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Senior School Certificate Examination

Compartment July 2019

Marking Scheme — Mathematics 65/1/1, 65/1/2, 65/1/3

General Instructions:

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. Evaluators will mark (✓) wherever answer is correct. For wrong answer 'X"be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
5. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled.
6. If a question does not have any parts, marks must be awarded in the left hand margin and encircled.
7. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
8. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
9. A full scale of marks 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
10. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 / 25 answer books per day.
11. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
12. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as (X) and awarded zero (0) Marks.
13. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
14. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
15. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
16. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 65/1/1
EXPECTED ANSWER/VALUE POINTS
SECTION A

Q.NO.		MARKS
1.	$A_{23} = -7$	1
2.	$\frac{dy}{dx} = -2x \cdot \cos x^2 \cdot \sin(\sin x^2)$	1
3.	Order = 2, Degree = 1	$\frac{1}{2}+\frac{1}{2}$
4.	Required length = $\sqrt{3^2+(-4)^2} = 5$	$\frac{1}{2}+\frac{1}{2}$
	OR	
	$\hat{n} = \frac{1}{3}(2\hat{i} - \hat{j} + 2\hat{k})$	$\frac{1}{2}$
	Equation of plane is $\vec{r} \cdot \hat{n} = d$ i.e. $\vec{r} \cdot \frac{1}{3}(2\hat{i} - \hat{j} + 2\hat{k}) = 5$ or $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 15$	$\frac{1}{2}$
	SECTION B	
5.	$fof(x) = f(f(x)) = f((3 - x^3)^{1/3})$	$\frac{1}{2}$
	$= [3 - \{(3 - x^3)^{1/3}\}^3]^{1/3} = x$	$1\frac{1}{2}$
6.	$A^{-1} = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$	$\frac{1}{2}$
	$(AB)^{-1} = B^{-1}.A^{-1}$	
	$= \begin{bmatrix} 3 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 12 & 19 \\ -3 & -5 \end{bmatrix}$	$1\frac{1}{2}$
7.	$I = \int \frac{1}{\sqrt{1-(x-1)^2}} dx$	1
	$= \sin^{-1}(x - 1) + C$	1

8.	$I = \int \frac{\sec^2 x}{(1-\tan x)^2} dx$	
	Put $1 - \tan x = t \Rightarrow \sec^2 x\ dx = -dt$	$\frac{1}{2}$
	$I = -\int \frac{dt}{t^2} = \frac{1}{t} + C = \frac{1}{1-\tan x} + C$	$1\frac{1}{2}$
	OR	
	$I = \int_0^1 x\,(1-x)^n dx$	
	$= \int_0^1 (1-x)\cdot x^n dx = \int_0^1 (x^n - x^{n+1}) dx$	$\frac{1}{2}+\frac{1}{2}$
	$= \left.\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$	$\frac{1}{2}$
	$= \frac{1}{n+1} - \frac{1}{n+2}$ or $\frac{1}{(n+1)(n+2)}$	$\frac{1}{2}$
9.	$y = b \cos(x + a)$	
	$\Rightarrow \frac{dy}{dx} = -b \sin(x + a)$	$\frac{1}{2}$
	$\frac{d^2y}{dx^2} = -b \cos(x + b)$	1
	$\Rightarrow \frac{d^2y}{dx^2} = -y$ or $\frac{d^2y}{dx^2} + y = 0$	$\frac{1}{2}$
10.	$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 8 \\ 0 & -1 & 1 \end{vmatrix} = 7\hat{i} - 4\hat{j} - 4\hat{k}$	1
	Required unit vector $= \frac{(\vec{a} \times \vec{b})}{\lvert \vec{a} \times \vec{b} \rvert}$	

	$= \frac{1}{9}(7\hat{i} - 4\hat{j} - 4\hat{k})$	1
	OR	
	$(\vec{a} + \lambda\vec{b}) \perp \vec{c} \Rightarrow (\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$	$\frac{1}{2}$
	$\Rightarrow [(2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k}] \cdot (3\hat{i} + \hat{j}) = 0$	$\frac{1}{2}$
	$\Rightarrow 3(2 - \lambda) + 1.(2 + 2\lambda) = 0 \Rightarrow \lambda = 8$	1
11.	$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\ \cancel{P(B)}}{\cancel{P(B)}}$	1
	$= 0.3$	1
	OR	
	Required probability $= \frac{3}{5} \times \frac{3}{7} + \frac{2}{5} \times \frac{4}{7} = \frac{17}{35}$	1+1
12.	Required probability = 1 – P(problem is not solved)	
	$= 1 - P(A' \cap B' \cap C')$	1
	$= 1 - P(A') \cdot P(B') \cdot P(C')$	$\frac{1}{2}$
	$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{3}{4}$	$\frac{1}{2}$
	SECTION C	
13.	For reflexive: As $ab = ba$ $\Rightarrow$ (a, b) R(a, b) $\therefore$ R is reflexive	1
	For symmetric: Let (a, b,) R (c, d) $\Rightarrow$ $ad = bc$ $\Rightarrow$ $cb = da$ $\Rightarrow$ (c, d) R(a, b) $\therefore$ R is symmetric	1

	For transitive: Let a, b, c, d, e, f $\in$ N Let (a, b) R(c, d) and (c, d) R(e, f) $\Rightarrow$ ad = bc and cf = de $\Rightarrow \quad d = \frac{cf}{e}$ $\therefore \quad a\left(\frac{cf}{e}\right) = bc$ $\Rightarrow$ acf = bce $\Rightarrow$ af = be $\Rightarrow$ (a, b) R(e, f) $\therefore$ R is transitive	$1\frac{1}{2}$
	Since R is reflexive, symmetric and transitive $\therefore$ R is an equivalence relation.	$\frac{1}{2}$
	OR	
	Let $x_1, x_2 \in R - \{2\}$ Let $f(x_1) = f(x_2)$ $\Rightarrow \quad \frac{x_1}{x_1 - 2} = \frac{x_2}{x_2 - 2} \Rightarrow x_1(x_2 - 2) = x_2(x_1 - 2)$ $\Rightarrow x_1 = x_2$ $\Rightarrow$ f is one-one.	2
	Now, gof(x) = g(f(x)), $x \in \mathbb{R} - \{2\}$ $= g\left(\frac{x}{x-2}\right)$ $= \frac{2\left(\frac{x}{x-2}\right)}{\frac{x}{x-2} - 1} = x$	2
14.	Put $x = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1} x$	1
	$\text{LHS} = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}\right)$	$\frac{1}{2}$

	$= \tan^{-1}\left(\dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right)$	1
	$= \tan^{-1}\left(\dfrac{1+\tan\theta}{1-\tan\theta}\right) = \tan^{-1}\left(\tan\left(\dfrac{\pi}{4}+\theta\right)\right)$	$\frac{1}{2}$
	$= \dfrac{\pi}{4}+\theta = \dfrac{\pi}{4}+\dfrac{1}{2}\cos^{-1}x = \text{RHS}$	1
15.	LHS: $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$	
	$= \begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$	2
	$= (x-y)(y-z)\begin{vmatrix} 0 & 1 & x+y \\ 0 & 1 & y+z \\ 1 & z & z^2 \end{vmatrix}$	1
	Expanding along C_1	
	$= (x-y)(y-z)(z-x) = \text{RHS}$	1
16.	$x^y \cdot y^x = x^x$	
	$\Rightarrow \quad y \log x + x \log y = x \log x$	1
	differentiate both sides w.r.t. x,	
	$\left(y \cdot \dfrac{1}{x} + \log x \cdot \dfrac{dy}{dx}\right) + \left(x \cdot \dfrac{1}{y} \cdot \dfrac{dy}{dx} + \log y \cdot 1\right) = x \cdot \dfrac{1}{x} + \log x \cdot 1$	2
	$\Rightarrow \quad \dfrac{y}{x} + \log\left(\dfrac{y}{x}\right) - 1 = -\left(\log x + \dfrac{x}{y}\right) \cdot \dfrac{dy}{dx}$	
	$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{1 - \dfrac{y}{x} - \log\left(\dfrac{y}{x}\right)}{\log x + \dfrac{x}{y}}$ or $\dfrac{y}{x}\left[\dfrac{x + x\log x - y - x\log y}{y \log x + x}\right]$	1
	OR	
	$\dfrac{dx}{d\theta} = 3a \sec^2\theta \cdot \sec\theta \tan\theta$	1

	$$\frac{dy}{d\theta} = 3a\ \tan^2\theta \cdot \sec^2\theta$$	1
	$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \sin\theta$$	1
	Also, $$\frac{d^2y}{dx^2} = \cos\theta \cdot \frac{d\theta}{dx}$$ $$= \frac{\cos\theta}{3a\tan\theta\sec^3\theta} \text{ or } \frac{\cos^5\theta}{3a\sin\theta}$$	1
17.	$y = a\cos(\log x) + b\sin(\log x)$	
	$$\frac{dy}{dx} = \frac{-a\sin(\log x)}{x} + \frac{b\cos(\log x)}{x}$$	1
	$$\Rightarrow \quad x \cdot \frac{dy}{dx} = -a\sin(\log x) + b\cos(\log x)$$	$\frac{1}{2}$
	differentiate both sides again w.r.t x,	
	$$x \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot 1 = \frac{-a\cos(\log x)}{x} - \frac{b\sin(\log x)}{x}$$	$1\frac{1}{2}$
	$$\Rightarrow \quad x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -\frac{y}{x}$$	$\frac{1}{2}$
	$$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$	$\frac{1}{2}$
18.	Given $ay^2 = x^3$	
	differentiate both sides wrt x	
	$$a \cdot 2y\frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$$	1
	$\therefore$ Slope of tangent at $(am^2, am^3) = \frac{3(am^2)^2}{2a(am^3)} = \frac{3}{2}m$	1

