Exercise 5B

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Question 1:
Determine k so that (3k – 2), (4k – 6) and (k + 2) are three consecutive terms of an AP.
Solution:
Given: (3k – 2), (4k – 6) and (k + 2) are three consecutive terms of an AP.
So, (4k – 6) – (3k – 2) = (k + 2) – (4k – 6)
2(4k – 6) = (k + 2) + (3k – 2)
8k – 12 = 4k + 0
8k – 4k = 0 + 12
or k = 3

Question 2:
Find the value of x for which the numbers (5x + 2), (4x – 1) and (x + 2) are in AP.
Solution:
Given: (5x + 2), (4x – 1) and (x + 2) are terms in AP.
So, d = (4x – 1) – (5x + 2) = (x + 2) – (4x – 1)
2(4x – 1) = (x + 2) + (5x + 2)
8x – 2 = 6x + 2 + 2
8x – 2 = 6x + 4
8x – 6x = 4 + 2
or x = 3
The value of x is 3.

Question 3:
If (3y – 1), (3y + 5) and (5y + 1) are three consecutive terms of an AP then find the value of y.
Solution:
Given: (3y – 1), (3y + 5) and (5y + 1) are 3 consecutive terms of an AP.
So, (3y + 5) – (3y – 1) – (5y + 1) – (3y + 5)
2(3y + 5) = 5y + 1 + 3y – 1
6y + 10 = 8y
8y – 6y = 10
2y = 10
Or y = 5
The value of y is 5.

Question 4:
Find the value of x for which (x + 2), 2x, (2x + 3) are three consecutive terms of an AP.
Solution:
Given: (x + 2), 2x, (2x + 3) are three consecutive terms of an AP.
So, 2x – (x + 2) = (2x + 3) – 2x

$2x - x - 2 = 2x + 3 - 2x$
$x - 2 = 3$
$x = 2 + 3 = 5$
The value of x is 5.

Question 5:
Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in AP.
Solution:
Assume that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in AP.
So, $(a^2 + b^2) - (a - b)^2 = (a + b)^2 - (a^2 + b^2)$
$(a^2 + b^2) - (a^2 + b^2 - 2ab) = a^2 + b^2 + 2ab - a^2 - b^2$
$2ab = 2ab$
Which is true.
Hence given terms are in AP.

Question 6:
Find three numbers in AP whose sum is 15 and product is 80.
Solution:
Let a – d, a, a + d are three numbers in AP.

So, their sum = $a - d + a + a + d = 15$

$3a = 15$
or $a = 5$
Again,
Their Product = $(a - d) \times a \times (a + d) = 80$
$a(a^2 - d^2) = 80$
$5(5^2 - d^2) = 80$
$25 - d^2 = 16$
$d^2 = 25 - 16 = 9 = (\pm 3)^2$
or $d = \pm 3$
$d = 3$ or $d = -2$
We have 2 conditions here:
At a = 5, d = 3
Numbers are: 2, 5 and 8

At a = 5 and d = -3
Numbers are : 8, 5, 2

Question 7:
The sum of three numbers in AP is 3 and their product is -35. Find the numbers.

Solution:

Let a – d, a, a + d are three numbers in AP.

their sum = a – d + a + a + d = 3

3a = 3
or a = 1

Again,
Their Product = (a – d) x a x (a + d) = -35
$a(a^2 - d^2) = -35$
$(1^2 - d^2) = -35$
or d = ±6
d = 6 or d = -6
We have 2 conditions here:
At a = 1, d = 6
Numbers are: -5, 1 and 7

At a = 1 and d = -6
Numbers are : 7, 1, -5

Question 8:
Divide 24 in three parts such that they are in AP and their product is 440.
Solution:
Let a – d, a, a + d are three numbers in AP.

their sum = a – d + a + a + d = 24

3a = 24
or a = 8

Again,
Their Product = (a – d) x a x (a + d) = 440
$a(a^2 - d^2) = 440$
$8(8^2 - d^2) = 440$
or d = ±3

Numbers are : (5, 8, 11) or (11, 8, 5)

Question 9:
The sum of three consecutive terms of an AP is 21 and the sum of the squares of these terms is 165. Find these terms.

