ISC Class 12 Maths Question Paper Solution 2017

MATHEMATICS

SECTION A (80 Marks)

Question 1

[10x3]

(i) If the matrix $\begin{pmatrix} 6 & -x^2 \\ 2x-15 & 10 \end{pmatrix}$ is symmetrix, find the value of x.

(ii) If $y - 2x - k = 0$ touches the conic $3x^2 - 5y^2 = 15$, find the value of k.

(iii) Prove that $\frac{1}{2}cos^{-1}\left(\frac{1-x}{1+x}\right) = tan^{-1}\sqrt{x}$

(iv) Using L'Hospital's Rule, evaluate:

$$\underset{x \to \pi/2}{lt}\left(x\ \ tanx - \frac{\pi}{4}.\sec x\right)$$

(v) Evaluate: $\int \frac{1}{x^2} sin^2\left(\frac{1}{x}\right)\ dx$

(vi) Evaluate:

$$\int_0^{\pi/4} \log(1 + tan\theta)\ d\theta$$

(vii) By using the data $\bar{x} = 25$, $\bar{y} = 30$, $b_{yx} = 1{\cdot}6$ and $b_{xy} = 0{\cdot}4$, find:

(a) The regression equation y on x.

(b) What is the most likely value of y when $x = 60$?

(c) What is the coefficient of correlation between x and y?

(viii) A problem is given to three students whose chances of solving it are $\frac{1}{4}, \frac{1}{5}\ and\ \frac{1}{3}$ respectively. Find the probability that the problem is solved.

(ix) If a + ib = $\frac{x+iy}{x-iy}$ prove that $a^2 + b^2 = 1\ and\ \frac{b}{a} = \frac{2xy}{x^2-y^2}$

(x) Solve: $\frac{dy}{dx} = 1 - xy + y - x$

Comments of Examiners

(i) Most candidates used the concept of singular matrix instead of symmetric matrix. Also, errors were made by some candidates in solving quadratic equation.

(ii) Many candidates while expressing the given equation into standard form of Hyperbola made mistakes. In addition, candidates used incorrect formula of condition of tangency. Some candidates used elimination method but made errors in solving quadratic equation to find the value of 'k' by equating discriminant.

(iii) Some candidates used $x = \cos 2\theta$ but left it half way through. Some candidates attempted it by writing in terms of $\tan^{-1}$ function but failed in the conversion.

(iv) This part of the question was well attempted by most candidates.

(v) Candidates made errors by making inappropriate substitution. Some candidates applied correct substitution $(1/x = t)$ but made errors in further simplification. Many attempted it by incorrect method of integration by parts so could not get the correct solution.

(vi) Many candidates made errors in applying the properties of definite integrals correctly and some candidates attempted it by integration by parts.

(vii) Some candidates considered incorrect regression equations due to lack of understanding of the regression concept.

(viii) Most candidates calculated complement of given probabilities but could not complete the solution due to lack of understanding of the concept of Probability.

(ix) Many candidates took rationalization factor correctly but errors were committed in the process of simplifying the expression. Some candidates not identify real and imaginary parts in the complex number.

(x) Many candidates could not separate the variables to solve the differential equation. Some candidates attempted it by linear equation method but made errors in calculating integrating factor and further. In most of the cases, solution had been written without a constant 'c'.

Suggestions for teachers

- Explain basic concepts of Singular and Symmetric matrices to students. Also provide practice on simple applications based on the above.
- Clarify tangency condition to the students giving sufficient number of examples. Give ample practice for solving different types of problems by applying the tangency condition.
- Explain clearly the conversion of inverse circular functions (one to another form) to students. Give adequate practice for conversion through diagram and by using formulae.
- Thorough practice for calculating Limits of Indeterminate Forms $\left(\frac{0}{0} \text{ or } \frac{\infty}{\infty}\right)$ by application of L'Hospital's Rule should be given to the students.
- Practice should also be given to the students for Graded questions of integration by substitution.
- Teach properties of definite integrals well to the students and give them adequate practice to learn to apply them appropriately.
- Explain clearly the Regression equations Y on X and X on Y and the basic difference between them. Give different types of problems for sufficient practice.
- More practice is required in simplifying the expressions by applying the concept of rationalization.
- Emphasize on mutually exclusive events and independent events. Give plenty of practice to understand the application of $P(A \cup B \cup C) = 1 - P(\bar{A}).P(\bar{B}).P(\bar{C})$.
- Adequate practice should be given on various types of differential equations.

MARKING SCHEME

Question 1

(i)	$\begin{pmatrix} 6 & -x^2 \\ 2x-15 & 10 \end{pmatrix} = \begin{pmatrix} 6 & 2x-15 \\ -x^2 & 10 \end{pmatrix}$ $\Rightarrow \ -x^2 = 2x - 15$ $x^2 + 2x - 15 = 0$ $x^2 + 5x - 3x + 15 = 0$ $(x + 5)\,(x - 3) = 0$ $x = \ -5 \ or \ 3$
(ii)	$3x^2 - 5y^2 = 15$ and Tangent: $y - 2x - k = 0$ $\frac{x^2}{5} - \frac{y^2}{3} = 1 \qquad y = 2x + k$ $\Rightarrow a^2 = 5,\ b^2 = 3,\ m = 2$ and constant $c = k$. Conditions for tangency: $c^2 = a^2\, m^2 - b^2$ $k^2 = 5 \cdot 4 - 3 = 17$ $k = \pm\sqrt{17}$
(iii)	LHS $= \frac{1}{2}\ cos^{-1}\left(\frac{1-x}{1+x}\right)$ Let $x = \tan^2\theta$ LHS $= \frac{1}{2}\ cos^{-1}\left(\frac{1-tan^2\theta}{1+tan^2\theta}\right)$ $= \frac{1}{2}\ cos^{-1}(cos2\theta) = \frac{1}{2} \cdot 2\theta = \theta$ $\tan^2 \theta = x$ $\Rightarrow tan\theta = \sqrt{x}$ $\theta = tan^{-1}\sqrt{x}$ *(Alternate correct method is also acceptable.)*
(iv)	$\underset{x \to \pi/2}{lt} \left(x \ tanx - \frac{\pi}{2}.\sec x\right) \quad (\infty - \infty)$ $\underset{x \to \pi/2}{lt} \left[\frac{x \ sinx}{cosx} - \frac{\pi}{2} \cdot \frac{1}{cosx}\right]$ $\underset{x \to \pi/2}{lt} \left[\frac{x \ sinx - \pi/2}{cos\ x}\right] \qquad \frac{0}{o}\ form$ $\underset{x \to \pi/2}{lt} \left[\frac{x \ cosx + \sin x}{-\ sinx}\right]$ $= \frac{1}{-1} = -1$
(v)	Evaluate: $\int \frac{1}{x^2} sin^2\left(\frac{1}{x}\right)\ dx$ Let $t = {}^1/_x$

