

Exercise 5A

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Question 1:

Show that each of the progressions given below is an AP. Find the first term, common difference and next term of each.

(i) 9, 15, 21, 27,

(ii) 11, 6, 1, -4,

(iii) -1, $-\frac{5}{6}$, $-\frac{2}{3}$, $-\frac{1}{2}$,

(iv) $\sqrt{2}$, $\sqrt{8}$, $\sqrt{18}$, $\sqrt{32}$,

(v) $\sqrt{20}$, $\sqrt{45}$, $\sqrt{80}$, $\sqrt{125}$,

Solution:

(i) 9, 15, 21, 27, ...

Here, $15 - 9 = 21 - 15 = 27 - 21 = 6$ (which is constant)

Common difference is 6

Or $d = 6$

Next term = $27 + d = 27 + 6 = 33$

(ii) 11, 6, 1, -4, ...

Here, $6 - 11 = 1 - 6 = -5$, $-4 - 1 = -5$ (which is constant)

d (common difference) = -5

Next term = $-4 - 5 = -9$

(iii) -1, $-\frac{5}{6}$, $-\frac{2}{3}$, $-\frac{1}{2}$,

$-\frac{5}{6} - (-1) = \frac{1}{6}$ and

$-\frac{2}{3} - (-\frac{5}{6}) = \frac{1}{6}$

d (common difference) = $\frac{1}{6}$

Next term = $-\frac{1}{2} + \frac{1}{6} = -\frac{1}{3}$

(iv) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$

$$\sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$$

$$\sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$$

d (common difference) = $\sqrt{2}$

Next term = $\sqrt{32} + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$

(v) $\sqrt{20}, \sqrt{45}, \sqrt{80}, \sqrt{125}, \dots$

$$\sqrt{20}, \sqrt{45}, \sqrt{80}, \sqrt{125}, \dots$$

$$\sqrt{45} - \sqrt{20} = 3\sqrt{5} - 2\sqrt{5} = \sqrt{5}$$

$$\sqrt{125} - \sqrt{80} = 5\sqrt{5} - 4\sqrt{5} = \sqrt{5}$$

d (common difference) = $\sqrt{5}$

Next term = $\sqrt{125} + \sqrt{5} = 5\sqrt{5} + \sqrt{5} = 6\sqrt{5}$ or $\sqrt{180}$

Question 2:

Find:

- (i) the 20th term of the AP 9, 13, 17, 21,
- (ii) the 35th term of the AP 20, 17, 14, 11,
- (iii) the 18th term of the AP $\sqrt{2}, \sqrt{18}, \sqrt{50}, \sqrt{98}, \dots$
- (iv) the 9th term of the AP $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \dots$
- (v) the 15th term of the AP -40, -15, 10, 35,

Solution:

(i) Given: AP is 9, 13, 17, 21,

Here, first term = $a = 9$

Common difference = $d = 13 - 9 = 4$

$$a_n = a + (n-1)d$$

$$a_{20} = 9 + (20-1)4$$

$$= 85$$

(ii) the 35th term of the AP 20, 17, 14, 11,

Given: AP is 20, 17, 14, 11,

Here, first term = $a = 20$

Common difference = $d = 17 - 20 = -3$

$$n = 35$$

$$a_n = a + (n-1)d$$

$$a_{35} = 20 + (35-1)(-3) =$$

$$-82$$

(iii) the 18th term of the AP $\sqrt{2}, \sqrt{18}, \sqrt{50}, \sqrt{98}, \dots$

Given: AP is $\sqrt{2}, \sqrt{18}, \sqrt{50}, \sqrt{98}, \dots$

or $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, \dots$

Here, first term = $a = \sqrt{2}$

Common difference = $d = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

$$n = 18$$

$$a_n = a + (n-1)d$$

$$a_{18} = \sqrt{2} + 34\sqrt{2} =$$

$$35\sqrt{2}$$

(iv) the 9th term of the AP $3/4, 5/4, 7/4, 9/4, \dots$

Given: AP is $3/4, 5/4, 7/4, 9/4, \dots$

Here, first term = $a = 3/4$

Common difference = $d = 5/4 - 3/4 = 1/2$

$n = 9$

$$a_n = a + (n-1)d$$

$$a_9 = 3/4 + (9 - 1)1/2$$

$$= 19/4$$

(v) the 15th term of the AP $-40, -15, 10, 35, \dots$

Given: AP is $-40, -15, 10, 35, \dots$

Here, first term = $a = -40$

Common difference = $d = -15 - (-40) = -15 + 40 = 25$

$n = 15$

$$a_n = a + (n-1)d$$

$$a_{15} = -40 + (15 - 1)25 =$$

$$310$$

Question 3:

(i) Find the 37th term of the AP $6, 7\frac{3}{4}, 9\frac{1}{2}, 11\frac{1}{4}, \dots$

(ii) Find the 25th term of the AP $5, 4\frac{1}{2}, 4, 3\frac{1}{2}, 3, \dots$

Solution:

(i) Given AP is

$$6, 7\frac{3}{4}, 9\frac{1}{2}, 11\frac{1}{4}, \dots$$

Here, first term = $a = 6$

Common difference:

$$d = 7\frac{3}{4} - 6 = 1\frac{3}{4} = \frac{7}{4}$$

Here, first term = $a = 6$

Common difference = $d = -15 - (-40) = -15 + 40 = 25$

$n = 37$

$$a_n = a + (n-1)d$$

$$a_{37} = 6 + (37-1)(7/4)$$

$$= 69$$

(ii) Given AP is $5, 4\frac{1}{2}, 4, 3\frac{1}{2}, 3, \dots$

Given AP is

$$5, 4\frac{1}{2}, 4, 3\frac{1}{2}, 3, \dots$$

Here, first term = $a = 5$

Common difference:

$$d = 4\frac{1}{2} - 5 = \frac{-1}{2}$$

$n = 25$

$$a_n = a + (n-1)d$$

$$a_{25} = 5 + (25-1)(-1/2)$$

$$= -7$$

Question 4:

Find the value of p for which the numbers $2p - 1$, $3p + 1$, 11 are in AP. Hence, find the numbers.

Solution:

If $2p - 1$, $3p + 1$, 11 are terms in AP, then

$$a_2 - a_1 = a_3 - a_2 \dots(1)$$

From given:

$$a_1 = 2p - 1$$

$$a_2 = 3p + 1$$

$$a_3 = 11$$

From (1), we get

$$(3p + 1) - (2p - 1) = 11 - (3p + 1)$$

$$3p + 1 - 2p + 1 = 11 - 3p - 1$$

$$p + 2 = 10 - 3p$$

$$4p = 8$$

$$p = 2$$

For $p = 2$, these terms are in AP.

Question 5:

Find the nth term of each of the following APs:

(i) 5, 11, 17, 23,

(ii) 16, 9, 2, -5,

Solution:

(i) AP is 5, 11, 17, 23,

Here, first term = $a = 5$

Common difference = $d = 11 - 5 = 6$

Now,

$$a_n = a + (n - 1)d$$

$$= 5 + (n - 1) 6$$

$$= 5 + 6n - 6$$

$$= (6n - 1)$$

(ii) AP is 16, 9, 2, -5,

Here, first term = $a = 16$

Common difference = $d = 9 - 16 = -7$

$$a_n = a + (n - 1)d$$

$$= 16 + (n - 1) (-7)$$

$$= 16 - 7n + 7$$

$$= (23 - 7n)$$

Question 6:

If the n th term of a progression is $(4n - 10)$ show that it is an AP. Find its

(i) first term,

(ii) common difference, and

(iii) 16th term.

Solution:

n^{th} term of AP is $4n - 10$ (Given)

Putting $n = 1, 2, 3, 4, \dots$, we get

$$\text{At } n = 1: 4n - 10 = 4 \times 1 - 10 = 4 - 10 = -6$$

$$\text{At } n = 2: 4n - 10 = 4 \times 2 - 10 = 8 - 10 = -2$$

$$\text{At } n = 3: 4n - 10 = 4 \times 3 - 10 = 12 - 10 = 2$$

$$\text{At } n = 4: 4n - 10 = 4 \times 4 - 10 = 16 - 10 = 6$$

We see that $-6, -2, 2, 6, \dots$ are in AP

(i) first term = -6

(ii) Common difference = $-2 - (-6) = 4$

(iii) 16th term:

Using formula: $a_n = a + (n - 1)d$

Here $n = 16$

$$a_{16} = -6 + (16 - 1)4 = 54$$

Question 7:

How many terms are there in the AP $6, 10, 14, 18, \dots, 174$?

