Exercise 5C

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Question 1: Find the sum of each of the following APs:
(i) 2, 7, 12, 17,..... to 19 terms.
(ii) 9, 7, 5, 3,.....to 14 terms.
(iii) -37, -33, -29,....12 terms.
(iv) 1/15 ,1/12,1/10,...to 11 terms
(v) 0.6, 1.7, 2.8,.... to 100 terms.

Solution:

Sum of n terms of AP formula:
$$S_n = \frac{n}{2}[2a + (n-1)d]$$ ......(1)
Where
First term = a
Common difference = d
Number of terms = n

(i) 2, 7, 12, 17,..... to 19 terms.

a = 2
d = 7 - 2 = 5
Using (1)

S_{19} = 19/2(2(2) + (19 - 1)5)

= (19)(4 + 90)/2
= (19 × 94)/2
= 893
Sum of 19 terms of this AP is 893.

(ii) 9, 7, 5, 3,.....to 14 terms.
a = 9
d = 7 - 9 - -2
Using (1)

S_{14} = 14/2 [2(9) + (14 - 1)(-2)] =
(7)(18 - 26)
= - 56

Sum of 14 terms of this AP is - 56.

(iii) -37, -33, -29,....12 terms.

a = -37
d = (-33) - (-37) = 4
Using (1)

S_{12} = 12/2 [2(-37) + (12 - 1)(4)]
= (6)(-74 + 44)
= 6 × (-30)
= - 180
Sum of 12 terms of this AP is - 180.

(iv) 1/15 ,1/12,1/10,...to 11 terms

a = 1/15
d = (1/12) - (1/15) = 1/60
Using (1)

S_{11} = 11/2 [2(1/15) + (11 - 1)(1/60)] =
(11/2) × [(2/15) + (1/6)]
=33/20
Sum of 11 terms of this AP is 33/20.

(v) 0.6, 1.7, 2.8,.... to 100 terms.

a = 0.6
d = 1.7 - 0.6 = 1.1
Using (1)

S_{100} = 100/2 [2(0.6) + (100 - 1)(1.1)]
= (50) × [1.2 + (99 × 1.1)]
= 50 × 110.1
= 5505
Sum of 100 terms of this AP is 5505.

Question 2: Find the sum of each of the following arithmetic series:
(i) 7 + 10 1/2 + 14 + … + 84
(ii) 34 + 32 + 30 + … + 10
(iii) (-5) + (-8) + (-11) + … + (-230)

(iv) 5 + (-41) + 9 + (-39) + 13 + (-37) + 17 + ... + (-5) + 81 + (-3)

Solution:

(i) 7 + 10 1/2 + 14 + ... + 84
First term = a = 7
Common difference = d = (21/2) - 7 = (7/2)
Last term = l = 84
Now, using formula:
84 = a + (n - 1)d
84 = 7 + (n - 1)(7/2)
77 = (n - 1)(7/2)
154 = 7n - 7
7n = 161
n = 23
Thus, there are 23 terms in AP.
Now,
Find Sum of these 23 terms:
S_{23} = 23/2 [2(7) + (23 - 1)(7/2)]
= (23/2) [14 + (22)(7/2)]
= (23/2) [91]
= 2093/2

Sum of 23 terms of this AP is 2093/2.

(ii) 34 + 32 + 30 + ... + 10
First term = a = 34
Common difference = d = 34 - 32 = - 2
Last term = l = 10
Now, using formula:
10 = a + (n - 1)d
10 = 34 + (n - 1)(-2)
10 - 34 = (n - 1)(-2)
n = 13
Thus, there are 13 terms in AP.
Now,
Find Sum of these 13 terms:
S_{13} = 13/2 [2(34) + (13 - 1)(-2)]
= (13/2) [68 + (12)(-2)]
= (13/2) x 44
= 286

Sum of 13 terms of this AP is 286.