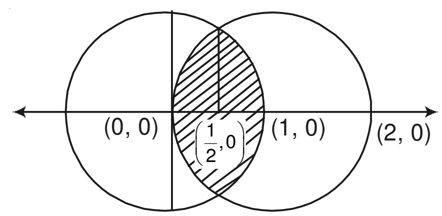
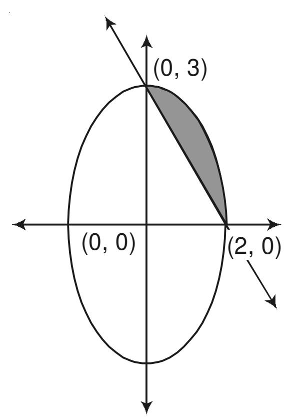
	Equation of tangent is $y - am^3 = \frac{3}{2}m(x - am^2)$ $\Rightarrow \quad 3mx - 2y = am^3$	2
19.	Put $\sin x = t \Rightarrow \cos x\, dx = dt$	1
	$I = \int \frac{dt}{(t+1)(t+2)}$	
	$= \int \left(\frac{1}{t+1} - \frac{1}{t+2} \right) dt$	1
	$= \log \lvert t+1 \rvert - \log \lvert t+2 \rvert + C$	$1\frac{1}{2}$
	$= \log \lvert \sin x + 1 \rvert - \log \lvert \sin x + 2 \rvert + C$ or $\log \left\lvert \frac{\sin x + 1}{\sin x + 2} \right\rvert + C$	$\frac{1}{2}$
20.	$I = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$ ...(1)	
	$I = \int_0^{\pi} \frac{(\pi - x) \cdot \sin x}{1 + \cos^2 x} dx$ ...(2)	1
	Adding (1) and (2)	
	$2I = \int_0^{\pi} \frac{\pi \sin x}{1 + \cos^2 x} dx$	
	$I = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx$	$\frac{1}{2}$
	Put $\cos x = t \Rightarrow -\sin x\, dx = dt$	$\frac{1}{2}$
	$\therefore \quad I = \frac{\pi}{2} \int_{-1}^{1} \frac{dt}{1 + t^2}$	1
	$= \frac{\pi}{2} [\tan^{-1} t]_{-1}^{1} = \frac{\pi^2}{4}$	$\frac{1}{2} + \frac{1}{2}$

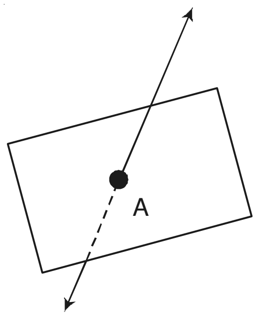
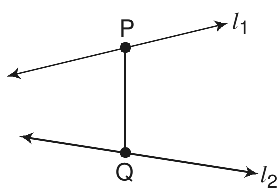
21.	Given equation can be written as $x\ dx = ye^y\sqrt{1+x^2}\ dy$ $\Rightarrow \int \frac{x}{\sqrt{1+x^2}} dx = \int y \cdot e^y dy$ $\Rightarrow \sqrt{1+x^2} = e^y\ (y-1) + C$ when $y = 1,\ x = 0 \Rightarrow C = 1$ $\therefore$ Required solution is $\sqrt{1+x^2} = e^y(y-1)+1$	 1 1+1 $\frac{1}{2}$ $\frac{1}{2}$
	OR Given differential equation can be written as $\frac{dy}{dx} = \frac{y}{x} + \frac{1}{\cos\left(\frac{y}{x}\right)}$ Put $\frac{y}{x} = v$ i.e. $y = vx$ $\Rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$ Given equation becomes $v + x\frac{dv}{dx} = v + \frac{1}{\cos v}$ $\Rightarrow \int \cos v\, dv = \int \frac{dx}{x}$ $\Rightarrow \sin v = \log\|x\| + c$ $\Rightarrow \sin\left(\frac{y}{x}\right) = \log\|x\| + c$	 $\frac{1}{2}$ $\left.\right\}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$
22.	A(1, 2, –1), B(3, – 1, 0), C(2, 3, 2), D(4, 0, 3) $\overrightarrow{AB} = 2\hat{i} - 3\hat{j} + \hat{k},\ \overrightarrow{AC} = \hat{i} + \hat{j} + 3\hat{k},\ \overrightarrow{AD} = 3\hat{i} - 2\hat{j} + 4\hat{k}$	 $1\frac{1}{2}$

	Consider = $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = \begin{vmatrix} 2 & -3 & 1 \\ 1 & 1 & 3 \\ 3 & -2 & 4 \end{vmatrix} = 2(10)+3(-5)+1(-5)=0$	1+1
	$\Rightarrow$ A, B, C and D are coplanar	$\frac{1}{2}$
23.	Let equation of line is $\vec{r}=(2\hat{i}+3\hat{j}-\hat{k})+\lambda(a\hat{i}+b\hat{j}+c\hat{k})$	1
	here, $3a + 4b + 2c = 0$ ...(1) $3a - 2b - 2c = 0$ ...(2)	1
	Solving (1) and (2) $\frac{a}{-8+4} = \frac{-b}{-6-6} = \frac{c}{-6-12} = \mu$	
	$\Rightarrow \frac{a}{2} = \frac{b}{-6} = \frac{c}{9} = -2\mu$	1
	$\therefore$ Requried equation of line is $\vec{r} = (2\hat{i}+3\hat{j}-\hat{k})+\lambda(2\hat{i}-6\hat{j}+9\hat{k})$	1
	SECTION D	
24.	$\|A\| = -2 \neq 0 \Rightarrow A^{-1}$ exists	$\frac{1}{2}$
	Now, $A_{11} = -1$, $A_{12} = 8$, $A_{13} = -5$ $A_{21} = 1$, $A_{22} = -6$, $A_{23} = 3$ $A_{31} = -1$, $A_{32} = 2$, $A_{33} = -1$	$2\frac{1}{2}$
	adj A = $\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$	
	$A^{-1} = \frac{1}{\|A\|} \cdot \text{adj } A = \frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$	1
	Given system of equations can be written as AX = B,	

	where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 10 \\ 9 \end{bmatrix}$	
	Now $AX = B \Rightarrow X = A^{-1}B$	1
	$= \frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \\ 9 \end{bmatrix}$	
	$= \frac{-1}{2}\begin{bmatrix} -4 \\ -2 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$	$\frac{1}{2}$
	$\therefore \quad x = 2,\ y = 1,\ z = 2$	$\frac{1}{2}$
	OR	
	$A = IA$	
	$\Rightarrow \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$	1
	$R_1 \to \frac{R_1}{3}$	
	$\Rightarrow \begin{bmatrix} 1 & \frac{-1}{3} & \frac{1}{3} \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$	
	$R_2 \to R_2 + 15R_1,\ R_3 \to R_3 - 5R_1$	
	$\Rightarrow \begin{bmatrix} 1 & \frac{-1}{3} & \frac{1}{3} \\ 0 & 1 & 0 \\ 0 & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 5 & 1 & 0 \\ \frac{-5}{3} & 0 & 1 \end{bmatrix} \cdot A$	
	$R_1 \to R_1 + \frac{1}{3}R_2,\ R_3 \to R_3 + \frac{1}{3}R_2$	

	$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 2 & \frac{1}{3} & 0 \\ 5 & 1 & 0 \\ 0 & \frac{1}{3} & 1 \end{bmatrix} \cdot A$ $R_3 \to 3R_3$ $\Rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & \frac{1}{3} & 0 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \cdot A$ $R_1 \to R_1 - \frac{1}{3}R_3$ $\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \cdot A$	4
	$\Rightarrow \quad A^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$	1
25.	Let Q(x, y) be the nearest point to P(1, 2)	
	Minimize $s = \sqrt{(x-1)^2 + (y-2)^2}$	1
	Let $l = s^2 = (x-1)^2 + (y-2)^2$ $\Rightarrow \quad l = x^2 + y^2 - 2x - 4y + 5$ ...(1)	1
	Also, $x^2 + y^2 = 80$ ...(2) from (1) and (2), $l = 85 - 2x - 4\sqrt{80 - x^2}$	
	$\frac{dl}{dx} = -2 - 4 \cdot \frac{1}{2\sqrt{80 - x^2}}(-2x) = -2 + \frac{4x}{\sqrt{80 - x^2}}$	1
	$\frac{dl}{dx} = 0 \Rightarrow x = 4, -4$ (rejected)	1

	$\frac{d^2 l}{dx^2} = \frac{4\sqrt{80-x^2} + \frac{4x^2}{\sqrt{80-x^2}}}{80-x^2} > 0$ at $x = 4$	1
	$\therefore$ (4, 8) is the nearest point.	1
26.	x-coordinate of the point of intersection of given cicles is $\frac{1}{2}$.	1
	Correct Figure	1
	Required Area $= 4 \cdot \int_0^{1/2} \sqrt{1-(x-1)^2}\, dx$ 	1
	$= 4\left[\frac{x-1}{2}\cdot\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}}$	2
	$= 4\left[\frac{-1}{4}\times\frac{\sqrt{3}}{2} + \frac{1}{2}\sin^{-1}\left(\frac{-1}{2}\right) - \frac{1}{2}\sin^{-1}(-1)\right]$	
	$= \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$	1
	OR	
	Given equation of ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ Correct Figure	1
	and equation of line $\frac{x}{2} + \frac{y}{3} = 1$ 	
	Point of intersection (2, 0) and (0, 3)	1
	Required Area $= \int_0^2 \frac{3}{2}\sqrt{4-x^2}\, dx - \int_0^2 \frac{3}{2}(2-x)\, dx$	1
	$= \frac{3}{2}\left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2 + \frac{3}{2}\cdot\left[\frac{(2-x)^2}{2}\right]_0^2$	2
	$= \frac{3}{2}(2\sin^{-1} 1) + \frac{3}{4}(-4)$	
	$= \frac{3\pi}{2} - 3$	1