	$dt = -\frac{1}{x^2}dx$ $\therefore -\int sin^2t\ dt = -\int \frac{1-cos2\ t}{2}\ dt$ $= \frac{-1}{2}\left[t - \frac{sin2\ t}{2}\right]$ $= +\frac{1}{4}\sin(^2/_x) - \frac{1}{2x} + c$
(vi)	Evaluate: $\int_0^{\pi/4} \log(1 + tan\theta)\ d\theta$ $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ I = $\int_0^{\pi/4} \log[1 + \tan\theta)]\ d\theta$ $= \int_0^{\pi/4} \log[1 + \tan(^\pi/_4 - \theta)]\ d\theta =$ $\int_0^{\pi/4} \log 2 - \log(1 + tan\ \theta)]\ d\theta$ I = + I =2I = $\int_0^{\pi/4} \log 2d\theta = \log 2[\theta]_0^{\pi/4}$ = log 2 . $^\pi/_4$ I = $^\pi/_8$.log2
(vii)	$\bar{x} = 25, \bar{y} = 30, b_{yx} = 1{\cdot}6, b_{xy} = 0{\cdot}4$ Regression equation y on x: $y - \bar{y} = b_{yx}(x - \bar{x})$ $y - 30 = \frac{16}{10}(x - 25)$ $y - 30 = 1{\cdot}6x - 40$ $y = 1{\cdot}6x - 10$ when x = 60 y = (1·6)(60)-10 = 96 – 10 = 86 Coefficient of correlation: r = $\pm\sqrt{b_{yx}.b_{xy}}$ $= \pm\sqrt{(1\cdot 6)(0\cdot 4)} = \pm\sqrt{0\cdot 64}$ = + 0·8 ∵ both regression coefficients +ve.
(viii)	$P(A) = \frac{1}{4},\ P(B) = \frac{1}{5}\ \ P(C) = \frac{1}{3}$ $P(\bar{A}) = 1 - \frac{1}{4}, P(\bar{B}) = 1 - \frac{1}{5}, P(\bar{C}) = 1 - \frac{1}{3}$ $= \frac{3}{4} \qquad = \frac{4}{5} \qquad = \frac{2}{3}$ Probability of none of them solve the problem: $\frac{\cancel{3}}{\cancel{4}}.\frac{\cancel{4}}{5}.\frac{2}{\cancel{3}} = \frac{2}{5}$ ∴ Probability of the problem is being solved = $1 - \frac{2}{5} = \frac{3}{5}$ *(Alternate correct method is also acceptable.)*

(ix)	a + ib = $\frac{x+iy}{x-iy}$ $\|a+ib\| = \left\|\frac{x+iy}{x-iy}\right\|$ $\sqrt{a^2+b^2} = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} = 1$ $\Rightarrow a^2 + b^2 = 1$ a + ib = $\frac{x+iy}{x-iy} \times \frac{x+iy}{x+iy}$ a + ib = $\frac{(x+iy)^2}{x^2+y^2} = \frac{x^2-y^2}{x^2+y^2} + i \quad \frac{2xy}{x^2+y^2}$ a = $\frac{x^2-y^2}{x^2+y^2}$, $\quad b = \frac{2xy}{x^2+y^2}$ $\therefore \frac{b}{a} = \frac{2xy}{x^2-y^2}$
(x)	$\frac{dy}{dx} = (1+y) - x(y+1)$ $= (1+y)(1-x)$ $\Rightarrow \frac{dy}{1+y} = (1-x)dx$ $\int \frac{dy}{1+y} = \int (1-x)dx + c$ $\log(1+y) = x - \frac{x^2}{2} + c$ *(Alternate correct method is also acceptable.)*

Question 2

(a) Using properties of determinants, prove that: **[5]**

$$\begin{vmatrix} a & b & b+c \\ c & a & c+a \\ b & c & a+b \end{vmatrix} = (a+b+c)(a-c)^2$$

(b) Given that: $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ $and\ B = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$, find AB. **[5]**

Using this result, solve the following system of equation:

$x - y = 3$, $2x + 3y + 4z = 17$ and $y + 2z = 7$

Comments of Examiners

(a) Some candidates did not apply the properties of determinants in the right order. Many candidates used $R_1 \rightarrow R_1 + R_2 + R_3$ but failed to take common (a+b+c). In addition, candidates used row/column operation second time but failed to simplify the determinant.

(b) Many candidates did not know how to use the product A.B to solve the given equations. Made errors in substituting $A^{-1} = \frac{1}{6}B$. Some candidates solved the equations by finding A^{-1} by conventional method i.e $\frac{adj\,A}{\det A}$. A few candidates solved it using Cramer's Rule.

Suggestions for teachers

- Explain every property of the determinants with proper examples. Teach this type of questions in the class by discussion method. Stress upon developing logical and reasoning skills to apply the correct property.
- Basic concepts of inverse of a matrix and its properties need to be explained with examples. Adequate practice should be given to students for solving equations by inverse matrix method.

MARKING SCHEME

Question 2

(a)

$$\begin{vmatrix} a & b & b+c \\ c & a & c+a \\ b & c & a+b \end{vmatrix}$$

$R_1 : R_1 + R_2 + R_3 = \begin{vmatrix} a+b+c & a+b+c & 2(a+b+c) \\ c & a & c+a \\ b & c & a+b \end{vmatrix}$

$(a+b+c) \begin{vmatrix} 1 & 1 & 2 \\ c & a & c+a \\ b & c & a+b \end{vmatrix}$

$C_1 : C_1 - C_2 \quad (a+b+c) \begin{vmatrix} 0 & 1 & 0 \\ c-a & a & c-a \\ b-c & c & a+b-2c \end{vmatrix}$

$C_3 : C_3 - 2C_2$

$(a+b+c)\,[-1\{(c-a)(a+b-2c) - (c-a)(b-c)\}]$

$(a+b+c)\,(a-c)\,[a+b-2c-b+c]$

$(a+b+c)\,(a-c)^2$

(b)

$A.B = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$

$= \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix} = 6I$

x – y = 3, 2x + 3y + 4z = 17, y + 2z = 7

$$= \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 17 \\ 7 \end{pmatrix}$$

AB = 6I

$$\Rightarrow A.\left(\frac{1}{6}B\right) = I$$

$$\Rightarrow A^{-1} = \frac{1}{6}B = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$x = A^{-1}\begin{pmatrix} 3 \\ 17 \\ 7 \end{pmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 12 \\ -2 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$x = 2,\ y = -1, z = 4$

Question 3

(a) Solve the equation for *x*: **[5]**

$$\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1}x,\ x \neq 0$$

(b) If A, B and C are the elements of Boolean algebra, simplify the expression (A′+B′) (A + C′) + B′ (B + C). Draw the simplified circuit. **[5]**

Comments of Examiners

(a) Many candidates made errors in applying the formula of $\sin^{-1} A + \sin^{-1} B$. In addition, errors took place in simplifying and solving higher degree algebraic equations. Some candidates converted all terms into Inverse tangent function form and could not handle the resulting equations.

