Exercise 5D

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Question 1: The first three terms of an AP are respectively (3y – 1), (3y + 5) and (5y + 1), find the value of y.

Solution:

Given: (3y – 1), (3y + 5) and (5y+ 1) are in AP

So, (3y + 5) – (3y – 1) = (5y + 1) – (3y + 5)

2 (3y + 5) = (5y + 1) + (3y – 1)

6y + 10 = 8y

8y – 6y = 10

2y = 10

Or y = 5

The value of y is 5.

Question 2: If k, (2k – 1) and (2k + 1) are the three successive terms of an AP, find the value of k.

Solution:

Given: k, (2k – 1) and (2k + 1) are the three successive terms of an AP.

So, (2k – 1) – k = (2k + 1) – (2k – 1)

2 (2k – 1) = 2k + 1 + k

4k – 2 = 3k + 1

4k – 3k = 1 + 2

or k = 3

The value of k is 3.

Question 3: If 18, a, (b – 3) are in AP, then find the value of (2a – b).

Solution:

Given: 18, a, (b – 3) are in AP

a – 18 = b – 3 – a

a + a – b = -3 + 18

2a – b = 15

Question 4: If the numbers a, 9, b, 25 form an AP, find a and b.

Solution:

Given: a, 9, b, 25 are in AP.

So, 9 – a = b – 9 = 25 – b

b – 9 = 25 – b

b + b = 22 + 9 = 34

or b = 17

And,

a – b = a – 9
9 + 9 = a + b
a + b = 18
a + 17 = 18
or a = 1
Answer: a = 18, b= 17

Question 5: If the numbers (2n – 1), (3n + 2) and (6n – 1) are in AP, find the value of n and the numbers.

Solution:
Given: (2n – 1), (3n + 2) and (6n – 1) are in AP
So, (3n + 2) – (2n – 1) = (6n – 1) – (3n + 2)
(3n + 2) + (3n + 2) = 6n – 1 + 2n – 1
6n + 4 = 8n – 2
8n – 6n = 4 + 2
Or n = 3

Numbers are:
2 x 3 – 1 = 5
3 x 3 + 2 = 11
6 x 3 – 1 = 17
Answer: (5, 11, 17) are required numbers.

Question 6: How many three-digit natural numbers are divisible by 7?
Solution:
3-digit natural numbers: 100, 101,…….. 990 and

3-digit natural numbers divisible by 7: 105, 112, 119, 126, …, 994

Here, a = 105, d= 7, l = 994

a_n = (l) = a + (n – 1) d
994 = 105 + (n – 1) x 7
994 – 105 = (n – 1) 7
n – 1 = 127
n = 128
Answer: There are 128 required numbers.

Question 7: How many three-digit natural numbers are divisible by 9?

Solution:

3-digit numbers: 100, 101,……,999

3-digit numbers divisible by 9 : 108, 117, 126, 135, …, 999

Here, a = 108, d= 9, l = 999
a_n (l) = a + (n – 1) d
999 = 108 + (n – 1) x 9
(n – 1) x 9 = 999 – 108 = 891
n – 1 = 99
n = 100

Question 8: If the sum of first m terms of an AP is ($2m^2 + 3m$) then what is its second term?
Solution:
Sum of first m terms of an AP = $2m^2 + 3m$ (given)
$S_m = 2m^2 + 3m$
Sum of one term = $S_1 = 2(1)^2 + 3 \times 1 = 2 + 3 = 5$ = first term
Sum of first two terms = $S_2 = 2(2)^2 + 3 \times 2 = 8 + 6 = 14$
Sum of first three terms = $S_3 = 2(3)^2 + 3 \times 3 = 18 + 9 = 27$
Now,
Second term = $a_2 = S_2 – S_1 = 14 – 5 = 9$

Question 9: What is the sum of first n terms of the AP, a, 3a, 5a,……

Solution:
AP is a, 3a, 5a,.....
Here, a = a, d = 2a

Sum = $S_n = n/2 [2a + (n-1)d]$

$= n/2[2a + 2an - 2a]$
$=an^2$

Question 10.: What is the 5th term from the end of the AP 2, 7, 12, …… 47?

Solution:

Given AP is 2, 7, 12, 17, …… 47
Here, a = 2, d = 7 – 2 = 5, l = 47
nth term from the end = l – (n – 1)d
5th term from the end = 47 – (5 – 1) 5 = 47 – 4 x 5 = 27

Question 11: If an denotes the nth term of the AP 2, 7, 12, 17, …, find the value of ($a_{30} - a_{20}$).

Solution:
Given AP is 2, 7, 12, 17,……..
Here, a = 2, d = 7 – 2 = 5
Now,
$a_n = a + (n - 1)\, d = 2 + (n - 1)\ 5 = 5n - 3$

$a_{30} = 2 + (30 - 1)\ 5 = 2 + 145 = 147$ and

$a_{20} = 2 + (20 - 1)\ 5 = 2 + 95 = 97$

$a_{30} - a_{20} = 147 - 97 = 50$

Question 12: The nth term of an AP is (3n + 5). Find its common difference.
Solution:
Nth term = $a_n = 3n + 5$ (given)
$a_{(n-1)} = 3\ (n - 1) + 5 = 3n + 2$
Common difference = $d = a_n - a_{(n-1)}$
$= (3n + 5) - (3n + 2)$
$= 3n + 5 - 3n - 2$
$= 3$
Therefore, common difference is 3.

Question 13: The nth term of an AP is (7 – 4n). Find its common difference.
Solution:
Nth term = $a_n = 7 - 4n$
$a_{(n-1)} = 7 - 4(n - 1) = 11 - 4n$
Common difference = $d = a_n - a_{(n-1)}$
$= (7 - 4n) - (11 - 4n)$
$= 7 - 4n - 11 + 4n$
$= -4$
Therefore, common difference is -4.