Solution:

Given: AP is $6, 10, 14, 18, \dots, 174$

Here, first term = $a = 6$

Common difference = $d = 10 - 6 = 4$

To find: the number of terms (n)

$$\text{Last term} = a + (n - 1)d$$

$$174 = 6 + (n - 1)4$$

$$174 - 6 = (n - 1)4$$

$$n - 1 = 168 / 4 = 42$$

$$n = 42 + 1 = 43$$

There are 43 terms.

Question 8:

How many terms are there in the AP 41, 38, 35, ..., 8?

Solution:

Given: AP is 41, 38, 35, ..., 8

Here, first term = $a = 41$

Last term = 8

Common difference = $d = 38 - 41 = -3$

To find: the number of terms (n)

Last term = $a + (n - 1)d$

$$8 = 41 + (n - 1)(-3)$$

$$8 - 41 = (n - 1)(-3)$$

$$n - 1 = 11$$

$$n = 11 + 1 = 12$$

There are 12 terms.

Question 9:

How many terms are there in the AP is 18, $15 \frac{1}{2}$, 13, ..., -47 ?

Solution:

Given: AP is 18, $15 \frac{1}{2}$, 13, ..., -47

Here, first term = $a = 18$

Last term = -47

Common difference = $d = -5/2$

To find: the number of terms (n)

Last term = $a + (n - 1)d$

$$-47 = 18 + (n - 1)(-5/2)$$

$$-47 - 18 = (n - 1)(-5/2)$$

$$n = 27$$

There are 27 terms.

Question 10:

Which term of the AP 3, 8, 13, 18, ... is 88?

Solution:

Let nth term is 88.

AP is 3, 8, 13, 18, ...

Here,

First term = $a = 3$

Common difference = $d = 8 - 3 = 5$

nth term of AP is $a_n = a + (n - 1)d$

Now,

$$88 = 3 + (n - 1)(5)$$

$$88 - 3 = (n - 1) \times 5$$

$$n - 1 = 88 / 5$$

$$\text{or } n = 17 + 1 = 18$$

Therefore: 88 is the 18th term.

Question 11:

Which term of the AP 72, 68, 64, 60, is 0?

Solution:

AP is 72, 68, 64, 60,

Let nth term is 0.

Here,

$$\text{First term} = a = 72$$

$$\text{Common difference} = d = 68 - 72 = -4$$

$$a_n = a + (n - 1)d$$

$$0 = 72 + (n - 1)(-4)$$

$$-72 = -4(n - 1)$$

$$n - 1 = 18$$

$$n = 18 + 1 = 19$$

Therefore: 0 is the 19th term.

Question 12:

Which term of the AP $5/6, 1, 1\frac{1}{6}, 1\frac{1}{3}, \dots$ is 3?

Solution:

$$\frac{5}{6}, 1, 1\frac{1}{6}, 1\frac{1}{3}, \dots$$

Here,

$$\text{First term} = a = 5/6$$

$$\text{Common difference} = d = 1 - 5/6 = 1/6$$

$$\text{Now: } a_n = a + (n - 1)d$$

$$3 = 5/6 + (n - 1)1/6$$

Let nth term is 3

$$\text{Now, } a_n = a + (n-1)d$$

$$3 = 5/6 + (n-1)1/6$$

$$n - 1 = 13$$

$$n = 13 + 1 = 14$$

Therefore, 3 is the 14th term.

Question 13:

Which term of the AP 21, 18, 15,... is -81?