(iii) (-5) + (-8) + (-11) + ... + (-230)
First term = a = -5
Common difference = d = - 8 - (-5) = - 3
Last term = l = -230
Now, using formula:
-230 = a + (n - 1)d
-230 = -5 + (n - 1)(-3)
-230 + 5 = (n - 1)(-3)
n = 76
Thus, there are 76 terms in AP.
Now,
Find Sum of these 76 terms:
S_{76} = 76/2 [2(-5) + (76 - 1)(-3)]
= 38 × [(-10) + (75)(-3)]
= - 8930

Sum of 76 terms of this AP is -8930.

(iv) 5 + (-41) + 9 + (-39) + 13 + (-37) + 17 + ... + (-5) + 81 + (-3)

The given series is combination of two AP's.
Let A_1 = 5 + 9 +13 + ...+ 77 + 81
and A_2 = -41 - 39 - 37 - ......(-3)

For A_1:

First term = a = 5
Common difference = d = 2
Last term = l = 81
Now, using formula:
81 = 5 + (n - 1)4
n = 20
Thus, there are 20 terms in AP.
Now,
Find Sum of these 76 terms:
S_{20} = n/2 [a + l]
= 20/2 (5 + 81)
= 860

Sum of 20 terms of this AP is 860.

For A_2:

First term = a = -41
Common difference = d = 2
Last term = l = -3
Now, using formula:
-3 = -41 + (n - 1)2
n = 20
Thus, there are 20 terms in AP.
Now,
Find Sum of these 76 terms:
S_{20} = n/2 [a + l]
= 20/2 (-41- 3)
=-440

Sum of 20 terms of this AP is -440.

Therefore, Sum of total terms: 860 - 440 = 420.

Question 3: Find the sum of first n terms of an AP whose nth term is (5 – 6n). Hence, find the sum of its first 20 terms.
Solution:

Given: a_n = 5 - 6n
Find some of the terms of AP:
Put n = 1, we get a_1 = - 1 = first term
Put n = 2, we get a_2 = - 7 = second term

Common difference = d = a_2 - a_1 = - 7 - (-1) = - 6
Sum of first n terms:
S_n= n/2 [2a + (n - 1)d]
= n/2[- 2 + (n - 1)(-6)]
= n(2 - 3n)
sum of first 20 terms:
S_{20} = 20/2[2(-1) + (20 - 1)(-6)]
= 10 [- 2 - 114]
= - 1160

Sum of its first 20 terms of AP is -1160.

Question 4:
The sum of the first n terms of an AP is $(3n^2 + 6n)$. Find the nth term and the 15th term of this AP.
Solution:
Given: $S_n = 3n^2 + 6n$

$S_1 = 3(1)^2 + 6 \times 1 = 3 + 6 = 9$
$S_2 = 3(2)^2 + 6 \times 2 = 12 + 12 = 24$
$T_2 = S_2 - S_1 = 24 - 9 = 15$
$d = 15 - 9 = 6$
$a = 9$
$T_n = a + (n-1)d = 9 + (n-1) \times 6$
$= 9 + 6n - 6 = 3 + 6n$
$= 6n + 3$
$T_{15} = 6 \times 15 + 3 = 90 + 3 = 93$

Question 5: The sum of the first n terms of an AP is given by $S_n = (3n^2 - n)$. Find its
(i) nth term,
(ii) first term and
(iii) common difference.

Solution:
$S_n = 3n^2 - n$
$S_1 = 3(1)^2 - 1 = 3 - 1 = 2$
$S_2 = 3(2)^2 - 2 = 12 - 2 = 10$
$a_1 = 2$
$a_2 = 10 - 2 = 8$

(i) $a_n = a + (n - 1) d$
$= 2 + (n - 1) \times 6$
$= 2 + 6n - 6$
$= 6n - 4$
(ii) First term = 2
(iii) Common difference = 8 – 2 = 6

Question 6: (i) The sum of the first n terms of an AP is $(5n^2/2 + 3n/2)$. Find the nth term and the 20th term of this AP.
(ii) The sum of the first n terms of an AP is $(3n^2/2 + 5n/2)$. Find its nth term and the 25th term.
Solution:

(i) $S_n = \frac{5n^2}{2} + \frac{3n}{2}$

$S_1 = \frac{5(1)^2}{2} + \frac{3(1)}{2} = \frac{5}{2} + \frac{3}{2} = \frac{8}{2} = 4$

$S_2 = \frac{5(2)^2}{2} + \frac{3(2)}{2} = \frac{20}{2} + \frac{6}{2}$

$= 10 + 3 = 13$

$T_2 = S_2 - S_1 = 13 - 4 = 9$

$T_1 = 4$

$d = 9 - 4 = 5$

Now, $T_n = a + (n - 1)d$

$= 4 + (n - 1) \times 5$

$= 4 + 5n - 5 = (5n - 1)$

$T_{20} = 5 \times 20 - 1 = 100 - 1 = 99$

(ii)

$S_n = \frac{3n^2}{2} + \frac{5n}{2}$

$S_1 = \frac{3(1)^2}{2} + \frac{5 \times 1}{2} = \frac{3}{2} + \frac{5}{2} = \frac{8}{2} = 4$

$S_2 = \frac{3(2)^2}{2} + \frac{5(2)}{2} = \frac{3 \times 4}{2} + \frac{5 \times 2}{2}$

$= \frac{12}{2} + \frac{10}{2} = 6 + 5 = 11$

$T_2 = S_2 - S_1 = 11 - 4 = 7$

$T_1 = 4$ or $a = 4$

$d = T_2 - T_1 = 7 - 4 = 3$

Now, $T_n = a + (n - 1)d$

$= 4 + (n - 1) \times 3 = 4 + 3n - 3$

$= 3n + 1$

$T_{25} = 3 \times 25 + 1 = 75 + 1 = 76$

Question 7: If mth term of an AP is 1/n and nth term is 1/m then find the sum of its first mn terms.

Solution:

Let a be first term and d be the common difference of an AP.

mth term = 1/n

so,

$a_m = a + (m – 1) d = 1/n$ ....(1)

nth term = 1/m

$a_n = a + (n – 1) d = 1/m$ ....(2)

Subtract (2) from (1)

$$(m-1)d-(n-1)d=\frac{1}{n}-\frac{1}{m}$$

$$d(n-n)=\frac{m-n}{mn}$$

d = 1/mn ....(3)

From (3) and (1), we get

$$a+(m-1)\times\frac{1}{mn}=\frac{1}{n}\Rightarrow a=\frac{1}{n}-\frac{m-1}{mn}$$

$$a=\frac{1}{mn}$$

Sum of first mn terms:

S_{mn} = mn/2 [2(1/mn) + (mn-1) (1/mn)]

= mn/2 [1/mn + 1]

= (1+mn)/2

Question 8:
How many terms of the AP 21, 18, 15, … must be added to get die sum 0?
Solution:
AP is 21, 18, 15,…
a = 21,
d = 18 – 21 = -3
Sum of terms = S_n = 0
(n/2) [2a + (n - 1)d] = 0
(n/2) [2(21) + (n - 1)(-3)] = 0
(n/2) [45 - 3n] = 0
[45 - 3n] = 0

n = 15 (number of terms)
Thus, 15 terms of the given AP sums to zero.

Question 9: How many terms of the AP 9, 17, 25, … must be taken so that their sum is 636?
Solution:
AP is 9, 17, 25,…
a = 9, d = 17 – 9 = 8

Sum of terms = S_n = 636

(n/2) [2a + (n - 1)d] = 636
(n/2) [2(9) + (n - 1)(8)] = 636
(n/2)[10 + 8n] = 636
$4n^2 + 5n - 636 = 0$ (which is a quadratic equation)

(n- 12)(4n + 53) = 0

Either (n- 12) = 0 or (4n + 53) = 0

n = 12 or n = - 53/4
Since n can't be negative and fraction, so
n = 12
Number of terms = 12 terms.