27.	 Given line $\frac{x-8}{4}=\frac{y-1}{1}=\frac{z-3}{8}=\lambda$	
	Any point on it is $(4\lambda + 8, \lambda + 1, 8\lambda + 3)$	1
	Let A $(4\lambda + 8, \lambda + 1, 8\lambda + 3)$	
	A lies on plane $2x + 2y + z = 3$	
	$\therefore\ 2(4\lambda + 8) + 2(\lambda + 1) + (8\lambda + 3) = 3$	2
	$\Rightarrow \lambda = -1$	
	$\therefore$ A(4, 0, –5)	1
	II part: Let angle between line and plane be θ.	
	Then, $\sin\theta = \frac{4(2)+1(2)+8(1)}{\sqrt{16+1+64}\sqrt{4+4+1}}=\frac{2}{3}$	$1\frac{1}{2}$
	$\Rightarrow \theta = \sin^{-1}\left(\frac{2}{3}\right)$	$\frac{1}{2}$
	OR	
	 Let $P(3\lambda + 7, 2\lambda + 5, \lambda + 3)$ and	$\frac{1}{2}$
	$Q(2\mu + 1, 4\mu - 1, 3\mu - 1)$	$\frac{1}{2}$
	Now, d.r′.s. of PQ = $3\lambda - 2\mu + 6, 2\lambda - 4\mu + 6, \lambda - 3\mu + 4$	1
	According to question,	
	$\frac{3\lambda-2\mu+6}{2} = \frac{2\lambda-4\mu+6}{2}=\frac{\lambda-3\mu+4}{1}$	1
	$\Rightarrow \lambda + 2\mu = 0$ and $2\mu = 2 \Rightarrow \mu = 1$	
	$\Rightarrow \lambda = -2\mu$	
	$\therefore\ \mu = 1, \lambda = -2$	1
	$\therefore$ P(1, 1, 1) and Q(3, 3, 2)	
	PQ = $\sqrt{(3-1)^2+(3-1)^2+(2-1)^2}=\sqrt{4+4+1}=3$	1
	Equation of PQ is $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-1}{1}$	1
28.	Let number of chairs be x and number of tables be y.	
	Maximize $z = 40x + 60y$	1

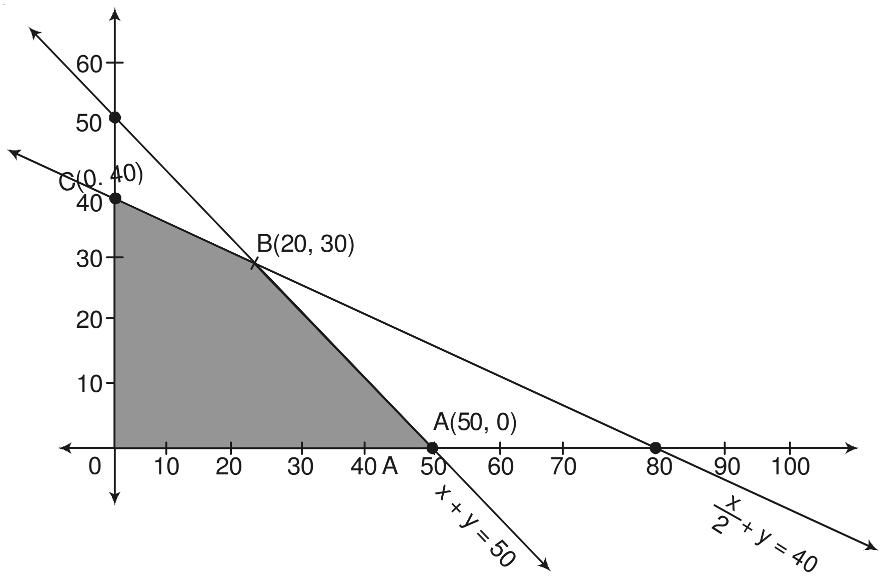
Subject to following constratins:

$$x + y \leq 50$$

$$\frac{x}{2} + y \leq 40$$

$$x \geq 0,\ y \geq 0$$

2



2

Corner point	z = 40x + 60y
A(50, 0)	2000
B(20, 30)	2600
C(0, 40)	2400

Number of chairs manufactured = 20

Number of tables manufactured = 30 1

Maximum profit = ₹ 2,600

29. Let E_1: Transferred ball is green

E_2: Transferred ball is red 1

A: Green ball is found

Here, $P(E_1) = \frac{2}{6}, P(E_2) = \frac{4}{6}$ 1

$P(A/E_1) = \frac{6}{9}, P(A/E_2) = \frac{5}{9}$ 1

	Using Baye's theorem.	
	$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)\cdot P(A/E_2)}$$	
	$$= \frac{\frac{2}{6}\times\frac{6}{9}}{\frac{2}{6}\times\frac{6}{9}+\frac{4}{6}\times\frac{5}{9}}$$	2
	$$= \frac{12}{12+20}=\frac{3}{8}$$	1

	QUESTION PAPER CODE 65/1/2 **EXPECTED ANSWER/VALUE POINTS** **SECTION A**	
1.	Required length $= \sqrt{3^2+(-4)^2} = 5$ OR $\hat{n} = \frac{1}{3}(2\hat{i}-\hat{j}+2\hat{k})$ Equation of plane is $\vec{r}\cdot\hat{n} = d$ i.e. $\vec{r}\cdot\frac{1}{3}(2\hat{i}-\hat{j}+2\hat{k}) = 5$ or $\vec{r}\cdot(2\hat{i}-\hat{j}+2\hat{k}) = 15$	$\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
2.	$A_{23} = -7$	1
3.	$\frac{dy}{dx} = \frac{e^x\cdot\cos(e^x)}{2\sqrt{\sin(e^x)}}$	1
4.	$\frac{dx}{dy} - \frac{2}{y}\cdot x = y^2e^{-y}$ I.F. $= e^{\int -\frac{2}{y}dy} = e^{-2\log y} = \frac{1}{y^2}$	$\frac{1}{2}$ $\frac{1}{2}$
5.	**SECTION B** $\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -1 & 8\\ 0 & -1 & 1\end{vmatrix} = 7\hat{i}-4\hat{j}-4\hat{k}$ Required unit vector $= \frac{(\vec{a}\times\vec{b})}{\lvert\vec{a}\times\vec{b}\rvert}$ $= \frac{1}{9}(7\hat{i}-4\hat{j}-4\hat{k})$ OR $(\vec{a}+\lambda\vec{b})\perp\vec{c} \Rightarrow (\vec{a}+\lambda\vec{b})\cdot\vec{c} = 0$	 1 1 $\frac{1}{2}$

	$\Rightarrow \quad [(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}]\cdot(3\hat{i}+\hat{j}) = 0$	$\frac{1}{2}$
	$\Rightarrow \quad 3(2-\lambda)+1.(2+2\lambda) = 0 \Rightarrow \lambda = 8$	1
6.	$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)\ \cancel{P(B)}}{\cancel{P(B)}}$	1
	$= 0.3$	1
	OR	
	Required probability $= \frac{3}{5}\times\frac{3}{7}+\frac{2}{5}\times\frac{4}{7}=\frac{17}{35}$	1+1
7.	Required probability = 1 – P(problem is not solved)	
	$= 1 - P(A' \cap B' \cap C')$	1
	$= 1 - P(A')\cdot P(B')\cdot P(C')$	$\frac{1}{2}$
	$= 1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=\frac{3}{4}$	$\frac{1}{2}$
8.	$A^{-1} = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$	$\frac{1}{2}$
	$(AB)^{-1} = B^{-1}.A^{-1}$	
	$= \begin{bmatrix} 3 & 2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}=\begin{bmatrix} 12 & 19 \\ -3 & -5 \end{bmatrix}$	$1\frac{1}{2}$
9.	$I = \int \frac{1}{\sqrt{1-(x-1)^2}}\,dx$	1
	$= \sin^{-1}(x-1)+C$	1
10.	$I = \int \frac{\sec^2 x}{(1-\tan x)^2}\,dx$	
	Put $1-\tan x = t \Rightarrow \sec^2 x\ dx = -dt$	$\frac{1}{2}$
	$I = -\int \frac{dt}{t^2}=\frac{1}{t}+C=\frac{1}{1-\tan x}+C$	$1\frac{1}{2}$