(b) Many candidates made errors after applying the distributive property and some candidates were not clear about addition and multiplication sign. Many candidates made errors in drawing the circuit diagram.

Suggestions for teachers

– Teach Inverse Circular Functions laws thoroughly to the students. Also, give them adequate practice for inter conversion of the functions. More practice should be given in solving different types of higher order equations.

- Explain laws of Boolean algebra comprehensively to the students and ample practice should be given on simplification of different types of Boolean expressions and then drawing their simplified circuits.

MARKING SCHEME

Question 3

(a)

$$\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$$

$$\sin^{-1}x + \sin^{-1}(1-x) = \frac{\pi}{2} - \sin^{-1}x$$

$$\sin^{-1}(1-x) = \frac{\pi}{2} - 2\sin^{-1}x$$

$$1 - x = \sin[\pi/2 - 2\sin^{-1}x]$$

$$1 - x = \cos(2\sin^{-1}x)$$

$$1 - x = \cos[\cos^{-1}(1-2x^2)]$$

$$\Rightarrow \quad 1 - x = 1 - 2x^2$$

$$2x^2 - x = 0$$

$$x(2x-1) = 0$$

$$\Rightarrow x = ½$$

(b)

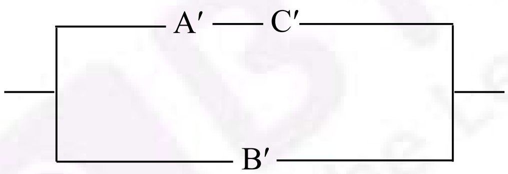
$(A' + B')(A + C') + B'(B + C)$

$A'A + A'C' + B'A + B'C' + B'B + B'C$ $\quad (A'A = 0,\ B'B = 0)$

$A'C' + B'A + B'(C' + C)$ $\quad (\because C' + C = 1)$

$A'C' + B'A + B'$

$A'C' + B'$



Question 4

(a) Verify Lagrange's mean value theorem for the function: **[5]**

$f(x) = x(1 - \log x)$ and find the value of 'c' in the interval $[1, 2]$

(b) Find the coordinates of the centre, foci and equation of directrix of the hyperbola $x^2 - 3y^2 - 4x = 8$. **[5]**

Comments of Examiners

(a) Many candidates got confused about closed interval and open interval while proving Lagrange's Mean Value theorem. Some candidates committed errors in solving logarithmic equations as they could not make a distinction between logarithmic function with base '10' and base 'e'.

(b) Many candidates failed to convert the given equation into standard form of conic section. The candidates also made errors while applying the concepts such as shifting the origin and translation of axes.

Suggestions for teachers

- Teach Lagrange's Mean Value theorem to the students in detail. Also discuss difference between 'closed' and 'open' intervals with their significance by sketching the curve etc.
- Explain basic concepts and properties of logarithmic functions with examples in the class.
- Explain to the students, conics, their equations, basic terms related to them thoroughly giving appropriate examples. Also, give adequate practice for simplification of general conic equation into standard conic equation, shifting of origin and translation of axes to compute foci, directrix and eccentricity, etc.

MARKING SCHEME

Question 4

(a)

(i) $f(x) = x(1 - logx)$ is continuous function in the closed interval [1,2]

(ii) $f^1(x) = x\left(\frac{-1}{x}\right)+(1-\log x)$

$= -\log x$

$f^1(x)$existing in open (1, 2)

(iii) $f(1) = 1(1 - \log 1) = 1$

$f(2) = 2(1 - log2) =$

$f(1) \neq f(2)$

$\because$ All three conditions of Lagrange's mean values are satisfied, there exists at least one value of 'c' such that:

$$f^1(c) = \frac{f(b) - f(a)}{b - a}$$

$$-\log c = \frac{2(1 - log2) - 1}{2 - 1}$$

$$-\log c = 2 - 2log2 - 1$$

$$\log c = -2log2 - 1$$

$$= 2log2 - \log e$$

$$\log c = log\, 4/e$$

$$c = 4/e \quad \epsilon(1,2)$$

(b)	$x^2 - 3y^2 - 4x = 8$ $x^2 - 4x + 4 - 3y^2 = 8 + 4$ $(x-2)^2 - 3y^2 = 12$ $\frac{(x-2)^2}{12} - \frac{y^2}{4} = 1$ $\Rightarrow a^2 = 12,\ b^2 = 4$ $b^2 = a^2(e^2 - 1)$ $4 = 12(e^2 - 1)$ $\frac{1}{3} + 1 = e^2$ $e^2 = {}^4/_3$ $e = \frac{2}{\sqrt{3}}$ ∴ Centre (2, 0) Focus $(X, Y) = (\pm ae, 0)$ $X = \pm ae,\ Y = 0$ $x - 2 = \pm\sqrt{12}.\frac{2}{\sqrt{3}}, \quad y = 0$ $x = \pm 4 + 2, \quad y = 0$ $x = 6 \ or -2, \quad y = 0$ (6, 0) and $(-2, 0)$ *Foci* Equation of directrix: $X = \pm {}^a/_e$ $x - 2 = \pm\sqrt{12}.\frac{\sqrt{3}}{2}$ $x - 2 = \pm 3$ $x = \pm 3 + 2$ $x = 5 \ or -1$

Question 5

(a) If y = *cos* (*sin x*), show that: **[5]**

$$\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$$

(b) Show that the surface area of a closed cuboid with square base and given volume is minimum when it is a cube. **[5]**

Comments of Examiners

(a) First order differentiation was attempted correctly by many candidates but some made errors in second order derivative and framing the required equation. Several candidates failed to differentiate using chain rule.

(b) Many candidates did not express volume and surface area of cube with square base by correct formula. Most of the candidates did not show the working of second order derivative and failed to prove $\frac{d^2S}{dx^2} > 0$.

Suggestions for teachers

- Ample practice of derivatives of all forms of functions should be given to the students.
- Ensure that the students are familiar with the terms area, perimeter, surface and volume of 2-dimensional and 3-dimensional figures referred to in the syllabus.
- Train students to express the function to be optimized in terms of a single variable by using the given data.