Solution:
Let a – d, a, a + d are three numbers in AP.

their sum = a – d + a + a + d = 21

3a = 21
or a = 7

Again,
Sum of squares = (a – d)^2 + a^2 +(a + d)^2 = 165

$$a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad = 165$$
$$3a^2 + 2d^2 = 165$$
$$3(7)^2 + 2d^2 = 165 \Rightarrow 3 \times 49 + 2d^2 = 165$$
$$147 + 2d^2 = 165 \Rightarrow 2d^2 = 165 - 147 = 18$$
$$d^2 = \frac{18}{2} = 9 = (\pm 3)^2$$
$$d = \pm 3$$

Numbers are : (4, 7, 10) or (10, 7, 4)

Question 10:
The angles of a quadrilateral are in AP whose common difference is 10°. Find the angles.

Solution:
Sum of angles of a quadrilateral = 360°
Common difference = 10 = d (say)
If the first number be a, then the next four numbers will be
a, a + 10, a + 20, a + 30

As per definition:
a + a + 10 + a + 20 + a + 30 = 360°
4a + 60 = 360
4a = 300
or a = 75°
Other angles:
a + 10 = 75 + 10 = 85
a + 20 = 75 + 20 = 95
a + 30 = 75 + 30 = 105

Therefore, Angles are 75°, 85°, 95°, 105°

Question 11:
Find four numbers in AP whose sum is 28 and the sum of whose squares is 216.
Solution:
Let a – 3d, a – d, a + d, a + 3d are the four numbers in AP.
Their sum = a – 3d + a – d + a + d + a + 3d = 28

Sum of their square = (a – 3d)^2 + (a – d)^2 + (a + d)^2 + (a + 3d)^2 = 216

$$a^2 + 9d^2 - 6ad + a^2 + d^2 - 2ad + a^2 + d^2 + 2ad + a^2 + 9d^2 + 6ad = 216$$
$$4a^2 + 20d^2 = 216$$
$$a^2 + 5d^2 = 54$$
$$(7)^2 + 5d^2 = 54 \Rightarrow 5d^2 + 49 = 54$$
$$5d^2 = 54 - 49 = 5 \Rightarrow d^2 = \frac{5}{5} = 1$$
$$d^2 = (\pm 1)^2$$
$$d = \pm 1$$

Numbers will be: (4, 6, 8,10) or (10, 8, 6, 4)

Question 12: Divide 32 into four parts which are the four terms of an AP such that the product of the first and the fourth terms is to the product of the second and the third terms as 7 : 15.

Solution:

Let a – 3d, a – d, a + d, a + 3d are the four numbers in AP.
Therefore:
$$-3d + a - d + a + d + a + 3d = 32$$
or a = 8

Again,
$$(a - 3d)\,(a + 3d) : (a - d)\,(a + d)$$
= 7:15

$(a^2 - 9d^2) : (a^2 - d^2) = 7 : 15$

$\frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15}$

$15a^2 - 135d^2 = 7d^2 - 7d^2$

$15a^2 - 7a = 135d^2 - 7d^2$

$8a^2 = 128d^2$

$d^2 = \frac{8a^2}{128} = \frac{8 \times 8^2}{128} = \frac{8 \times 64}{128} = 4 = (\pm 2)^2$

d = ±2

four parts are: a - 6, a - 2, a+2 amd a + 6

which implies,
Number are: (2, 6, 8, 10, 14) or (14, 10, 6, 2)

Question 13: The sum of first three terms of an AP is 48. If the product of first and second terms exceeds 4 times the third term by 12. Find the AP.
Solution:
Let a – d, a, a + d are the three terms in AP
So,
Sum = a – d + a + a + d = 48
3a = 48
Or a = 16

And,
a(a – d) = 4 (a + d) + 12
16 (16 – d) = 4(16 + d) + 12
256 – 16d = 64 + 4d + 12 = 4d + 76
256 – 76 = 4d + 16d
180 = 20d
d = 9
Which implies:
Numbers are : (7, 16, 25)