Solution:

AP is 21, 18, 15,

Let nth term -81

Here, $a = 21$, $d = 18 - 21 = -3$

$$a_n = a + (n - 1)d$$

$$-81 = 21 + (n - 1)(-3)$$

$$-81 - 21 = (n - 1)(-3)$$

$$-102 = (n - 1)(-3)$$

$$n = 34 + 1 = 35$$

Therefore, -81 is the 35th term

Question 14:

Which term of the AP 3, 8, 13, 18, ... will be 55 more than its 20th term?

Solution:

Given AP is 3, 8, 13, 18,...

First term = $a = 3$

Common difference = $d = 8 - 3 = 5$

And $n = 20$ and a_{20} be the 20th term, then

$$a_{20} = a + (n - 1)d$$

$$= 3 + (20 - 1) 5$$

$$= 3 + 95$$

$$= 98$$

The required term = $98 + 55 = 153$

Now, 153 be the n th term, then

$$a_n = a + (n - 1)d$$

$$153 = 3 + (n - 1) \times 5$$

$$153 - 3 = 5(n - 1)$$

$$150 = 5(n - 1)$$

$$n - 1 = 30$$

$$n = 31$$

Required term will be 31st term.

Question 15:

Which term of the AP 5, 15, 25,... will be 130 more than its 31st term?

Solution:

AP is 5, 15, 25,...

First term = $a = 5$

Common difference = $d = 15 - 5 = 10$

Find 31st term:

$$a_{31} = a + (n - 1)d$$

$$= 5 + (31 - 1)10$$

$$= 5 + 30 \times 10$$

$$= 305$$

Required term = $305 + 130 = 435$

Now, say 435 be the n th term, then

$$a_n = a + (n - 1)d$$

$$435 = 5 + (n - 1)10$$

$$435 - 5 = (n - 1)10$$

$$n - 1 = 43$$

$$n = 44$$

The required term will be 44th term.

Question 16:

If the 10th term of an AP is 52 and 17th term is 20 more than its 13th term, find the AP

Solution:

Let a be the first term and d be the common difference, then

$$T_{10} = a + (n - 1)d = a + 9d = 52$$

$$T_{17} = a + 16d \text{ and } T_{13} = a + 12d$$

$$\therefore T_{17} - T_{13} = 20$$

$$(a + 16d) - (a + 12d) = 20$$

$$a + 16d - a - 12d = 20$$

$$\Rightarrow 4d = 20$$

$$d = \frac{20}{4} = 5$$

$$52 = a + 9d = a + 9 \times 5$$

$$52 = a + 45$$

$$\therefore a = 52 - 45 = 7$$

Now, AP will be 7, 12, 17, 22, ...

Question 17:

Find the middle term of the AP 6, 13, 20, ..., 216

Solution:

AP is 6, 13, 20, ..., 216

$a = 6$, $d = 7$ and $l = 216$

Let $T_n = l = a + (n - 1)d$

$$216 - 6 = (n - 1)7 \Rightarrow \frac{210}{7} = n - 1$$

$$n - 1 = 30 \Rightarrow n - 30 + 1 = 31$$

This AP has 31 terms

$$\text{Middle term} = \frac{31+1}{2} = 16\text{th term}$$

$$a_{16} = 6 + (16 - 1)7 = 6 + 105 = 111$$

Therefore, midterm of the AP is 111.

Question 18:

Find the middle term of the AP 10, 7, 4, (-62)

Solution:

AP is 10, 7, 4,, (-62)

$a = 10$,

$$d = 7 - 10 = -3,$$

and

$$l = -62$$

$$\text{Now, } a_n = l = a + (n - 1)d$$

$$-62 = 10 + (n - 1) \times (-3)$$

$$-62 - 10 = -3(n - 1)$$

$$-72 = -3(n - 1)$$

$$\text{Or } n = 24 + 1 = 25$$

Middle term = $(25 + 1) / 2$ th = 13th term

Find the 13th term using formula, we get

$$a_{13} = 10 + (13 - 1)(-3) = 10 - 36 = -26$$

Question 19:

Find the sum of two middle most terms of the AP $-4/3, -1, -2/3, \dots, 4 1/3$.