For minimum value *f'(x)=0* and *f''(x) > 0*.

MARKING SCHEME

Question 5

(a)	$y = \cos(sinx)\ show\ that:$ $\frac{d^2y}{dx^2} + tanx.\frac{dy}{dx} + y.\ \ cos^2x = 0$ $\frac{dy}{dx} = -\sin(sinx)\,.cosx$ $\frac{d^2y}{dx^2} = -[cos^2x.\cos(sinx) - \sin(sinx)\,.sinx]$ $\frac{d^2y}{dx^2} = -cos^2x.y + sinx.\sin(sinx)$ $\frac{d^2y}{dx^2} = -cos^2x.y - \sin x.\frac{1}{cosx}.\frac{dy}{dx}$ $\frac{d^2y}{dx^2} + \ tanx\frac{dy}{dx} + y\ cos^2x = 0$
(b)	Let the length and breadth be x units (square base), height of cuboid be = h unit. $\therefore V = x^2h \qquad \Rightarrow \ h = {}^{v}/_{x^2}$ S = $2(x^2 + xh + xh)$ = $(2x^2) + 4xh$ S = $2x^2 + 4x.\frac{v}{x^2}$ = $2x^2 + 4v.\frac{1}{x}$ $\frac{ds}{dx} = 4x - 4v.\frac{1}{x^2}$ $\frac{ds}{dx} = 0 \quad \therefore \quad 4x = \frac{4v}{x^2}$ $x^3 = v$

	$x^3 = x^2h$ $\Rightarrow x = h$ $\frac{d^2s}{dx^2} = 4 + \frac{8v}{x^3}$ at x = h $\frac{d^2s}{dx^2} = 4 + \frac{8v}{h^3} > 0$ $\therefore$ Surace area is minimum when $x = h$ i.e. When it is a cube, surface area will be minimum.

Question 6

(a) Evaluate: **[5]**

$$\int \frac{\sin 2x}{(1+\sin x)\,(2+\sin x)}\,dx$$

(b) Draw a rough sketch of the curve $y^2 = 4x$ and find the area of the region enclosed by the curve and the line $y = x$. **[5]**

Comments of Examiners

(a) Some candidates made errors in substitution. In most of the cases, candidates who used appropriate substitution did not split the function into partial fractions to find the integral of each term.

(b) Many candidates got the correct solution for this question. A few candidates solved the question with incorrect limits. Candidates made mistakes in sketching the curve, finding limits and required area bounded by the curves.

Suggestions for teachers

- Give students a thorough understanding and good practice of Integration using substitution, partial fractions and special integrals. Instruct students not to leave their answer in terms of a new variable.
- Train students to sketch the curve to understand the requirement like the area required to be found, the points of intersection and the limits of the definite integral. Adequate practice of solving (sketching) different types of curves should be given.

MARKING SCHEME

Question 6

(a)

$$\int \frac{sin2x}{(1+sinx)\,(2+sinx)}\,dx$$

$$= \int \frac{2sinx\ cosx}{(1+sinx)\,(2+sinx)}\,dx$$ Let $sin\ x = t$

Cos $x = dt$

$$= \int \frac{2tdt}{(1+t)2+t)}$$

$$\frac{t}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t}$$

$t = A(2+t) + B(1+t)$

$-1 = +A \Rightarrow A = -1$

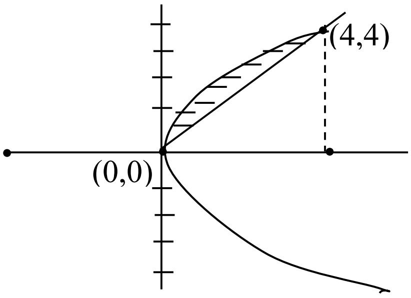
B = 2

	$= \int \left(\frac{1}{1+t} + \frac{2}{2+t}\right) dt$ $= 2[-\log(1+t) + 2\log(2+t)]$ $= -2\log(1+sinx) + 4\log(2+sinx) + c$ $(Alternate\ correct\ method\ is\ also\ acceptable)$
(b)	$y^2 = 4x$

X	0	1	1	4	+4
y	0	2	-2	4	-4

y = x

X	0	1	2	4
y	0	1	2	4



Area of the shaded region

$= 2\int_0^4 \sqrt{x}\ dx - \int_0^4 x\ dx$

$= 2.\frac{2}{3}.\left[x^{3/2}\right]_0^4 - \frac{1}{2}[x^2]_0^4$

$= \frac{4}{3}[4^{3/2}] - \frac{1}{2}[4^2]$

$= \frac{32}{3} - 8 = \frac{8}{3}$ sq units

Question 7

(a) Calculate the Spearman's rank correlation coefficient for the following data and interpret the result: **[5]**

X	35	54	80	95	73	73	35	91	83	81
Y	40	60	75	90	70	75	38	95	75	70

(b) Find the line of best fit for the following data, treating *x* as dependent variable (Regression equation *x* on *y*): **[5]**

X	14	12	13	14	16	10	13	12
Y	14	23	17	24	18	25	23	24

Hence, estimate the value of *x* when *y* = 16.

Comments of Examiners

(a) Many candidates made errors in computing ranks for the scores. Some candidates did not have an idea of repetition of rank and correction factor. Many used wrong formula for Spearman's correlation coefficient. Most of the candidates did not interpret the result.

(b) Many candidates made errors in finding the regression equation X on Y. Candidates were not clear about the formulae used for b_{yx} and b_{xy}. Many candidates failed to get the correct value of x for $y = 16$. Some candidates started solving with the help of normal equations but did not get the result.

Suggestions for teachers

- Provide plenty of practice for solving problems on correlation.
- Train students to use the appropriate formula as per the requirement of the question. Accuracy levels are to be maintained at highest level as such simplifications involve simple calculations only.