	OR	
	$I = \int_0^1 x\,(1-x)^n\,dx$	
	$= \int_0^1 (1-x)\cdot x^n dx = \int_0^1 (x^n - x^{n+1})dx$	$\frac{1}{2}+\frac{1}{2}$
	$= \left.\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$	$\frac{1}{2}$
	$= \frac{1}{n+1} - \frac{1}{n+2}$ or $\frac{1}{(n+1)(n+2)}$	$\frac{1}{2}$
11.	$fof(x) = f(f(x))$	
	$= f\left(\frac{4x+3}{6x-4}\right)$	1
	$= \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4} = \frac{34x}{34} = x$	1
12.	$y = a \sin(3x - b)$	
	$\frac{dy}{dx} = 3a\cos(3x-b)$	1
	$\frac{d^2y}{dx^2} = -9a\sin(3x-b)$	$\frac{1}{2}$
	$\Rightarrow \quad \frac{d^2y}{dx^2} = -9y$ or $\quad \frac{d^2y}{dx^2} + 9y = 0$	$\frac{1}{2}$
	SECTION C	
13.	A(1, 2, –1), B(3, – 1, 0), C(2, 3, 2), D(4, 0, 3)	
	$\overrightarrow{AB} = 2\hat{i} - 3\hat{j} + \hat{k},\ \overrightarrow{AC} = \hat{i} + \hat{j} + 3\hat{k},\ \overrightarrow{AD} = 3\hat{i} - 2\hat{j} + 4\hat{k}$	$1\frac{1}{2}$

	Consider = $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = \begin{vmatrix} 2 & -3 & 1 \\ 1 & 1 & 3 \\ 3 & -2 & 4 \end{vmatrix} = 2(10)+3(-5)+1(-5)=0$	1+1
	$\Rightarrow$ A, B, C and D are coplanar	$\frac{1}{2}$
14.	Let equation of line is $\vec{r}=(2\hat{i}+3\hat{j}-\hat{k})+\lambda(a\hat{i}+b\hat{j}+c\hat{k})$	1
	here, $3a+4b+2c=0$ ...(1) $3a-2b-2c=0$ ...(2)	1
	Solving (1) and (2) $\frac{a}{-8+4}=\frac{-b}{-6-6}=\frac{c}{-6-12}=\mu$	
	$\Rightarrow \frac{a}{2}=\frac{b}{-6}=\frac{c}{9}=-2\mu$	1
	$\therefore$ Requried equation of line is	
	$\vec{r}=(2\hat{i}+3\hat{j}-\hat{k})+\lambda(2\hat{i}-6\hat{j}+9\hat{k})$	1
15.	Given equation can be written as $x\,dx = ye^y\sqrt{1+x^2}\,dy$	
	$\Rightarrow \int \frac{x}{\sqrt{1+x^2}}dx = \int y\cdot e^y dy$	1
	$\Rightarrow \sqrt{1+x^2}=e^y(y-1)+C$	1+1
	when $y=1,\ x=0 \Rightarrow C=1$	$\frac{1}{2}$
	$\therefore$ Required solution is $\sqrt{1+x^2}=e^y(y-1)+1$	$\frac{1}{2}$
	OR	
	Given differential equation can be written as $\frac{dy}{dx}=\frac{y}{x}+\frac{1}{\cos\left(\frac{y}{x}\right)}$	$\frac{1}{2}$

	Put $\frac{y}{x} = v$ i.e. $y = vx$ $\Rightarrow \quad \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$	1
	Given equation becomes $v + x\frac{dv}{dx} = v + \frac{1}{\cos v}$	$\frac{1}{2}$
	$\Rightarrow \quad \int \cos v \, dv = \int \frac{dx}{x}$	$\frac{1}{2}$
	$\Rightarrow \quad \sin v = \log \lvert x \rvert + c$	1
	$\Rightarrow \quad \sin\left(\frac{y}{x}\right) = \log \lvert x \rvert + c$	$\frac{1}{2}$
16.	For reflexive: As $ab = ba$ $\Rightarrow \quad (a, b) R(a, b) \qquad \therefore$ R is reflexive	1
	For symmetric: Let $(a, b,) R (c, d)$ $\Rightarrow \quad ad = bc$ $\Rightarrow \quad cb = da$ $\Rightarrow \quad (c, d) R(a, b) \qquad \therefore$ R is symmetric	1
	For transitive: Let $a, b, c, d, e, f \in N$ Let $(a, b) R(c, d)$ and $(c, d) R(e, f)$ $\Rightarrow \quad ad = bc$ and $cf = de$ $\Rightarrow \quad d = \frac{cf}{e}$ $\therefore \quad a\left(\frac{cf}{e}\right) = bc$ $\Rightarrow \quad acf = bce \Rightarrow af = be$ $\Rightarrow \quad (a, b) R(e, f) \qquad \therefore$ R is transitive	$1\frac{1}{2}$

	Since R is reflexive, symmetric and transitive $\therefore$ R is an equivalence relation. OR Let $x_1, x_2 \in R - \{2\}$ Let $f(x_1) = f(x_2)$ $\Rightarrow \quad \frac{x_1}{x_1 - 2} = \frac{x_2}{x_2 - 2} \Rightarrow x_1(x_2 - 2) = x_2(x_1 - 2)$ $\Rightarrow x_1 = x_2$ $\Rightarrow$ f is one-one. Now, $gof(x) = g(f(x)), \quad x \in \mathbb{R} - \{2\}$ $= g\left(\frac{x}{x-2}\right)$ $= \frac{2\left(\frac{x}{x-2}\right)}{\frac{x}{x-2} - 1} = x$	$\frac{1}{2}$ 2 2
17.	LHS: $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$ $= \begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$ $= (x-y)(y-z) \begin{vmatrix} 0 & 1 & x+y \\ 0 & 1 & y+z \\ 1 & z & z^2 \end{vmatrix}$ Expanding along C_1 $= (x-y)(y-z)(z-x) = \text{RHS}$	2 1 1
18.	$x^y \cdot y^x = x^x$ $\Rightarrow \quad y \log x + x \log y = x \log x$ differentiate both sides w.r.t. x,	 1

	$\left(y\cdot\frac{1}{x}+\log x\cdot\frac{dy}{dx}\right)+\left(x\cdot\frac{1}{y}\cdot\frac{dy}{dx}+\log y\cdot 1\right) = x\cdot\frac{1}{x}+\log x\cdot 1$	2
	$\Rightarrow \quad \frac{y}{x}+\log\left(\frac{y}{x}\right)-1=-\left(\log x+\frac{x}{y}\right)\cdot\frac{dy}{dx}$	
	$\Rightarrow \quad \frac{dy}{dx} = \frac{1-\frac{y}{x}-\log\left(\frac{y}{x}\right)}{\log x+\frac{x}{y}}$ or $\frac{y}{x}\left[\frac{x+x\log x-y-x\log y}{y\log x+x}\right]$	1
	OR	
	$\frac{dx}{d\theta} = 3a\sec^2\theta\cdot\sec\theta\tan\theta$	1
	$\frac{dy}{d\theta} = 3a\tan^2\theta\cdot\sec^2\theta$	1
	$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}=\sin\theta$	1
	Also, $\frac{d^2y}{dx^2} = \cos\theta\cdot\frac{d\theta}{dx}$	
	$= \frac{\cos\theta}{3a\tan\theta\sec^3\theta}$ or $\frac{\cos^5\theta}{3a\sin\theta}$	1
19.	Given $ay^2 = x^3$	
	differentiate both sides wrt x	
	$a\cdot 2y\frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$	1
	$\therefore$ Slope of tangent at $(am^2, am^3) = \frac{3(am^2)^2}{2a(am^3)}=\frac{3}{2}m$	1
	Equation of tangent is	
	$y-am^3 = \frac{3}{2}m(x-am^2)$	
	$\Rightarrow \quad 3mx-2y = am^3$	2

20.	Put $\sin x = t \Rightarrow \cos x\, dx = dt$	1
	$I = \int \frac{dt}{(t+1)(t+2)}$	
	$= \int \left(\frac{1}{t+1} - \frac{1}{t+2} \right) dt$	1
	$= \log \lvert t+1 \rvert - \log \lvert t+2 \rvert + C$	$1\frac{1}{2}$
	$= \log \lvert \sin x + 1 \rvert - \log \lvert \sin x + 2 \rvert + C$ or $\log \left\lvert \frac{\sin x + 1}{\sin x + 2} \right\rvert + C$	$\frac{1}{2}$
21.	RHS $= \frac{1}{2} \cos^{-1}\left(\frac{1-x}{1+x} \right)$	
	Put $x = \tan^2 \theta$ or $\sqrt{x} = \tan\theta$	1
	RHS $= \frac{1}{2} \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta} \right) = \frac{1}{2}(2\theta)$	2
	$= \theta = \tan^{-1}\sqrt{x} =$ LHS	1
22.	$y = \sin^{-1} x - \cos^{-1} x$	
	$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \left(\frac{-1}{\sqrt{1-x^2}} \right) = \frac{2}{\sqrt{1-x^2}}$	2
	$\Rightarrow \sqrt{1-x^2} \cdot \frac{dy}{dx} = 2$	$\frac{1}{2}$
	differentiate again wrt x,	
	$\sqrt{1-x^2} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{1}{2\sqrt{1-x^2}}(-2x) = 0$	1
	$\Rightarrow (1-x^2) \cdot \frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$	$\frac{1}{2}$
23.	$\int_3^5 (\lvert x-3 \rvert + \lvert x-4 \rvert + \lvert x-5 \rvert)\, dx$	

$$= \int_3^5 (x-3)dx + \int_3^4 (4-x)dx + \int_4^5 (x-4)dx - \int_3^5 (x-5)dx$$ $1\frac{1}{2}$

$$= \left.\frac{(x-3)^2}{2}\right]_3^5 - \left.\frac{(4-x)^2}{2}\right]_3^4 + \left.\frac{(x-4)^2}{2}\right]_4^5 - \left.\frac{(x-5)^2}{2}\right]_3^5$$ $1\frac{1}{2}$