Question 10: How many terms of the AP 63, 60, 57, 54, … must be taken so that their sum is 693? Explain the double answer.
Solution:
AP is 63, 60, 57, 54,…
Here, a = 63, d = 60 – 63 = -3 and sum = S_n = 693

$$693 = \frac{n}{2}[2 \times 63 + (n-1)(-3)]$$
$$693 \times 2 = n(126 - 3n + 3)$$
$$1386 = n(129 - 3n)$$
$$1386 = 129n - 3n^2$$
$$3n^2 - 129n + 1386 = 0$$
$$n^2 - 43n + 462 = 0$$

Which is a quadratic equation

$n^2 - 21n - 22n + 462 = 0$
$n(n - 21) - 22(n - 21) = 0$
$(n - 21)(n - 22) = 0$
Either, $n - 21 = 0$, then $n = 21$
or $n - 22 = 0$, then $n = 22$
Number of terms = 21 or 22
$T_{22} = a + (n - 1)d$
$= 63 + (22 - 1)(-3)$
$= 63 + 21 \times (-3) = 63 - 63 = 0$
Which shows that, 22th term of AP is zero.

Number of terms are 21 or 22. So there will be no effect on the sum.

Question 11:
How many terms of the AP 20, 19 1/3 , 18 2/3 , … must be taken so that their sum is 300? Explain the double answer.
Solution:

AP is $20, 19\frac{1}{3}, 18\frac{2}{3}, \ldots$

Here, a = 20, d = -2/3 and sum = s_n = 300 (for n number of terms)

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$300 = \frac{n}{2}\left[2\times 20+(n-1)\left(\frac{-2}{3}\right)\right]$$

$$600 = n\left[40-\frac{2}{3}n+\frac{2}{3}\right]$$

$$600 = n\left[\frac{122}{3}-\frac{2}{3}n\right]$$

$$600 = \frac{n(122-2n)}{3}$$

$1800 = 122n - 2n^2$
$2n^2 - 122n + 1800 = 0$
$n^2 - 61n + 900 = 0$
$n^2 - 25n - 36n + 900 = 0$

n(n-25) - 36(n-25) = 0

(n - 25)(n - 36) = 0
Either (n - 25) = 0 or (n - 36) = 0

n = 25 or n = 36

For n = 25

$$300 = \frac{25}{2}\left[2 \times 20 + (25-1)\left(\frac{-2}{3}\right)\right]$$

$$= \frac{25}{2}\left[40 + 24 \times \left(\frac{-2}{3}\right)\right]$$

$$= \frac{25}{2}[40 - 16] = \frac{25}{2} \times 24 = 300$$

Which is true.

For n = 36

$$300 = \frac{36}{2}\left[2 \times 20 + (36-1)\left(\frac{-2}{3}\right)\right]$$

$$= 18\left[40 + 35 \times \frac{-2}{3}\right]$$

$$= 18\left[40 - \frac{70}{3}\right] = 18 \times \frac{120-70}{3}$$

= 300
Which is true.
Result is true for both the values of n

So both the numbers are correct.
Therefore, Sum of 11 terms is zero. (36-25 = 11)

Question 12: Find the sum of all odd numbers between 0 and 50.
Solution:
Odd numbers between 0 and 50 are 1, 3, 5, 7, 9, …, 49
Here, a = 1, d = 3 – 1 = 2, l = 49

$49 = 1 + (n - 1) \times 2$

$49 - 1 = 2(n - 1) \Rightarrow 2(n - 1) = 48$

$n - 1 = \frac{48}{2} = 24$

$n = 24 + 1 = 25$

Sum of odd numbers:
= n/2(a + l)
= 25/2 (l + 49)
= 25/2(50)
= 625

Question 13: Find the sum of all natural numbers between 200 and 400 which are divisible by 7.
Solution:
Natural numbers between 200 and 400 which are divisible by 7 are 203, 210, 217,......,399.

