cosC - sinA sinC

= (1/2)(√3/2) - (1/2)(√3/2)

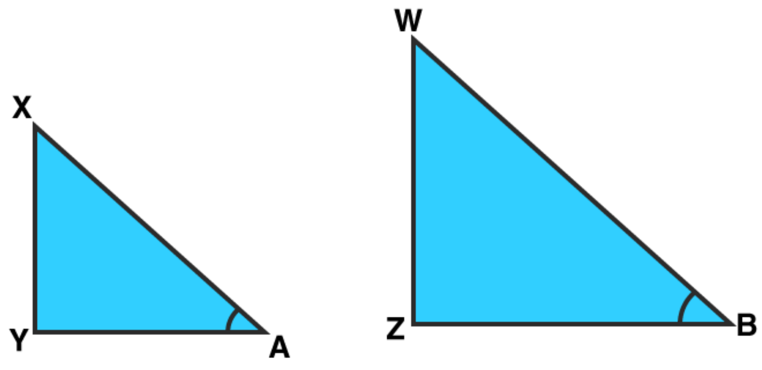
= (√3/4) - (√3/4)

= 0

=RHS

Question 30:
If ∠A and ∠B are acute angles such that sin A = sin B, then prove that ∠A = ∠B.

Solution:
Consider two right triangles XAY and WBZ such that sin A = sin B



To Prove: ∠A = ∠B

From figures:

sin A = XY/XA and sin B = WZ / WB

XY/XA = WZ / WB = k (say)

or XY/WZ = XA/ WB ...(1)

sin A = sin B (Given)

We have,

XY = WZ k and XA = WB k ...(2)

By Pythagoras: Apply on both the triangles

WB^2 = WZ^2 + BZ^2

BZ^2 = WB^2 - WZ^2

And,

XA^2 = XY^2 + AY^2

AY^2 = XA^2 - XY^2

Find: AY/BZ

$$\frac{AY}{BZ} = \frac{\sqrt{XA^2 - XY^2}}{\sqrt{WB^2 - WZ^2}} = \frac{\sqrt{k^2 WB^2 - k^2 WZ^2}}{\sqrt{WB^2 - WZ^2}} = \frac{k\sqrt{WB^2 - WZ^2}}{\sqrt{WB^2 - WZ^2}}$$

AY/BZ = k ...(3)

From equations (1), (2) and (3), we get

$$\frac{XY}{WZ} = \frac{XA}{WB} = \frac{AY}{BZ}$$
$$\Rightarrow \Delta XYA \sim \Delta WZB$$
$$\Rightarrow \angle A = \angle B$$

Question 31:
If ∠A and ∠B are acute angles such that tan A = tan B, the prove that ∠A=∠B.

Solution:
Consider ΔABC to be a right angled triangle.
angle C = 90 degree

tan A = BC/AC and

tan B = AC/BC

Given: tanA = tanB

So, BC/AC = AC/BC
BC^2 = AC^2
BC = AC
Which implies, ∠ A = ∠ B (using triangle opposite and equal angles property)

Question 32:
In a right ΔABC , right-angled at B, if tan A = 1 , then verify that 2sinA·cosA=1.

Solution:
Consider ΔABC to be a right angled triangle at B.
angle C = 90 degree

Given: tan A = 1 ...(1)

tan A = 1 = BC/AB

AB = BC

Again, tan A = sin A/cos A

sin A = cos A ...using (1)

By Pythagoras theorem:

AC^2 = BC^2 + AB^2

$AC^2 = 2BC^2$

$(AC/BC)^2 = 2$

Or $AC/BC = \sqrt{2}$

$\text{cosec}A = \sqrt{2}$

or $\sin A = 1/\sqrt{2}$
and $\cos A = 1/\sqrt{2}$

Now,

$2 \sin A \cos A = 2(1/\sqrt{2})(1/\sqrt{2})$

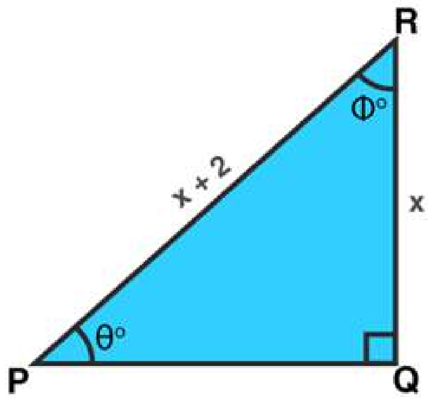
$= 2(1/2)$

$= 1$

= RHS

Question 33:
In the figure of $\triangle PQR$, $\angle P=\theta°$ and $\angle R=\phi°$.
Find
(i) $\sqrt{(x+1)} \cot \phi$
(ii) $\sqrt{(x^3 + x^2)} \tan \theta$
(iii) $\cos \theta$

Solution:



Δ PQR is a right angled triangle.

By Pythagoras theorem:

$PR^2 = RQ^2 + PQ^2$

$(x + 2)^2 = x^2 + PQ^2$

$PQ^2 = 4 + 4x$

or $PQ = 2\sqrt{(x+1)}$

Now,

cot ϕ = QR/PQ = x/2(√(x+1)

tan θ = QR/PQ = x/2(√(x+1)

(i)

√(x+1) cot ϕ = √(x+1) {x/2(√(x+1)} = x/2

(ii)

√(x^3 + x^ 2) tan θ = √(x^3 + x^ 2) {x/2(√(x+1)} = x^2/2

(iii) cos θ = PQ/PR = 2(√(x+1) / (x+2)

Question 34:
If x = cosecA+cosA and y=cosecA-cosA , then prove that

$$\left(\frac{2}{x+y}\right)^2+\left(\frac{x-y}{2}\right)^2-1=0.$$

Solution:

x = cosecA+cosA and y=cosecA-cosA

x + y = cosecA + cosA + cosecA-cosA = 2cosecA

x - y = cosecA + cosA - cosecA + cosA = 2cos A

LHS:

$$\left(\frac{2}{x+y}\right)^2+\left(\frac{x-y}{2}\right)^2-1$$

$$=\left(\frac{2}{2\ \text{cosecA}}\right)^2+\left(\frac{2\ \text{cosA}}{2}\right)^2-1$$

= sin^2 + cos^2 A – 1

(Using trig property: sin^2 + cos^2 A = 1)

= 1 – 1

= 0

Question 35:
If x = cot A + cos A and y = cot A – cos A, prove that

$$\left(\frac{x-y}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2 = 1.$$

Solution:

x = cot A + cos A and y = cot A – cos A

x + y = cotA + cosA + cotA - cosA = 2cotA

x - y = cotA + cosA - (cotA - cosA) = 2cosA

LHS:

$$\left(\frac{x-y}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2$$

$$= \left(\frac{2\cos A}{2}\right)^2 + \left(\frac{2\cos A}{2\cot A}\right)^2$$

= cos^2 A + sin^2 A
(Using trig property: sin^2 + cos^2 A = 1)

= 1

= RHS

Hence proved.