$$= 2 + \frac{1}{2} + \frac{1}{2} + 2 = 5$$ 1

SECTION D

24. Let number of chairs be x and number of tables be y.

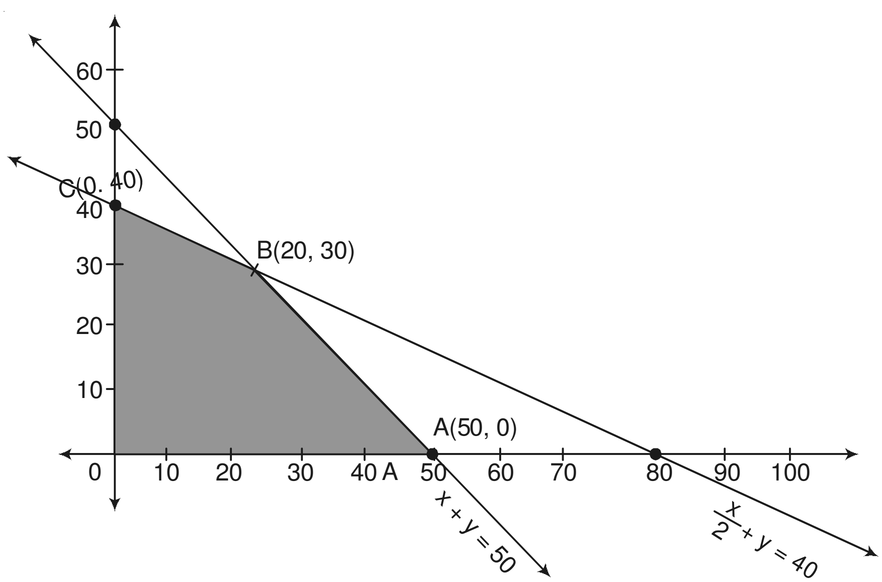
Maximize $z = 40x + 60y$ 1

Subject to following constratins:

$$x + y \le 50$$

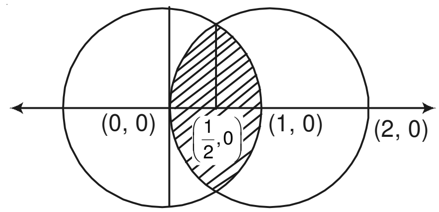
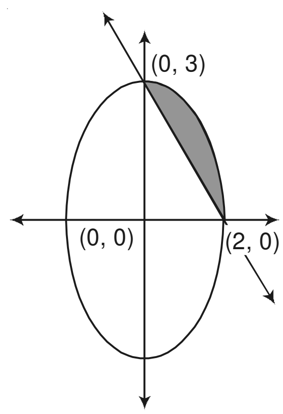
$$\frac{x}{2} + y \le 40$$ 2

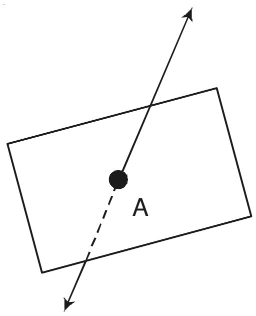
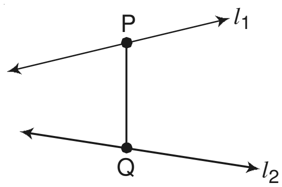
$$x \ge 0, \; y \ge 0$$



2

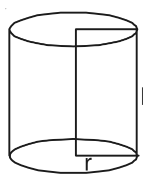
Corner point	$z = 40x + 60y$
A(50, 0)	2000
B(20, 30)	2600
C(0, 40)	2400

	Number of chairs manufactured = 20 Number of tables manufactured = 30 Maximum profit = ₹ 2,600	1
25.	x-coordinate of the point of intersection of given cicles is $\frac{1}{2}$.	1
	Correct Figure	1
		
	Required Area = $4 \cdot \int_0^{1/2} \sqrt{1-(x-1)^2}\, dx$	1
	$= 4\left[\frac{x-1}{2} \cdot \sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}}$	2
	$= 4\left[\frac{-1}{4} \times \frac{\sqrt{3}}{2} + \frac{1}{2}\sin^{-1}\left(\frac{-1}{2}\right) - \frac{1}{2}\sin^{-1}(-1)\right]$	
	$= \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$	1
	OR	
	Given equation of ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ Correct Figure	1
		
	and equation of line $\frac{x}{2} + \frac{y}{3} = 1$	
	Point of intersection (2, 0) and (0, 3)	1
	Required Area = $\int_0^2 \frac{3}{2}\sqrt{4-x^2}\, dx - \int_0^2 \frac{3}{2}(2-x)\, dx$	1
	$= \frac{3}{2}\left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2 + \frac{3}{2} \cdot \left[\frac{(2-x)^2}{2}\right]_0^2$	2
	$= \frac{3}{2}(2\sin^{-1}1) + \frac{3}{4}(-4)$	
	$= \frac{3\pi}{2} - 3$	1

26.	 Given line $\frac{x-8}{4}=\frac{y-1}{1}=\frac{z-3}{8}=\lambda$	
	Any point on it is $(4\lambda + 8, \lambda + 1, 8\lambda + 3)$	1
	Let A $(4\lambda + 8, \lambda + 1, 8\lambda + 3)$	
	A lies on plane $2x + 2y + z = 3$	
	$\therefore\ 2(4\lambda + 8) + 2(\lambda + 1) + (8\lambda + 3) = 3$	2
	$\Rightarrow \lambda = -1$	
	$\therefore$ A(4, 0, –5)	1
	II part: Let angle between line and plane be θ.	
	Then, $\sin\theta = \frac{4(2)+1(2)+8(1)}{\sqrt{16+1+64}\,\sqrt{4+4+1}}=\frac{2}{3}$	$1\frac{1}{2}$
	$\Rightarrow \theta = \sin^{-1}\left(\frac{2}{3}\right)$	$\frac{1}{2}$
	OR	
	 Let $P(3\lambda + 7, 2\lambda + 5, \lambda + 3)$ and	$\frac{1}{2}$
	$Q(2\mu + 1, 4\mu - 1, 3\mu - 1)$	$\frac{1}{2}$
	Now, d.r′.s. of PQ = $3\lambda - 2\mu + 6, 2\lambda - 4\mu + 6, \lambda - 3\mu + 4$	1
	According to question,	
	$\frac{3\lambda-2\mu+6}{2} = \frac{2\lambda-4\mu+6}{2}=\frac{\lambda-3\mu+4}{1}$	1
	$\Rightarrow \lambda + 2\mu = 0$ and $2\mu = 2 \Rightarrow \mu = 1$	
	$\Rightarrow \lambda = -2\mu$	
	$\therefore\ \mu = 1, \lambda = -2$	1
	$\therefore$ P(1, 1, 1) and Q(3, 3, 2)	
	PQ = $\sqrt{(3-1)^2+(3-1)^2+(2-1)^2}=\sqrt{4+4+1}=3$	1
	Equation of PQ is $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-1}{1}$	1
27.	$\lvert A\rvert = -2 \neq 0 \Rightarrow A^{-1}$ exists	$\frac{1}{2}$

	Now, $A_{11} = -1,\ A_{12} = 8,\ A_{13} = -5$ $A_{21} = 1,\ A_{22} = -6,\ A_{23} = 3$ $A_{31} = -1,\ A_{32} = 2,\ A_{33} = -1$	$2\frac{1}{2}$
	$\text{adj } A = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$	
	$A^{-1} = \frac{1}{\lvert A \rvert} \cdot \text{adj } A = \frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$	1
	Given system of equations can be written as AX = B, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 10 \\ 9 \end{bmatrix}$	
	Now $AX = B \Rightarrow X = A^{-1}B$	1
	$= \frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \\ 9 \end{bmatrix}$	
	$= \frac{-1}{2}\begin{bmatrix} -4 \\ -2 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$	$\frac{1}{2}$
	$\therefore \quad x = 2,\ y = 1,\ z = 2$	$\frac{1}{2}$
	OR	
	A = IA	
	$\Rightarrow \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$	1

	$R_1 \to \frac{R_1}{3}$	
	$\Rightarrow \begin{bmatrix} 1 & \frac{-1}{3} & \frac{1}{3} \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$	
	$R_2 \to R_2 + 15R_1,\ R_3 \to R_3 - 5R_1$	
	$\Rightarrow \begin{bmatrix} 1 & \frac{-1}{3} & \frac{1}{3} \\ 0 & 1 & 0 \\ 0 & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 5 & 1 & 0 \\ \frac{-5}{3} & 0 & 1 \end{bmatrix} \cdot A$	
	$R_1 \to R_1 + \frac{1}{3}R_2,\ R_3 \to R_3 + \frac{1}{3}R_2$	
	$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 2 & \frac{1}{3} & 0 \\ 5 & 1 & 0 \\ 0 & \frac{1}{3} & 1 \end{bmatrix} \cdot A$	4
	$R_3 \to 3R_3$	
	$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & \frac{1}{3} & 0 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \cdot A$	
	$R_1 \to R_1 - \frac{1}{3}R_3$	
	$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \cdot A$	
	$\Rightarrow A^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$	1