Question 14: Write the next term of the AP √8, √18, √32, ……..

Solution:
Given AP is √8, √18, √32,……..
Above AP can be written as:
2√2 , 3√2, 4 √2, ……

Here a = 2√2 and d = √2

Next term = 4√2 + √2 = 5√2 = √50

Question 15: Write the next term of the AP √2, √8, √18,....

Solution:
Given AP is √2, √8, √18,....

Can be written as:

√2, 2√2, 3√2,.....

First term = √2

Common difference = 2√2 - √2 = √2

Next term = 3√2 + √2 = 4√2 = √32

Question 16: Which term of the AP 21, 18, 15,.... is zero?

Solution:
Given AP is 21, 18, 15,....

First term = a = 21
Common difference = d = 18-21 = -3
Last term = l = 0
l = a + (n – 1) d
0 = 21 + (n – 1)(-3)
0 = 21 – 3n + 3
24 – 3n = 0
Or n = 8
Answer: Zero is the 8th term.

Question 17: Find the sum of first n natural numbers.
Solution:
First n natural numbers: 1, 2, 3, 4, 5, …, n
Here, a = 1, d = 1

Sum = S_n = n/2 [2a + (n-1)d]

= n/2 [2(1) + (n-1)(1)]

= n(n+1)/2

Question 18: Find the sum of first n even natural numbers.

Solution:

First n even natural numbers: 2, 4, 6, 8, 10,....,n
Here, a = 2, d = 4 – 2 = 2

Sum = S_n = n/2 [2a + (n-1)d]

= n/2 [2(2) + (n-1)(2)]

= n(n+1)

Question 19: The first term of an AP is p and its common difference is q. Find its 10th term.

Solution:
Given:
First term =a = p and
Common difference = d =q
Now,
a_{10} = a + (n – 1) d
= p + (10 – 1)q
= (p + 9q)

Question 20: If 4/5, a, 2 are in AP, find the value of a.

Solution:

AP terms: 4/5, a, 2 (given)
Then,

a - 4/5 = 2 - a

a = 7/5

Question 21: If (2p + 1), 13, (5p – 3) are in AP, find the value of p.
Solution:
Given, 2p + 1, 13, 5p – 3 are in AP
Then,
13 – (2p + 1) = (5p – 3) – 13
13 – 2p – 1 = 5p – 3 – 13
12 – 2p = 5p – 16
p = 4
The value of p is 4.

Question 22: If (2p – 1), 7, 3p are in AP, find the value of p.
Solution:
Given, (2p – 1), 7, 3p are in AP
Then,
7 – (2p – 1) = 3p – 7
7 – 2p + 1 = 3p – 7
5p = 15
p = 3
The value of p is 3.

Question 23: If the sum of first p terms of an AP is ($ap^2 + bp$), find its common difference.

Solution:

Sum of first p terms = $S_p = (ap^2 + bp)$

Sum of one term = $S_1 = a(1)^2 + b(1) = a+b$ = first term
Sum of first two terms = $S_2 = a(2)^2 + b \times 2 = 4a + 2b$

We know that, second term = $a_2 = S_2 - S_1$

$= (4a + 2b) - (a + b)$
$= 3a + b$

Now, $d = a_2 - a_1$

$= 3a + b - (a+b)$
$= 2a$

Answer: Common difference is 2a.

Question 24: If the sum of first n terms is ($3n^2 + 5n$), find its common difference.

Solution:

Sum of first n terms = $S_n = (3n^2 + 5n)$

Sum of one term = $S_1 = 3(1)^2 + 5(1) = 8$ = first term
Sum of first two terms = $S_2 = 3(2)^2 + 5(2) = 22$

We know that, second term = $a_2 = S_2 - S_1$

=22 - 8
= 14

=> a_2 = 14

Now, $d = a_2 - a_1$

= 14 - 8 = 6

Answer: Common difference is 6.

Question 25: Find an AP whose 4th term is 9 and the sum of its 6th and 13th terms is 40.

Solution:

Let a be the first term and d be the common difference.
Given:
4th term = $a_4 = 9$

Sum of 6th and 13th terms = $a_6 + a_{13} = 40$

Now,

$a_4 = a + (4-1)d$
$9 = a + 3d$

$a = 9 - 3d$ ....(1)

And
$a_6 + a_{13} = 40$

a + 5d + a + 12d = 40
2a +17d = 40

2(9 - 3d) + 17d = 40 (using (1))

d = 2

From (1): a = 9 - 6 = 3

Required AP = 3,5,7,9,.....

Question 26: What is the common difference of an AP in which $a_{27} - a_7 = 84$?

Solution:
Given: $a_{27} - a_7 = 84$

[a + 26d] - [a + 6d] = 84

20d = 84
d = 4.2

Question 27: If 1 + 4 + 7 + 10 + … + x = 287, find the value of x.

Solution:
Given: 1 + 4 + 7 + 10 + … + x = 287

Here a = 1, d = 3 and $S_n = 287$

Sum = $S_n = n/2 [2a + (n-1)d]$

$287 = n/2 [2 + (n-1)3]$

$574 = 3n^2 - n$
Which is a quadratic equation.

Solve $3n^2 - n - 574 = 0$

$3n^2 - 42n + 41n - 574 = 0$

3n(n - 14) + 41(n-14) = 0

(3n + 41)(n-14) = 0

Either (3n + 41) = 0 or (n-14) = 0

n = -41/3 or n = 14

Since number of terms cannot be negative, so result is n = 14.

=> Total number of terms in AP are 14.

Which shows, $x = a_{14}$

or x = a + 13d

or x = 1 + 39

or x = 40

The value of x is 40.