Solution:

Given AP is $-4/3, -1, -2/3, \dots, 13/3$

$$\text{Here, } a = -\frac{4}{3}, d = -1 - \left(-\frac{4}{3}\right) = -1 + \frac{4}{3}$$

$$\frac{1}{3}, l = 4\frac{1}{3} = \frac{13}{3}$$

$$T_n = l = \frac{13}{3} = a + (n - 1)d$$

$$\frac{13}{3} = \frac{-4}{3} + (n - 1)\frac{1}{3}$$

$$\frac{13}{3} + \frac{4}{3} = (n - 1)\frac{1}{3}$$

$$\frac{17}{3} = (n-1)\frac{1}{3} \Rightarrow n-1 = \frac{17}{3} \times \frac{3}{1} = 17$$

$$n = 17 + 1 = 18$$

$\frac{13}{3}$ is the 18th term

Middle terms will be: $(18/2)$ th + $(18/2 + 1)$ th
 $= 9$ th + 10 th term

Now,

$$a_9 + a_{10} = a + 8d + a + 9d$$

$$= 2a + 17d$$

$$= 2(-4/3) + 17(1/3)$$

$$= 3$$

Question 20:

Find the 8th term from the end of the AP 7, 10, 13,, 184.

Solution:

Given: AP is 7, 10, 13, ..., 184

$a = 7$, $d = 10 - 7 = 3$ and $l = 184$

n^{th} term from the end $= l - (n - 1)d$

Now,

8th term from the end be

$$184 - (8 - 1)3 = 184 - 21 = 163$$

Question 21:

Find the 6th term from the end of the AP 17, 14, 11, ..., (-40).

Solution:

Given: AP is 17, 14, 11, ..., (-40)

$a = 17$, $d = 14 - 17 = -3$, $l = -40$

6th term from the end $= l - (n - 1)d$

$$= -40 - (6 - 1)(-3)$$

$$= -40 - (5 \times (-3))$$

$$= -40 + 15$$

$$= -25$$

Question 22:**Is 184 a term of the AP 3, 7, 11, 15,?****Solution:**

Given AP is 3, 7, 11, 15, ...

$$a = 3, d = 7 - 3 = 4$$

Let 184 be the n th term of the AP

$$a_n = a + (n - 1)d$$

$$184 = 3 + (n - 1) \times 4$$

$$184 - 3 = (n - 1) \times 4$$

$$181 / 4 = n - 1$$

$$n = 181 / 4 + 1 = 185/4 \text{ (Which is in fraction)}$$

Therefore, 184 is not a term of the given AP.

Question 23:**Is -150 a term of the AP 11, 8, 5, 2,....?****Solution:**

Given AP is AP 11, 8, 5, 2,...

Here $a = 11, d = 8 - 11 = -3$

Let -150 be the n th term of the AP

$$a_n = a + (n - 1)d$$

$$-150 = 11 + (n - 1)(-3)$$

or $n = 164/3$

Which is a fraction.

Therefore, -150 is not a term of the given AP.

Question 24:**Which term of the AP 121, 117, 113,... is its first negative term?****Solution:**Let n th of the AP 121, 117, 113,... is negative. Let T_n be the n th term then

$$T_n < 0$$

Here, $a = 121, d = 117 - 121 = -4$

$$T_n = 121 + (n - 1)(-4)$$

$$121 - 4n + 4 < 0$$

$$125 - 4n < 0$$

$$125 < 4n \Rightarrow 4n > 125$$

$$n > \frac{125}{4} \Rightarrow n > 31 \frac{1}{4}$$

Therefore, 32nd term will be the 1st negative term.

Question 25:

Which term of the AP $20, 19 \frac{1}{4}, 18 \frac{1}{2}, 17 \frac{3}{4}, \dots$ is its first negative term?