28.	Let E_1: Transferred ball is white E_2: Transferred ball is black A: white ball is found	1
	Here, $P(E_1) = \frac{3}{7}, P(E_2) = \frac{4}{7}$	1
	$P(A/E_1) = \frac{6}{9}, P(A/E_2) = \frac{5}{9}$	1
	Using Baye's theorem $P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$	
	$= \frac{\frac{3}{7}\times\frac{6}{9}}{\frac{3}{7}\times\frac{6}{9}+\frac{4}{7}\times\frac{5}{9}}$	2
	$= \frac{18}{18+20} = \frac{9}{19}$	1
29.	 Given: Surface area = S = $2\pi rh + 2\pi r^2$	
	$\Rightarrow h = \frac{S - 2\pi r^2}{2\pi r}$	1
	Maximize volume when h = 2r	
	Now, $V = \pi r^2 h = \pi r^2\left(\frac{S-2\pi r^2}{2\pi r}\right)$	$1\frac{1}{2}$
	$\Rightarrow V = \frac{1}{2}(Sr - 2\pi r^3)$	
	$\frac{dV}{dr} = \frac{1}{2}(S - 6\pi r^2)$	1
	$\frac{dV}{dr} = 0 \Rightarrow S = 6\pi r^2 \Rightarrow 2\pi rh + 2\pi r^2 = 6\pi r^2$	$1\frac{1}{2}$
	$\Rightarrow h = 2r$	
	$\frac{d^2V}{dr^2} = \frac{1}{2}(-12\pi r) = -6\pi r < 0$	1
	$\therefore$ Volume is maximum when h = 2r	

	QUESTION PAPER CODE 65/1/3 **EXPECTED ANSWER/VALUE POINTS** **SECTION A**	
1.	$A_{23} = -7$	1
2.	Required length = $\sqrt{3^2+(-4)^2} = 5$ OR $\hat{n} = \frac{1}{3}(2\hat{i}-\hat{j}+2\hat{k})$ Equation of plane is $\vec{r}\cdot\hat{n}=d$ i.e. $\vec{r}\cdot\frac{1}{3}(2\hat{i}-\hat{j}+2\hat{k})=5$ or $\vec{r}\cdot(2\hat{i}-\hat{j}+2\hat{k})=15$	$\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
3.	$ax + by = 0$ differentiate wrt x, $a+b\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-a}{b}$ differenitate again, $\frac{d^2y}{dx^2} = 0$, which is required equation	 $\frac{1}{2}$ $\frac{1}{2}$
4.	$\sin^2 x + \cos^2 y = 1$ differentiate wrt x, $2\sin x\cdot\cos x + 2\cos y\cdot(-\sin y)\cdot\frac{dy}{dx} = 0$ $\Rightarrow \sin 2x = \sin 2y\frac{dy}{dx}$ $\Rightarrow \frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}$	 $\frac{1}{2}$ $\frac{1}{2}$
	SECTION B	
5.	$I = \int\frac{\sec^2 x}{(1-\tan x)^2}dx$	

	Put $1 - \tan x = t \Rightarrow \sec^2 x\ dx = -dt$	$\frac{1}{2}$
	$I = -\int \frac{dt}{t^2} = \frac{1}{t} + C = \frac{1}{1 - \tan x} + C$	$1\frac{1}{2}$
	OR	
	$I = \int_0^1 x\,(1-x)^n\,dx$	
	$= \int_0^1 (1-x) \cdot x^n dx = \int_0^1 (x^n - x^{n+1})dx$	$\frac{1}{2} + \frac{1}{2}$
	$= \left.\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$	$\frac{1}{2}$
	$= \frac{1}{n+1} - \frac{1}{n+2}$ or $\frac{1}{(n+1)(n+2)}$	$\frac{1}{2}$
6.	$y = b\cos(x + a)$	
	$\Rightarrow \frac{dy}{dx} = -b\sin(x + a)$	$\frac{1}{2}$
	$\frac{d^2y}{dx^2} = -b\cos(x + b)$	1
	$\Rightarrow \frac{d^2y}{dx^2} = -y$ or $\frac{d^2y}{dx^2} + y = 0$	$\frac{1}{2}$
7.	$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 8 \\ 0 & -1 & 1 \end{vmatrix} = 7\hat{i} - 4\hat{j} - 4\hat{k}$	1
	Required unit vector $= \frac{(\vec{a} \times \vec{b})}{\lvert \vec{a} \times \vec{b} \rvert}$	
	$= \frac{1}{9}(7\hat{i} - 4\hat{j} - 4\hat{k})$	1

	OR	
	$(\vec{a}+\lambda\vec{b}) \perp \vec{c} \Rightarrow (\vec{a}+\lambda\vec{b})\cdot\vec{c} = 0$	$\frac{1}{2}$
	$\Rightarrow \quad [(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}]\cdot(3\hat{i}+\hat{j}) = 0$	$\frac{1}{2}$
	$\Rightarrow \quad 3(2-\lambda) + 1.(2+2\lambda) = 0 \Rightarrow \lambda = 8$	1
8.	$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)\ \cancel{P(B)}}{\cancel{P(B)}}$	1
	$= 0.3$	1
	OR	
	Required probability $= \frac{3}{5}\times\frac{3}{7}+\frac{2}{5}\times\frac{4}{7}=\frac{17}{35}$	1+1
9.	Required probability = 1 – P(problem is not solved)	
	$= 1 - P(A' \cap B' \cap C')$	1
	$= 1 - P(A')\cdot P(B')\cdot P(C')$	$\frac{1}{2}$
	$= 1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=\frac{3}{4}$	$\frac{1}{2}$
10.	$fof(x) = f(f(x)) = f((3-x^3)^{1/3})$	$\frac{1}{2}$
	$= [3 - \{(3-x^3)^{1/3}\}^3]^{1/3} = x$	$1\frac{1}{2}$
11.	$[1\ \ 2\ \ 1]\begin{bmatrix}4\\x\\2x\end{bmatrix}=O$	1
	$\Rightarrow \quad [4+2x+2x] = O \Rightarrow x = -1$	1
12.	$I = \int \frac{dx}{\sqrt{9-(x+2)^2}}$	1
	$= \sin^{-1}\left(\frac{x+2}{3}\right)+C$	1

	SECTION C	
13.	Put $\sin x = t \Rightarrow \cos x\, dx = dt$	1
	$I = \int \frac{dt}{(t+1)(t+2)}$	
	$= \int \left(\frac{1}{t+1} - \frac{1}{t+2} \right) dt$	1
	$= \log \lvert t+1 \rvert - \log \lvert t+2 \rvert + C$	$1\frac{1}{2}$
	$= \log \lvert \sin x + 1 \rvert - \log \lvert \sin x + 2 \rvert + C$ or $\log \left\lvert \frac{\sin x + 1}{\sin x + 2} \right\rvert + C$	$\frac{1}{2}$
14.	$I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$ ...(1)	
	$I = \int_0^{\pi} \frac{(\pi - x)\cdot \sin x}{1+\cos^2 x} dx$ ...(2)	1
	Adding (1) and (2)	
	$2I = \int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x} dx$	
	$I = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx$	$\frac{1}{2}$
	Put $\cos x = t \Rightarrow -\sin x\, dx = dt$	$\frac{1}{2}$
	$\therefore \quad I = \frac{\pi}{2} \int_{-1}^{1} \frac{dt}{1+t^2}$	1
	$= \frac{\pi}{2} [\tan^{-1} t]_{-1}^{1} = \frac{\pi^2}{4}$	$\frac{1}{2} + \frac{1}{2}$
15.	Given equation can be written as	
	$x\, dx = ye^y \sqrt{1+x^2}\, dy$	

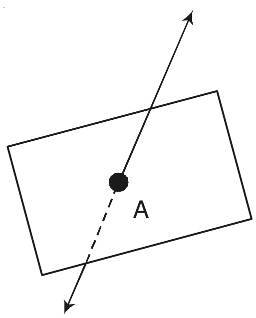
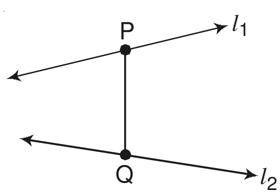
	$\Rightarrow \quad \int \frac{x}{\sqrt{1+x^2}} dx = \int y \cdot e^y dy$	1
	$\Rightarrow \quad \sqrt{1+x^2} = e^y (y - 1) + C$	1+1
	when $y = 1,\ x = 0 \Rightarrow C = 1$	$\frac{1}{2}$
	$\therefore$ Required solution is $\sqrt{1+x^2} = e^y (y-1)+1$	$\frac{1}{2}$
	OR	
	Given differential equation can be written as	
	$\frac{dy}{dx} = \frac{y}{x} + \frac{1}{\cos\left(\frac{y}{x}\right)}$	$\frac{1}{2}$
	Put $\frac{y}{x} = v$ i.e. $y = vx$ $\Rightarrow \quad \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$	1
	Given equation becomes	
	$v + x\frac{dv}{dx} = v + \frac{1}{\cos v}$	$\frac{1}{2}$
	$\Rightarrow \quad \int \cos v\, dv = \int \frac{dx}{x}$	$\frac{1}{2}$
	$\Rightarrow \quad \sin v = \log \lvert x \rvert + c$	1
	$\Rightarrow \quad \sin\left(\frac{y}{x}\right) = \log \lvert x \rvert + c$	$\frac{1}{2}$
16.	Let equation of line is $\vec{r} = (2\hat{i} + 3\hat{j} - \hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$	1
	here, $3a + 4b + 2c = 0 \quad ...(1)$ $3a - 2b - 2c = 0 \quad ...(2)$	1
	Solving (1) and (2)	
	$\frac{a}{-8+4} = \frac{-b}{-6-6} = \frac{c}{-6-12} = \mu$	