Here,AP is 203, 210, 217,......,399

First term = a = 203,
Common difference = d = 7 and
Last term = l = 399

We know that, l = a + (n-1)d

$399 = 203 + (n - 1) \times 7$

$399 - 203 = 7(n - 1) \Rightarrow 7(n - 1) = 196$

$n - 1 = \frac{196}{7} = 28$

$n = 28 + 1 = 29$

$n = 29$

There are total 29 terms.
Now, find the sum of all 29 terms:

$$S_{29} = \frac{n}{2}[a + l]$$

$$= \frac{29}{2}[203 + 399]$$

$$= \frac{29}{2} \times 602 = 8729$$

Sum of all 29 terms is 8729.

Question 14: Find the sum of first forty positive integers divisible by 6.
Solution:
First forty positive integers which are divisible by 6 are
6, 12, 18, 24,...... to 40 terms
Here, a = 6, d = 12 – 6 = 6, and n = 40.

Sum of 40 terms:

$$S_{40} = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{40}{2}[2 \times 6 + (40 - 1) \times 6]$$

$$= 20[12 + 39 + 6] = 20[12 + 234]$$

$$= 20 \times 246 = 4920$$

Question 15: Find the sum of the first 15 multiples of 8.
Solution:
First 15 multiples of 8 are as given below:
8, 16, 24, 32,..... to 15 terms
Here, a = 8, d = 16 – 8 = 8, and n = 15

Sum of 15 terms:

$$S_{15} = \frac{15}{2}[2 \times 8 + (15 - 1) \times 8]$$

$$= \frac{15}{2}[16 + 14 \times 8] = \frac{15}{2}[16 + 112]$$

$$= \frac{15}{2} \times 128 = 960$$

Question 16: Find the sum of all multiples of 9 lying between 300 and 700.
Solution:
Multiples of 9 lying between 300 and 700 = 306, 315, 324, 333, …, 693

Here, a = 306, d = 9, and l = 693

We know that, l = a + (n-1)d

$693 = 306 + (n-1) \times 9$

$(n-1) \times 9 = 693 - 306 = 387$

$n - 1 = \frac{387}{9} = 43$

$n = 43 + 1 = 44$

There are 44 terms.

Find the sum of 44 terms:

$S_{44} = \frac{n}{2}[a + l]$

$= \frac{44}{2}[306 + 693]$

$= 22 \times 999 = 21978$

Question 17: Find the sum of all three-digit natural numbers which are divisible by 13.
Solution:
3-digit natural numbers: 100, 101, 102,......., 999.

3-digit natural numbers divisible by 13:
104, 117, 130,........., 988
Which is an AP.

Here, a = 104, d = 13, l = 988
l = a + (n – 1) d

988 = 104 + (n-1)13

n = 69
There are 69 terms.

Find the sum of 69 terms:

$S_n = n/2(a + l)$

= 69/2(104 + 988)

= 37674

Question 18: Find the sum of first 100 even natural numbers which are divisible by 5.

Solution:
Even natural numbers: 2, 4, 6, 8, 10, …
Even natural numbers divisible by 5: 10, 20, 30, 40, … to 100 terms

Here, a = 10, d = 20 – 10 = 10, and n = 100

$S_n = n/2(2a + (n-1)d)$

= 100/2 [2 x 10 + (100-1)10]

= 50500

Question 19: Find the sum of the following.
(4 – 1/n) + (4-2/n) + (4– 3/n) + ….

Solution:

Given sum can be written as (4 + 4 + 4 + 4+....) - (1/n, 2/n, 3/n, ...)

Now, We have two series:
First series: = 4 + 4 + 4 + ... up to n terms
= 4n

Second series: 1/n, 2/n, 3/n, ...
Here, first term = a = 1/n
Common difference = d = (2/n) - (1/n) = (1/n)

Sum of n terms formula:
$S_n =$ n/2[2a + (n - 1)d]

Sum of n terms of second series:
S_n = n/2 [2(1/n) + (n - 1)(1/n)]
= n/2 [(2/n) + 1 - (1/n)]
= (n + 1)/2