Solution:

AP is $20, 19 \frac{1}{4}, 18 \frac{1}{2}, 17 \frac{3}{4}, \dots$

$$a = 20, d = -3/4$$

Let nth term be the 1st negative term of the AP

$$a_n < 0$$

$$a_n = a + (n - 1)d$$

$$20 + (n - 1)\left(\frac{-3}{4}\right) < 0$$

$$20 - \frac{3}{4}n + \frac{3}{4} < 0 \Rightarrow 20 \frac{3}{4} - \frac{3}{4}n < 0$$

$$20 \frac{3}{4} < \frac{3}{4}n \Rightarrow \frac{83}{4} \times \frac{4}{3} < n$$

$$\frac{83}{3} < n \Rightarrow n > \frac{83}{3} \Rightarrow n > 27 \frac{2}{3}$$

Therefore, 28th term will be the 1st negative term.

Question 26:

The 7th term of an AP is -4 and its 13th term is -16. Find the AP.

Solution:

Let us say a be the first term and d be the common difference of an AP

$$a_n = a + (n - 1)d$$

$$a_7 = a + (7 - 1)d$$

$$= a + 6d = -4 \dots\dots\dots(1)$$

$$\text{And } a_{13} = a + 12d = -16 \dots\dots\dots(2)$$

Subtracting equation (1) from (2), we get

$$6d = -16 - (-4) = -12$$

$$\text{From (1), } a + 6d = -4$$

$$a + (-12) = -4$$

$$a = -4 + 12 = 8$$

$$a = 8, d = -2$$

AP will be 8, 6, 4, 2, 0,

Question 27:

The 4th term of an AP is zero, Prove that its 25th term is triple its 11th term.

Solution:

Let a be the first term and d be the common difference of an AP.

$$a_4 = a + (n - 1)d$$

$$= a + (4 - 1)d$$

$$= a + 3d$$

Since 4th term of an AP is zero.

$$a + 3d = 0$$

$$\text{or } a = -3d \dots(1)$$

Similarly,

$$a_{25} = a + 24d = -3d + 24d = 21d \dots(2)$$

$$a_{11} = a + 10d = -3d + 10d = 7d \dots(3)$$

From (2) and (3), we have

$$a_{25} = 3 \times a_{11}$$

Hence proved.

Question 28:

If the sixth term of an AP is zero then show that its 33rd term is three times its 15th term.

Solution:

Sixth term of an AP is zero

that is $a_6 = 0$

$$a + 5d = 0$$

$$a = -5d$$

Now, $a_{15} = a + (n - 1)d$

$$a + (15 - 1)d = -5d + 14d = 9d$$

$$\text{and } a_{33} = a + (n - 1)d = a + (33 - 1)d = -5d + 32d = 27d$$

Now, $a_{33} : a_{15}$

$$27d : 9d$$

$$3 : 1$$

Which shows that $a_{33} = 3(a_{15})$

Hence proved.

Question 29:

The 4th term of an AP is 11. The sum of the 5th and 7th terms of this AP is 34. Find its common difference.

Solution:

Let a be the first term and d be the common difference of an AP.

$$a_n = a + (n - 1)d$$

$$a_4 = a + (4 - 1)d = a + 3d$$

$$a + 3d = 11 \dots\dots\dots(1)$$

$$\text{Now, } a_5 = a + 4d \text{ and } a_7 = a + 6d$$

$$\text{Now, } a_5 + a_7 = a + 4d + a + 6d = 2a + 10d$$

$$2a + 10d = 34$$

$$a + 5d = 17 \dots\dots\dots(2)$$

Subtracting (1) from (2), we get

$$2d = 17 - 11 = 6$$

$$d = 3$$

The common difference = 3

Question 30:

The 9th term of an AP is -32 and the sum of its 11th and 13th terms is -94. Find the common difference of the AP.

Solution:

Let a be the first term and d be the common difference of an AP.

$$\text{nth term} = a_n = a + (n - 1)d$$

Given: 9th term of an AP is -32 and the sum of its 11th and 13th terms is -94

Now,

$$a_9 = a + 8d = -32 \dots(1)$$

$$a_{11} = a + 10d$$

$$a_{13} = a + 12d$$

Sum of 11th and 13th terms:

$$a_{11} + a_{13} = a + 10d + a + 12d$$

$$-94 = 2a + 22d$$

$$\text{or } a + 11d = -47 \dots(2)$$

Subtracting (1) from (2), we have

$$3d = -47 + 32 = -15$$

$$\text{or } d = -5$$

Common difference is -5.