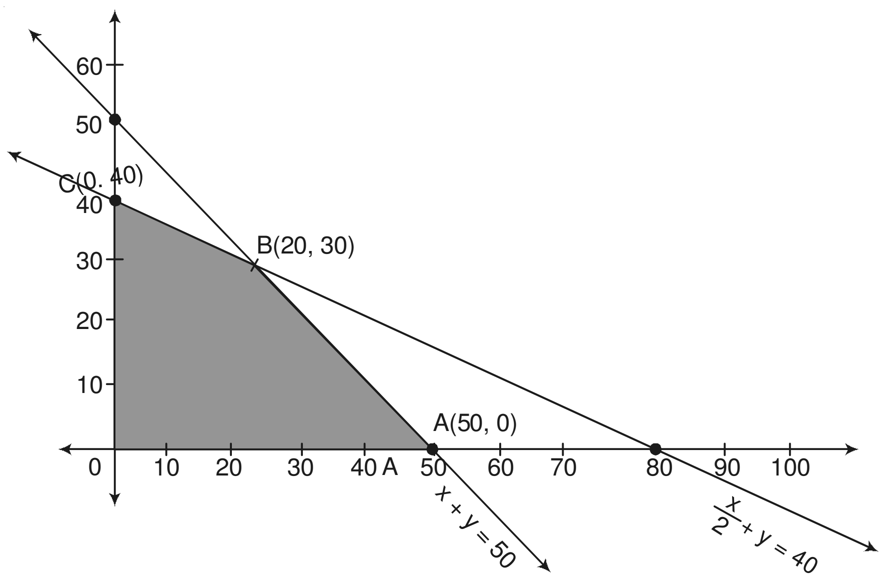
	$\Rightarrow \quad \frac{a}{2} = \frac{b}{-6} = \frac{c}{9} = -2\mu$	1
	$\therefore$ Requried equation of line is	
	$\vec{r} = (2\hat{i} + 3\hat{j} - \hat{k}) + \lambda(2\hat{i} - 6\hat{j} + 9\hat{k})$	1
17.	For reflexive:	
	As $ab = ba$	
	$\Rightarrow \quad (a, b)\ R(a, b) \qquad \therefore$ R is reflexive	1
	For symmetric:	
	Let $(a, b,)\ R\ (c, d)$	
	$\Rightarrow \quad ad = bc$	
	$\Rightarrow \quad cb = da$	
	$\Rightarrow \quad (c, d)\ R(a, b) \qquad \therefore$ R is symmetric	1
	For transitive:	
	Let $a, b, c, d, e, f \in N$	
	Let $(a, b)\ R(c, d)$ and $(c, d)\ R(e, f)$	
	$\Rightarrow \quad ad = bc$ and $cf = de$	
	$\Rightarrow \quad d = \frac{cf}{e}$	$1\frac{1}{2}$
	$\therefore \quad a\left(\frac{cf}{e}\right) = bc$	
	$\Rightarrow \quad acf = bce \Rightarrow af = be$	
	$\Rightarrow \quad (a, b)\ R(e, f) \qquad \therefore$ R is transitive	
	Since R is reflexive, symmetric and transitive $\therefore$ R is an equivalence relation.	$\frac{1}{2}$
	OR	
	Let $x_1, x_2 \in R - \{2\}$	
	Let $f(x_1) = f(x_2)$	2
	$\Rightarrow \quad \frac{x_1}{x_1 - 2} = \frac{x_2}{x_2 - 2} \Rightarrow x_1(x_2 - 2) = x_2(x_1 - 2)$	

	$\Rightarrow x_1 = x_2$ $\Rightarrow$ f is one-one. Now, $gof(x) = g(f(x)), \quad x \in \mathbb{R} - \{2\}$ $= g\left(\dfrac{x}{x-2}\right)$ $= \dfrac{2\left(\dfrac{x}{x-2}\right)}{\dfrac{x}{x-2} - 1} = x$	2
18.	$y = a \cos (\log x) + b \sin (\log x)$ $\dfrac{dy}{dx} = \dfrac{-a \sin (\log x)}{x} + \dfrac{b \cos (\log x)}{x}$	1
	$\Rightarrow \quad x \cdot \dfrac{dy}{dx} = -a \sin (\log x) + b \cos (\log x)$	$\frac{1}{2}$
	differentiate both sides again w.r.t x, $x \cdot \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} \cdot 1 = \dfrac{-a \cos (\log x)}{x} - \dfrac{b \sin (\log x)}{x}$	$1\frac{1}{2}$
	$\Rightarrow \quad x \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = -\dfrac{y}{x}$	$\frac{1}{2}$
	$\Rightarrow \quad x^2 \dfrac{d^2y}{dx^2} + x \dfrac{dy}{dx} + y = 0$	$\frac{1}{2}$
19.	Put $x = \cos 2\theta \Rightarrow \theta = \dfrac{1}{2} \cos^{-1} x$	1
	LHS $= \tan^{-1}\left(\dfrac{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}\right)$	$\frac{1}{2}$
	$= \tan^{-1}\left(\dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right)$	1
	$= \tan^{-1}\left(\dfrac{1+\tan\theta}{1-\tan\theta}\right) = \tan^{-1}\left(\tan\left(\dfrac{\pi}{4} + \theta\right)\right)$	$\frac{1}{2}$

	$= \dfrac{\pi}{4} + \theta = \dfrac{\pi}{4} + \dfrac{1}{2}\cos^{-1} x = \text{RHS}$	1
20.	$x^y \cdot y^x = x^x$	
	$\Rightarrow \quad y \log x + x \log y = x \log x$	1
	differentiate both sides w.r.t. x,	
	$\left(y \cdot \dfrac{1}{x} + \log x \cdot \dfrac{dy}{dx}\right) + \left(x \cdot \dfrac{1}{y} \cdot \dfrac{dy}{dx} + \log y \cdot 1\right) = x \cdot \dfrac{1}{x} + \log x \cdot 1$	2
	$\Rightarrow \quad \dfrac{y}{x} + \log\left(\dfrac{y}{x}\right) - 1 = -\left(\log x + \dfrac{x}{y}\right) \cdot \dfrac{dy}{dx}$	
	$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{1 - \dfrac{y}{x} - \log\left(\dfrac{y}{x}\right)}{\log x + \dfrac{x}{y}}$ or $\dfrac{y}{x}\left[\dfrac{x + x\log x - y - x \log y}{y \log x + x}\right]$	1
	OR	
	$\dfrac{dx}{d\theta} = 3a \sec^2 \theta \cdot \sec \theta \tan \theta$	1
	$\dfrac{dy}{d\theta} = 3a \tan^2 \theta \cdot \sec^2 \theta$	1
	$\dfrac{dy}{dx} = \dfrac{\left(\dfrac{dy}{d\theta}\right)}{\left(\dfrac{dx}{d\theta}\right)} = \sin\theta$	1
	Also, $\quad \dfrac{d^2y}{dx^2} = \cos\theta \cdot \dfrac{d\theta}{dx}$	
	$= \dfrac{\cos\theta}{3a \tan\theta \sec^3\theta}$ or $\dfrac{\cos^5\theta}{3a \sin\theta}$	1
21.	$\left.\begin{array}{l} \overrightarrow{BA} = \hat{i} + (x-4)\hat{j} + 4\hat{k} \\ \overrightarrow{BC} = \hat{i} + 0\hat{j} - 3\hat{k} \\ \overrightarrow{BD} = 3\hat{i} + 3\hat{j} - 2\hat{k} \end{array}\right\}$	$1\frac{1}{2}$
	$[\overrightarrow{BA} \quad \overrightarrow{BC} \quad \overrightarrow{BD}] = 0$	

	$\Rightarrow \quad \begin{vmatrix} 1 & x-4 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$	1
	$\Rightarrow \quad 1(9) - (x - 4) \cdot 7 + 4(3) = 0$	
	$\Rightarrow \quad x = 7$	$1\frac{1}{2}$
22.	$y^2 = 4ax$ differentiating, both sides w.r.t. x,	
	$2y\frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{2a}{y}$	1
	$\therefore$ slope of tangent at $(at^2, 2at) = \frac{2a}{2at} = \frac{1}{t}$	1
	Equation of tangent is: $y - 2at = \frac{1}{t}(x - at^2)$ $\Rightarrow \quad x - ty + at^2 = 0$	1
	Equation of normal is $y - 2at = -t(x - at^2)$ $\Rightarrow \quad tx + y - 2at - at^3 = 0$	1
23.	LHS: $C_1 \to C_1 + C_3$	
	$= \begin{vmatrix} \alpha+\beta+\gamma & \alpha^2 & \beta+\gamma \\ \alpha+\beta+\gamma & \beta^2 & \gamma+\alpha \\ \alpha+\beta+\gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$	1
	$= (\alpha+\beta+\gamma)\begin{vmatrix} 1 & \alpha^2 & \beta+\gamma \\ 1 & \beta^2 & \gamma+\alpha \\ 1 & \gamma^2 & \alpha+\beta \end{vmatrix}$	$\frac{1}{2}$
	$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$	
	$= (\alpha+\beta+\gamma)\begin{vmatrix} 0 & \alpha^2-\beta^2 & -(\alpha-\beta) \\ 0 & \beta^2-\gamma^2 & -(\beta-\gamma) \\ 1 & \gamma^2 & \alpha+\beta \end{vmatrix}$	1