MARKING SCHEME

Question 7

(a)

x	y	R_1	R_2	$D=R_1-R_2$	D^2
35	40	1·5	2	-0·5	0·25
54	60	3	3	0	0
80	75	6	7	-1	1
95	90	10	9	1	1
73	70	4·5	4·5	0	0
73	75	4·5	7	-2·5	6·25
35	38	1·5	1	0·5	0·25
91	95	9	10	-1	1
83	75	8	7	1	1
81	70	7	4·5	2·5	6·25
					$\sum D^2=17$

$$C.F = \frac{1}{12}\sum(m^3 - m)$$

$$= \frac{1}{12}[(2^3-2)+(2^3-2)+(2^3-2)+(3^3-3)]$$

$$= \frac{1}{12}[18+24] = 3\cdot 5$$

$$R = 1 - \frac{6[\sum D^2 + C.F]}{n(n^2-1)} = 1 - \frac{6[17+3\cdot5]}{10\times99} = \frac{867}{990} = 0\cdot875$$

Very high positive correlation

(b)

x	y	dx	dy	dx^2	dy^2	dx.dy
14	14	1	-7	1	49	-7
12	23	-1	2	1	4	-2
13	17	0	-4	0	16	0
14	24	1	3	1	9	3

16	18	3	-3	9	9	-9
10	25	-3	4	9	16	-12
13	23	0	2	0	4	0
12	24	-1	3	1	9	-3

$\sum x = 104$ $\sum y = 168$ $\sum dx = 0$ $\sum dy = 0$ $\sum dx^2 = 22$ $\sum dy^2 = 116$ $\sum dxdy = 30$

$\bar{x} = 13$ $\bar{y} = 21$

$b_{xy} = \frac{\sum dxdy}{\sum dy^2} = \frac{-30}{116}$

Required regression line: $x - \bar{x} = b_{xy}(y - \bar{y})$

$$x - 13 = \frac{-30}{116}(y - 21)$$

$$116x + 30y - 2138 = 0$$

When $y = 16$ $x = 14 \cdot 29$

Question 8

(a) In a class of 60 students, 30 opted for Mathematics, 32 opted for Biology and 24 opted for both Mathematics and Biology. If one of these students is selected at random, find the probability that: **[5]**

(i) The student opted for Mathematics or Biology.

(ii) The student has opted neither Mathematics nor Biology.

(iii) The student has opted Mathematics but not Biology.

(b) Bag A contains 1 white, 2 blue and 3 red balls. Bag B contains 3 white, 3 blue and 2 red balls. Bag C contains 2 white, 3 blue and 4 red balls. One bag is selected at random and then two balls are drawn from the selected bag. Find the probability that the balls drawn are white and red. **[5]**

Comments of Examiners

(a) Many candidates used Venn diagrams but incorrect values of $P(A \cap B)$. Some candidates noted as $P(A \cup B) = \frac{24}{60}$ instead of $P(A \cap B) = \frac{24}{60}$ Most of the candidates could not write the required combinations correctly. Many candidates wrote incorrectly $P(A \cup B) = P(A) + P(B)$.
Some candidates did not understand third part of the question.

(b) Many candidates did not write this part correctly. They mixed the probabilities of one white and one red. Many candidates wrote the result without multiplying the result with 1/3.

Suggestions for teachers

- Explain the probability properties/laws and their applications thoroughly.
- Lay stress on making the students understand the problem and identify the cases that satisfy the situation, conditions, conditional probability and condition-based problems.

MARKING SCHEME

Question 8

(a) Let A: event that candidates opted Mathematics.
Let B: event that candidates opted Biology.

P(A) = $\frac{30}{60}$ $P(B) = \frac{32}{60}$ $P(A \cap B) = \frac{24}{60}$

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{30}{60} + \frac{32}{60} - \frac{24}{60} = \frac{19}{30}$

(ii) $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - \frac{19}{30} = \frac{11}{30}$

(iii) Probability of student has opted Mathematics but not Biology

$= P(A) - P(A \cap B)$

$= \frac{30}{60} - \frac{24}{60} = \frac{6}{60} = \frac{1}{10}$

(b) Probability of selecting each bag = 1/3.

	A	**B**	**C**
White	1	3	2
Blue	2	3	3
Red	3	2	4
Prob of (1W&1R	$\frac{1 \times 3C_1}{6C_2}$	$\frac{3C_1 \times 2C_1}{8C_2}$	$\frac{2C_1 \times 4C_1}{9C_2}$

The probability: $\frac{1}{3}\left[\frac{1\times 3C_1}{6C_2} + \frac{3C_1\times 2C_1}{8C_2} + \frac{2C_1\times 4C_1}{9C_2}\right]$

$= \frac{1}{3}\left[\frac{1\times 3}{15} + \frac{3\times 2}{28} + \frac{2\times 4}{36}\right]$

$= \frac{1}{3}\left(\frac{1}{5} + \frac{3}{14} + \frac{2}{9}\right) = \frac{401}{1890} = 0.212$

Question 9

(a) Prove that locus of z is circle and find its centre and radius if $\frac{z-i}{z-1}$ is purely imaginary. **[5]**

(b) Solve: $(x^2 - yx^2)dy + (y^2 + xy^2)\,dx = 0$ **[5]**

Comments of Examiners

(a) Many candidates took $\frac{Z-i}{Z-1} = ik$ but could not simplify it further. Some candidates took $\frac{Z-1}{Z+1}$ instead of $\frac{Z-i}{Z-1}$. Many candidates made errors in simplification and identifying the real part, and equating it to zero. Some candidates made mistakes while noting down the centre and with correct radius.

(b) Many candidates attempted it by the method of Homogeneous equations by substituting y=VX but could not proceed further. Many candidates made errors in separating the variables *x* and *y* and a few candidates did not write the constant C.

Suggestions for teachers

- The concept of Rationalization done in previous classes must be revised.
- Familiarise students with locus of the standard equation of Circle, Ellipse, Hyperbola etc. Give more practice for sketching of straight lines and curves (circle, conics, etc.).
- Clarify all the forms of differential equations and provide sufficient practice in solving problems based on these equations.

MARKING SCHEME

Question 9

(a)	Let z = x + iy $\frac{z-i}{z-1} = \frac{x+iy-i}{x+iy-1} = \frac{x+i(y-1)}{(x-1)+iy}$ Rationalise denominator $= \frac{x+i(y-1)}{(x-1)+iy} \times \frac{(x-1)-iy}{(x-1)-iy}$ $= \frac{x(x-1)+y(y-1)-ixy+i(x-1)(y-1)}{(x-1)^2+y^2}$ $\because \frac{z-i}{z-1}$ is purely imaginary Real part = 0 $\therefore\ x(x-1)+y(y-1)=0$ $x^2+y^2-x-y=0$ which is a circle having its centre $(\frac{1}{2},\frac{1}{2})$ and radius = $\frac{1}{\sqrt{2}}$
(b)	$(x^2-yx^2)\,dy + (y^2+xy^2)dx = 0$ $x^2(1-y)dy + y^2(1+x)dx = 0$ $\frac{1-y}{y^2}dy + \frac{1+x}{x^2}dx = 0$ $\int \frac{1-y}{y^2}dy + \int \frac{1+x}{x^2}dx = C$ $\frac{-1}{y} - \log y\ \frac{-1}{x} + \log x = C$ $\log(x/y) - \frac{1}{x} + \frac{1}{y} + C$