Hence,
Sum of n terms of the given series = Sum of n terms of first series - Sum of n terms of second series
= 4n - (n + 1)/2
= (8n - n - 1)/2
= 1/2 (7n - 1)

Question 20: In an AP, it is given that $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where S_n denotes the sum of its first n terms.
Solution:
Let a be the first term and d be the common difference of the AP.

$$S_5 = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{5}{2}[2a + (5-1)d]$$

$$= \frac{5}{2}[2a + 4d]$$

Similarly,

$$S_7 = \frac{7}{2}[2a + 6d]$$

$$\text{and } S_{10} = \frac{10}{2}[2a + 9d] = 5(2a + 9d)$$

$$\text{Now, } \frac{5}{2}(2a + 4d) + \frac{7}{2}(2a + 6d) = 167$$

$5a + 10d + 7a + 21d = 167$
$12a + 31d = 167$ ...(i)
and $5(2a + 9d) = 235$
$2a + 9d = 47$ ...(ii)
Multiply (i) by (i) and (ii) by 6,

$$12a + 31d = 167$$
$$12a + 54d = 282$$
$$- \quad - \quad -$$

$$-23d = -115 \quad \Rightarrow d = \frac{-115}{-23} = 5$$

From (*i*), $12a + 31 \times 5 = 167$

$12a + 155 = 16 \Rightarrow 12a = 167 - 155$

$$12a = 12 \Rightarrow a = \frac{12}{12} = 1$$

Which implies: a = 1 and d = 5

Therefore, the AP is 1, 6, 11, 16,……

Question 21: In an AP, the first term is 2, the last term is 29 and the sum of all the terms is 155. Find the common difference.

Solution:
Let d be the common difference.
Given:
first term = a = 2
last term = l = 29
Sum of all the terms = S_n = 155

S_n = n/2[a + l]
155 = n/2[2 + 29]
n = 10

There are 10 terms in total.

Therefore, 29 is the 10th term of the AP.

Now, 29 = a + (10 - 1)d
29 = 2 + 9d
27 = 9d
d = 3

The common difference is 3.

Question 22: In an AP, the first term is -4, the last term is 29 and the sum of all its terms is 150. Find its common difference.

Solution:
Let d be the common difference.
Given:
first term = a = -4
last term = l = 29
Sum of all the terms = S_n = 150

S_n = n/2[a + l]
150 = n/2[-4 + 29]
n = 12

There are 12 terms in total.

Therefore, 29 is the 12th term of the AP.

Now, 29 = -4 + (12 - 1)d
29 = -4 + 11d
d = 3

The common difference is 3.

Question 23: The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution:
Let n be the total number of terms.
Given:
First term = a = 17
Last term = l = 350
Common difference = d = 9

l = a + (n-1)d

350 = 17 + (n-1)9

n = 38
Again,
S_n = n/2[a + l]

= 38/2[17 + 350]
= 6973

There are 38 terms in total and their sum is 6973.

Question 24: The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find the common difference and the number of terms.

Solution:
Let n be the total number of terms and d be the common difference.
Given:
first term = a = 5
last term = l = 45
Sum of all terms = S_n = 400

S_n = n/2[a + l]

400 = n/2[5 + 45]

n/2 = 400/50
n = 16

There are 16 terms in the AP.

Therefore, 45 is the 16th term of the AP.
45 = a + (16 - 1)d
45 = 5 + 15d
40 = 15d
15d = 40
d = 8/3
Common difference = d = 8/3

Common difference is 8/3 and the number of terms are 16.

Question 25: In an AP, the first term is 22, nth term is -11 and sum of first n terms is 66. Find n and hence find the common difference.

Solution:
Let n be the total number of terms and d be the common difference.
Given:
first term = a = 22

nth term = -11
Sum of all terms = S_n = 66

S_n = n/2[a + l]

66 = n/2[22 + (-11)]

n = 12

There are 12 terms in the AP.

Since nth term is -11, so
a_n = a + (n - 1)d
-11 = 22 + (12-1)d
d = -3

Therefore, Common difference is -3 and the number of terms are 12.