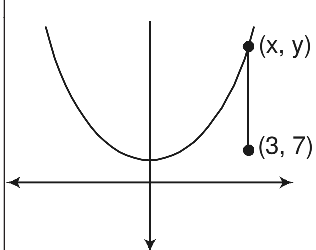
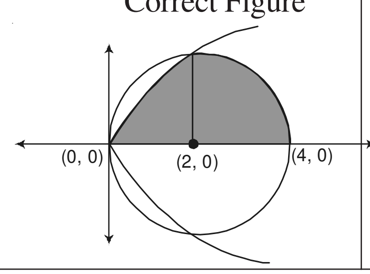
	$= (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)\begin{vmatrix} 0 & \alpha+\beta & -1 \\ 0 & \beta+\gamma & -1 \\ 1 & \gamma^2 & \alpha+\beta \end{vmatrix}$	$\frac{1}{2}$
	Expanding along C_1	
	$= (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma) = \text{RHS}$	1
	SECTION D	
24.		
	Given line $\frac{x-8}{4}=\frac{y-1}{1}=\frac{z-3}{8}=\lambda$	
	Any point on it is $(4\lambda+8, \lambda+1, 8\lambda+3)$	1
	Let A $(4\lambda+8, \lambda+1, 8\lambda+3)$	
	A lies on plane $2x+2y+z=3$	
	$\therefore\ 2(4\lambda+8)+2(\lambda+1)+(8\lambda+3)=3$	2
	$\Rightarrow \lambda = -1$	
	$\therefore$ A$(4, 0, -5)$	1
	II part: Let angle between line and plane be θ.	
	Then, $\sin\theta = \frac{4(2)+1(2)+8(1)}{\sqrt{16+1+64}\,\sqrt{4+4+1}}=\frac{2}{3}$	$1\frac{1}{2}$
	$\Rightarrow \theta = \sin^{-1}\left(\frac{2}{3}\right)$	$\frac{1}{2}$
	OR	
		
	Let P$(3\lambda+7, 2\lambda+5, \lambda+3)$ and	$\frac{1}{2}$
	Q$(2\mu+1, 4\mu-1, 3\mu-1)$	$\frac{1}{2}$
	Now, d.r′.s. of PQ $= 3\lambda-2\mu+6,\ 2\lambda-4\mu+6,\ \lambda-3\mu+4$	1
	According to question,	
	$\frac{3\lambda-2\mu+6}{2}=\frac{2\lambda-4\mu+6}{2}=\frac{\lambda-3\mu+4}{1}$	1
	$\Rightarrow \lambda+2\mu=0$ and $2\mu=2 \Rightarrow \mu=1$	
	$\Rightarrow \lambda=-2\mu$	

	$\therefore\ \mu = 1,\ \lambda = -2$	1
	$\therefore$ P(1, 1, 1) and Q(3, 3, 2)	
	$PQ = \sqrt{(3-1)^2+(3-1)^2+(2-1)^2} = \sqrt{4+4+1} = 3$	1
	Equation of PQ is $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-1}{1}$	1
25.	Let number of chairs be x and number of tables be y.	
	Maximize z = 40x + 60y	1
	Subject to following constratins:	
	$x + y \le 50$ $\frac{x}{2} + y \le 40$ $x \ge 0,\ y \ge 0$	2
		2
	Corner point / z = 40x + 60y: A(50, 0) 2000; B(20, 30) 2600; C(0, 40) 2400	
	Number of chairs manufactured = 20 Number of tables manufactured = 30 Maximum profit = ₹ 2,600	1

Corner point	z = 40x + 60y
A(50, 0)	2000
B(20, 30)	2600
C(0, 40)	2400

26.	Let E_1: Transferred ball is green E_2: Transferred ball is red A: Green ball is found	1
	Here, $P(E_1) = \frac{2}{6}, P(E_2) = \frac{4}{6}$	1
	$P(A/E_1) = \frac{6}{9}, P(A/E_2) = \frac{5}{9}$	1
	Using Baye's theorem. $P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$	
	$= \frac{\frac{2}{6}\times\frac{6}{9}}{\frac{2}{6}\times\frac{6}{9}+\frac{4}{6}\times\frac{5}{9}}$	2
	$= \frac{12}{12+20} = \frac{3}{8}$	1
27.	$\lvert A\rvert = -2 \neq 0 \Rightarrow A^{-1}$ exists	$\frac{1}{2}$
	Now, $A_{11} = -1,\ A_{12} = 8,\ A_{13} = -5$ $A_{21} = 1,\ A_{22} = -6,\ A_{23} = 3$ $A_{31} = -1,\ A_{32} = 2,\ A_{33} = -1$	$2\frac{1}{2}$
	$\text{adj } A = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$	
	$A^{-1} = \frac{1}{\lvert A\rvert}\cdot \text{adj } A = \frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$	1
	Given system of equations can be written as AX = B, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 10 \\ 9 \end{bmatrix}$	

	Now $AX = B \Rightarrow X = A^{-1}B$	1
	$$= \frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \\ 9 \end{bmatrix}$$	
	$$= \frac{-1}{2}\begin{bmatrix} -4 \\ -2 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$	$\frac{1}{2}$
	$\therefore \quad x = 2,\ y = 1,\ z = 2$	$\frac{1}{2}$
	OR	
	$A = IA$	
	$$\Rightarrow \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$	1
	$R_1 \to \frac{R_1}{3}$	
	$$\Rightarrow \begin{bmatrix} 1 & \frac{-1}{3} & \frac{1}{3} \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$	
	$R_2 \to R_2 + 15R_1,\ R_3 \to R_3 - 5R_1$	
	$$\Rightarrow \begin{bmatrix} 1 & \frac{-1}{3} & \frac{1}{3} \\ 0 & 1 & 0 \\ 0 & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 5 & 1 & 0 \\ \frac{-5}{3} & 0 & 1 \end{bmatrix} \cdot A$$	
	$R_1 \to R_1 + \frac{1}{3}R_2,\ R_3 \to R_3 + \frac{1}{3}R_2$	
	$$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 2 & \frac{1}{3} & 0 \\ 5 & 1 & 0 \\ 0 & \frac{1}{3} & 1 \end{bmatrix} \cdot A$$	4

	$R_3 \to 3R_3$ $\Rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & \frac{1}{3} & 0 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \cdot A$ $R_1 \to R_1 - \frac{1}{3} R_3$ $\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \cdot A$ $\Rightarrow A^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$	1
28.	 $S = \sqrt{(x-3)^2 + (y-7)^2}$	1
	$\Rightarrow P = S^2 = (x-3)^2 + x^4$	
	$\frac{dP}{dx} = 2(x-3) + 4x^3$	1
	$\frac{dP}{dx} = 0 \Rightarrow 2x^3 + x - 3 = 0$	1
	$\Rightarrow (x-1)(2x^2 + 2x + 3) = 0 \Rightarrow x = 1$	1
	$\frac{d^2P}{dx^2} = 6x^2 + 1 > 0$ at $x = 1$	1
	$\Rightarrow x = 1$ will give minimum distance.	
	Minimum distance $= \sqrt{4+1} = \sqrt{5}$	1
29.	$x^2 + y^2 = 4x \Rightarrow (x-2)^2 + y^2 = 4$ ...(1) Correct Figure  	1
	also, $y^2 = 2x$ ...(2) Solving (1) and (2) point of intersections are (0, 0) and (2, ±2).	1

Required Area

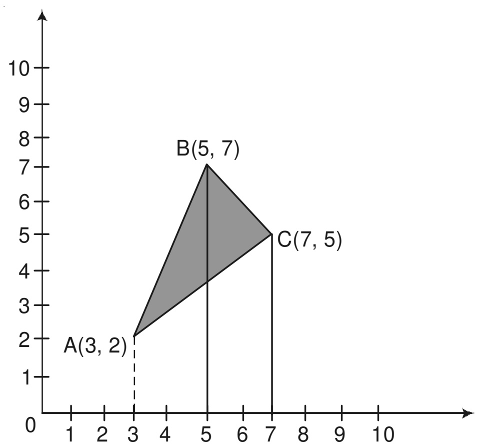
$$= \int_0^2 \sqrt{2}\cdot\sqrt{x}\, dx + \int_2^4 \sqrt{4-(x-2)^2}\, dx$$ 1

$$= \frac{2}{3}\sqrt{2}\,[x^{3/2}]_0^2 + \left[\frac{x-2}{2}\sqrt{4-(x-2)^2} + 2\sin^{-1}\left(\frac{x-2}{2}\right)\right]_2^4$$ 2

$$= \frac{2}{3}\sqrt{2}\cdot 2^{3/2} + [2\sin^{-1}1 - 2\sin^{-1}0]$$

$$= \frac{8}{3} + \pi$$ 1

OR



Correct Figure 1

Equation of AB: $y = \frac{1}{2}(5x-11)$

Equation of BC: $y = -x + 12$ $1\frac{1}{2}$

Equation of AC: $y = \frac{1}{4}(3x-1)$

Required area

$$= \int_3^5 \frac{1}{2}(5x-11)dx + \int_5^7 (-x+12)dx - \int_3^7 \frac{1}{4}(3x-1)dx$$ 1

$$= \frac{1}{2}\frac{(5x-11)^2}{2\times 5}\Bigg]_3^5 + \frac{(12-x)^2}{-2}\Bigg]_5^7 - \frac{1}{4}\cdot\frac{(3x-1)^2}{6}\Bigg]_3^7$$ $1\frac{1}{2}$

$$= \frac{1}{20}(196-16) - \frac{1}{2}(25-49) - \frac{1}{24}(400-64)$$

$$= 7$$ 1