SECTION B (20 Marks)

Question 10

(a) If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors of equal magnitude, prove that $(\vec{a} + \vec{b} + \vec{c})$ is equally inclined with vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$. **[5]**

(b) Find the value of λ for which the four points with position vectors $6\hat{\imath} - 7\hat{\jmath}$, $16\hat{\imath} - 19\hat{\jmath} - 4\hat{k}$, $\lambda\hat{\jmath} - 6\hat{k}$ and $2\hat{\imath} - 5\hat{\jmath} + 10\hat{k}$ are coplanar. **[5]**

Comments of Examiners

(a) Many candidates used $\vec{a} \times (\vec{a} + \vec{b} + \vec{c})$ instead of $\vec{a} \cdot (\vec{a} + \vec{b} + \vec{c})$ and could not simplify further. Many candidates were unable to express the formula for $\cos\alpha = \frac{\vec{a} \cdot (\vec{a}+\vec{b}+\vec{c})}{|\vec{a}| \cdot |\vec{a}+\vec{b}+\vec{c}|}$.

(b) Some candidates made errors in finding the vectors $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$. Some candidates expanded the scalar triple product determinant incorrectly. A few candidates did not know the basic difference between position vector and vector.

Suggestions for teachers

- Explain in detail about vectors [Scalar (dot) product of vectors, Cross product of vectors, their properties, Scalar triple product, -Proofs of Formulae (Using Vectors)- Sine rule, Cosine rule, Projection formula, Area of a triangle etc.]
- Teach Scalar triple product and its applications with the help of practical examples.
- Advise students to read the instructions given in the question carefully.

MARKING SCHEME

Question 10

(a) $\vec{a}, \vec{b}$ *and* $\vec{c}$ are mutually perpendicular.

$\therefore \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$

$|\vec{a}| = |\vec{b}| = |\vec{c}| = k \ (constant)$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c})$$
$$= \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{c} + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$
$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 = k^2 + k^2 + k^2 = 3k^2$$
$$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}\,k$$

Let $(\vec{a} + \vec{b} + \vec{c})$ make angles α, β and γ with vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$ respectively, then

$$(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{a} = |\vec{a} + \vec{b} + \vec{c}| \cdot |\vec{a}| \cos\alpha$$
$$(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{b} = |\vec{a} + \vec{b} + \vec{c}| \cdot |\vec{b}| \cos\beta$$
$$(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{c} = |\vec{a} + \vec{b} + \vec{c}| \cdot |\vec{c}| \cos\gamma$$

$$\cos\alpha = \frac{\vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}}{\sqrt{3}\,k \cdot k} = \frac{k^2}{\sqrt{3}\,k^2} = \frac{1}{\sqrt{3}}$$
$$\cos\beta = \frac{\vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{b} \cdot \vec{c}}{\sqrt{3}\,k \cdot k} = \frac{k^2}{\sqrt{3}\,k^2} = \frac{1}{\sqrt{3}}$$
$$\cos\gamma = \frac{\vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} + \vec{c} \cdot \vec{c}}{\sqrt{3}\,k \cdot k} = \frac{k^2}{\sqrt{3}\,k^2} = \frac{1}{\sqrt{3}}$$
$$\therefore \alpha = \beta = \gamma = \cos^{-1}\left(1/\sqrt{3}\right)$$

(b)	$\overrightarrow{OA}= 6\hat{\imath} - 7\hat{\jmath}$ $\overrightarrow{OB}=16\hat{\imath} - 19\hat{\jmath} -4\hat{k}$ $\overrightarrow{OC}= \lambda\hat{\jmath} - 6\hat{k}$ $\overrightarrow{OD}= 2\hat{\imath} - 5\hat{\jmath} +10\hat{k}$ $\overrightarrow{AB}= \overrightarrow{OB} -\overrightarrow{OA} = 10\hat{\imath} - 12\hat{\jmath} - 4\hat{k}$ $\overrightarrow{AC}= \overrightarrow{OC} -\overrightarrow{OA} = -6\hat{\imath} + (\lambda+7)\hat{\jmath} - 6\hat{k}$ $\overrightarrow{AD}= \overrightarrow{OD} -\overrightarrow{OA}= -4\hat{\imath} + 2\hat{\jmath} + 10\hat{k}$ $\because$ Points A, B, C and D are coplanar $[\overrightarrow{AB}\ \ \overrightarrow{AC}\ \ \overrightarrow{AD}] = 0$ $\therefore \begin{vmatrix} 10 & -12 & -4 \\ -6 & \lambda+7 & -6 \\ -4 & 2 & 10 \end{vmatrix} = 0$ $10[\ 10\ (\lambda+ 7) + 12] + 12\ [-60 - 24] - 4[-12 + 4(\lambda + 7)] = 0$ $10[\ 10\ \lambda+ 82] + 12[-84] - 4[4\lambda + 16] = 0$ $84\lambda = 252$ $\lambda = 3$

Question 11

(a) Show that the lines $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ *and* $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ intersect. Find the coordinates of their point of intersection. **[5]**

(b) Find the equation of the plane passing through the point $(1,-2, 1)$ and perpendicular to the line joining the points A (3, 2, 1) and B (1, 4, 2). **[5]**

Comments of Examiners

(a) Many candidates formed correct equations from the general points and some of them found the values of constants λ and μ correctly. However, they did not do the verification to prove that the lines are intersecting. Many candidates used incorrect formula to find the shortest distance, to prove the lines are intersecting. Some candidates applied vector method and proved lines are intersecting but failed to calculate the point of intersection.

(b) Many candidates made errors in applying the basic concept that the direction ratios of normal to the plane and direction ratios of line perpendicular to the plane are proportional.

Suggestions for teachers

- Explain three-dimensional geometry thoroughly especially:
  - the different methods for finding the shortest distance between two lines.
  - if two lines intersect, the method to find the coordinates of their point of intersection.
  - the concept of expressing the equation of line and plane in cartesian and vector form and vice versa.
  - different types and methods of finding the equation of a plane and straight line in various forms.
  - condition for a line and plane to be perpendicular, condition for two planes to be perpendicular.

MARKING SCHEME

Question 11

(a)	$\frac{x-4}{1} = \frac{y+3}{-4} = \frac{Z+1}{7} = \lambda$ ----(1) and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \mu$ ---- (2) ⇒ General point on (1) P, $(\lambda+4, -4\lambda - 3\ \ 7\lambda - 1)$ General point on (2) Q, $(2\mu + 1, -3\mu - 1, 8\mu - 10)$ If the lines intersect for some values of λ and μ the points P and Q coincide. $\therefore\ \lambda + 4 = 2\mu + 1 \Rightarrow \lambda - 2\mu = -3$ ----(i) $-4\lambda - 3 = -3\mu - 1 \Rightarrow -4\lambda + 3\mu = 2$ ----(ii) $7\lambda - 1 = 8\mu - 10 \Rightarrow 7\lambda - 8\mu = -9$ ------ (iii) Solving (i) and (ii) λ = 1 and μ = 2 and Substituting λ and μ in equation (iii) $7\cdot1 - 8\cdot2 = -9$ Satisfied the (iii) equation. i.e. the lines intersect point of intersection is P $(5, -7, 6)$
(b)	Any plane passing through the point $(1, -2, 1)$ $a(x-1) + b(y+2) + c(z-1) = 0$ D.R. of normal to the plane: a, b, c This plane is perpendicular to the line joining points A (3, 2, 1), B (1, 4, 2) $\therefore D.\ 'R'of\ line\ perpendicular\ to\ plane: 3-1, 2-4, 1-2\ i.e., 2-2, -1$ ∵ D.'R' of normal to the plane and D.R' line perpendicular to the plane are proportional. $\frac{a}{2} = \frac{b}{-2} = \frac{c}{-1} = k$ a = $2k$, b = $-2k$, c = $-k$ ∴ Required equation of the plane is: $2k\,(x-12k(y+2) - k(z-1) = 0$ $\Rightarrow 2x - 2 - 2y - 4 - z + 1 = 0$ $2x - 2y - z - 5 = 0$

Question 12

(a) A fair die is rolled. If face 1 turns up, a ball is drawn from Bag A. If face 2 or 3 turns up, a ball is drawn from Bag B. If face 4 or 5 or 6 turns up, a ball is drawn from Bag C. Bag A contains 3 red and 2 white balls, Bag B contains 3 red and 4 white balls and Bag C contains 4 red and 5 white balls. The die is rolled, a Bag is picked up and a ball is drawn. If the drawn ball is red, what is the probability that it is drawn from Bag B? **[5]**

(b) An urn contains 25 balls of which 10 balls are red and the remaining green. A ball is drawn at random from the urn, the colour is noted and the ball is replaced. If 6 balls are drawn in this way, find the probability that: **[5]**

(i) All the balls are red.

(ii) Not more than 2 balls are green.

(iii) Number of red balls and green balls are equal.

Comments of Examiners

(a) Some candidates made errors in computing the probabilities of selecting Bag A, Bag B and Bag C as 1/3 instead of 1/6, 2/6 and 3/6 respectively and were unable to get the result. Some candidates did not apply the theorem correctly.

(b) Many candidates made errors in computing the values of p and q and applying the binomial probability formula $P(X = r) = nC_r p^r q^{n-r}$. Most of the candidates were not clear in writing the correct Binomial Distribution.

Suggestions for teachers

- Emphasize that probabilities are ratios, not numbers. Clarify the concept of conditional probability which helps the students to understand the advanced concept of Bayes' theorem.
- Ensure extensive drill on concepts mean and variance of binomial probability distribution for thorough understanding.

MARKING SCHEME

Question 12

(a)	E_1: Event that bag A is picked E_2: Event that bag B is picked E_3: Event that bag C is picked $P(E_1) = {}^1/_6 \quad P(E_2) = {}^2/_6 \quad P(E_3) = {}^3/_6$ $P(R/E_1) = \frac{3}{5} \quad P(R/E_2) = \frac{3}{7} \quad P(R/E_3) = \frac{4}{9}$ By applying Bay's theorem: $P(E_2/R) = \frac{P(E_2).P(R/E_2)}{P(E_1)\,P((R/E_1) + P(E_2).P(R/E_2) + P(E_3).P(R/E_3)}$ $= \frac{\frac{2}{6}\cdot\frac{3}{7}}{\frac{1}{6}\cdot\frac{3}{5}+\frac{2}{6}\cdot\frac{3}{7}+\frac{3}{6}\cdot\frac{4}{9}} \qquad = \frac{90}{293}$
(b)	Number of red balls: 10, number of green balls: 15 $P(R) = \frac{10}{25} \quad P(G) = \frac{15}{25} = \frac{3}{5}$ $= \frac{2}{5} \quad P = {}^2/_5 \quad q = {}^3/_5$ Number of ways of drawing a red ball: X = 0, 1, 2, 3, 4, 5, 6 $P(x=r) = nc_r.P^r.q^{n-r}$ i) All are red $P(X=6) = 6C_6 \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^0 = \left(\frac{2}{5}\right)^6$ ii) Not more than 2 are green: $\Rightarrow = P(x=4) + P(x=5) + P(x=6)$ $P(x \geq 4) = 6C_4 \left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right)^2 + 6C_5 \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right)^1 + 6C_6 \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^0$

	iii) Equal number of balls $P(x=3) = 6C_3 \left(\frac{2}{5}\right)^3 . \left(\frac{3}{5}\right)^3$

SECTION C (20 Marks)

Question 13

(a) A machine costs₹ 60,000 and its effective life is estimated to be 25 years. A sinking fund is to be created for replacing the machine at the end of its life time when its scrap value is estimated as ₹5,000. The price of the new machine is estimated to be 100% more than the price of the present one. Find the amount that should be set aside at the end of each year, out of the profits, for the sinking fund if it accumulates at an interest of 6% per annum compounded annually. **[5]**

(b) A farmer has a supply of chemical fertilizer of type A which contains 10% nitrogen and 6% phosphoric acid and of type B which contains 5% nitrogen and 10% phosphoric acid. After soil test, it is found that at least 7 kg of nitrogen and same quantity of phosphoric acid is required for a good crop. The fertilizer of type A costs ₹5.00 per kg and the type B costs ₹8.00 per kg. Using Linear programming, find how many kilograms of each type of the fertilizer should be bought to meet the requirement and for the cost to be minimum. Find the feasible region in the graph. **[5]**

Comments of Examiners

(a) Some candidates made errors in calculating the price of a new machine after 25 years. Some candidates applied incorrect formula. Many candidates made simplification errors in computing the value of 'a'.

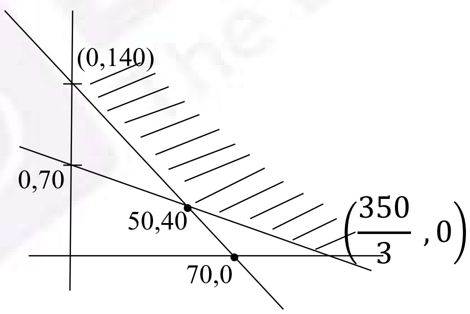
(b) Many candidates made errors in calculating the conditions using constraints. Some candidates were unable to represent feasible reason on the graph as such they may not have clarity of the concept of solving linear equations graphically. Some candidates solved it in terms of decimals and mostly went incorrect in simplifications.

Suggestions for teachers

- Explain the difference between the present value of an annuity and the amount of an annuity and the use of the appropriate formula.
- Emphasize on basic concept of sinking fund, present value, maturity value.
- The optimum function and all possible constraints in the form of in-equations must be put down from what is stated in the problem.
- Train students to solve the constraints equations in pairs to obtain all feasible points leading to optimal value (maximum or minimum) of desired function.

MARKING SCHEME

Question 13

(a)	Amount required at the end of 25 years = 60,000 + 60,000 – 5,000 = 1,15,000 n = 25, i= $\frac{6}{100} = 0 \cdot 06$ A = $\frac{a}{i}\ [(1+i)^n - 1]$ 1,15,000 = $\frac{a}{0\cdot 06}\ [(1 \cdot 06)^{25} - 1]$ a $[(1 \cdot 06)^{25} - 1]$ = 115000×0· 06 =6900 a(3.291871)= 6900, and a = 2096.073
(b)	Let x kg of type A fertilizer and y kg of type B fertilizer required. Type A cost per kg = Rs 5, type B cost per kg = Rs.8 C = 5x + 8y subject to the following constraints: 10% of x + 5% of y ≥ 7 and 6% of x + 10% of y ≥ 7, $x \geq 0$ *and* $y \geq 0$ $10x + 5y \geq 700$ $2x + y \geq 140 - - - - - - 1$ $6x + 10y \geq 700$ $3x + 5y \geq 350 - - - - 2$ $2x + y \geq 140 \Rightarrow x = 70, y = 0 \Rightarrow (70, 0)$ $\Rightarrow x = 0, y = 140 \Rightarrow (0, 140)$ $3x + 5y \geq 350 \Rightarrow x = \frac{350}{3}, y = 0 \Rightarrow (\frac{350}{3}, 0)$ $\Rightarrow x = 0, y = 70 \Rightarrow (0, 70)$ Solving 2x + y = 140 and 3x + 5y = 350 we get x = 50 and y = 40 (50, 40)  The shaded region is the feasible region. C = 5x + 8y At $(\frac{350}{3}, 0)$ C = $5 \cdot \frac{350}{3} + 0 = 583\frac{1}{3}$ (50, 40) C = 5×50+8×40 = 570 (0, 140) C = 5×0+8×140 = 1120 So, it is minimum at (50, 40) ∴ Minimum cost is Rs 570 for 50 kg of type A and 40 kg of type B fertilisers procured.

Question 14

(a) The demand for a certain product is represented by the equation $p = 500 + 25x - \frac{x^2}{3}$ in rupees where x is the number of units and p is the price per unit. Find: **[5]**

(i) Marginal revenue function.

(ii) The marginal revenue when 10 units are sold.

(b) A bill of ₹60,000 payable 10 months after date was discounted for ₹57,300 on 30th June,2007. If the rate of interest was 11¼% per annum, on what date was the bill drawn? **[5]**

Comments of Examiners

(a) Many candidates made errors in applying the concept of Revenue Function. Some candidates made errors in differentiating the revenue function. In addition, calculation errors made in finding the Marginal Revenue function at x=10.

(b) Most candidates could not solve this question completely. Many candidates erred in calculating the value 'n'. Some candidates attempted to solve by taking true discount instead of banker's discount. Some candidates had no idea of calculating the date, when the bill was discounted.

Suggestions for teachers

- Explain the application of 1st order & 2nd order differentiation in Commerce and Economics thoroughly to develop a better understanding.
- Clarify each term like Cost function, Breakeven point, Profit function, demand function Revenue function and Marginal revenue function, etc.
- All relevant terms and formulae need to be taught well for complete understanding. In addition, good practice of different types of questions needs to be given.

MARKING SCHEME

Question 14

(a) Demand function $P = 500 + 25x - \frac{x^2}{3}$

Revenue for function = R(x) = P.x

$= (500 + 25x - \frac{x^2}{3})x$

$= 500x + 25x^2 - \frac{x^3}{3}$

Marginal revenue function:

$= \frac{dy}{dx}(R)$

$= 500 + 50x - \frac{x^3}{3}$

$= 500 + 50x - x^2$

Marginal revenue when x = 10

$= 500+500 - 100 = 900$

(b)	Bill amount = Rs 60,000 Discounted on 30th June 2007 Period of the bill = 10 months Discounted value of the bill = 57300 Bankers discounts = 2700 BD = A n i; $2700 = 60,000 \times n \times \frac{45}{4} \times \frac{1}{100}$ $n = 2700 \times \frac{4}{45} \times \frac{1}{600} = \frac{2}{5}$ of year = 146 days Due date of Bill: 30th June 2007 + 146 days July 31 August 31 September 30 October 31 November 23 146 ∴ Due date of the Bill = 23rd November ∴ Nominal due date = 20th November ∴ Date of drawing the Bill is 20th January.

Question 15

(a) The price relatives and weights of a set of commodities are given below: **[5]**

Commodity	A	B	C	D
Price relatives	125	120	127	119
Weights	x	$2x$	y	$y+3$

If the sum of the weights is 40 and weighted average of price relatives index number is 122, find the numerical values of x and y.

(b) Construct 3 yearly moving averages from the following data and show on a graph against the original data: **[5]**

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Annual sales in lakhs	18	22	20	26	30	22	24	28	32	35

Comments of Examiners

(a) Some candidates made errors in obtaining the equations in terms of x and y. Many candidates used incorrect formula for weighted average of price relatives index number. In addition, some candidates could not solve the two equations to find the values of x and y.

(b) Most candidates calculated moving averages correctly but plotting of moving averages was not correct. Some candidates did not use the averages. In some cases, the graphs drawn were not neat.

Suggestions for teachers

- Train students to identify whether the question is based on aggregate or average method. Thorough knowledge of formula should be given to students.
- For plotting, moving averages correct to one decimal place is sufficient. The axes should be labelled. Plotting and sketching should be as neat as possible and the graph should be given a caption.
- Extensive practice is necessary in plotting points, choosing coordinate axes.
- Advise students to read the question carefully and choose the correct scale.

MARKING SCHEME

Question 15

(a)

Commodity	PR=R	Weights	RW
A	125	x	$125x$
B	120	$2x$	$240x$
C	127	y	$127y$
D	119	$y+3$	$119y+357$

$3x + 2y + 3 = 40$

$3x + 2y = 37$ ------------------- (1)

Index number: $122 = \frac{365x+246y+357}{40}$

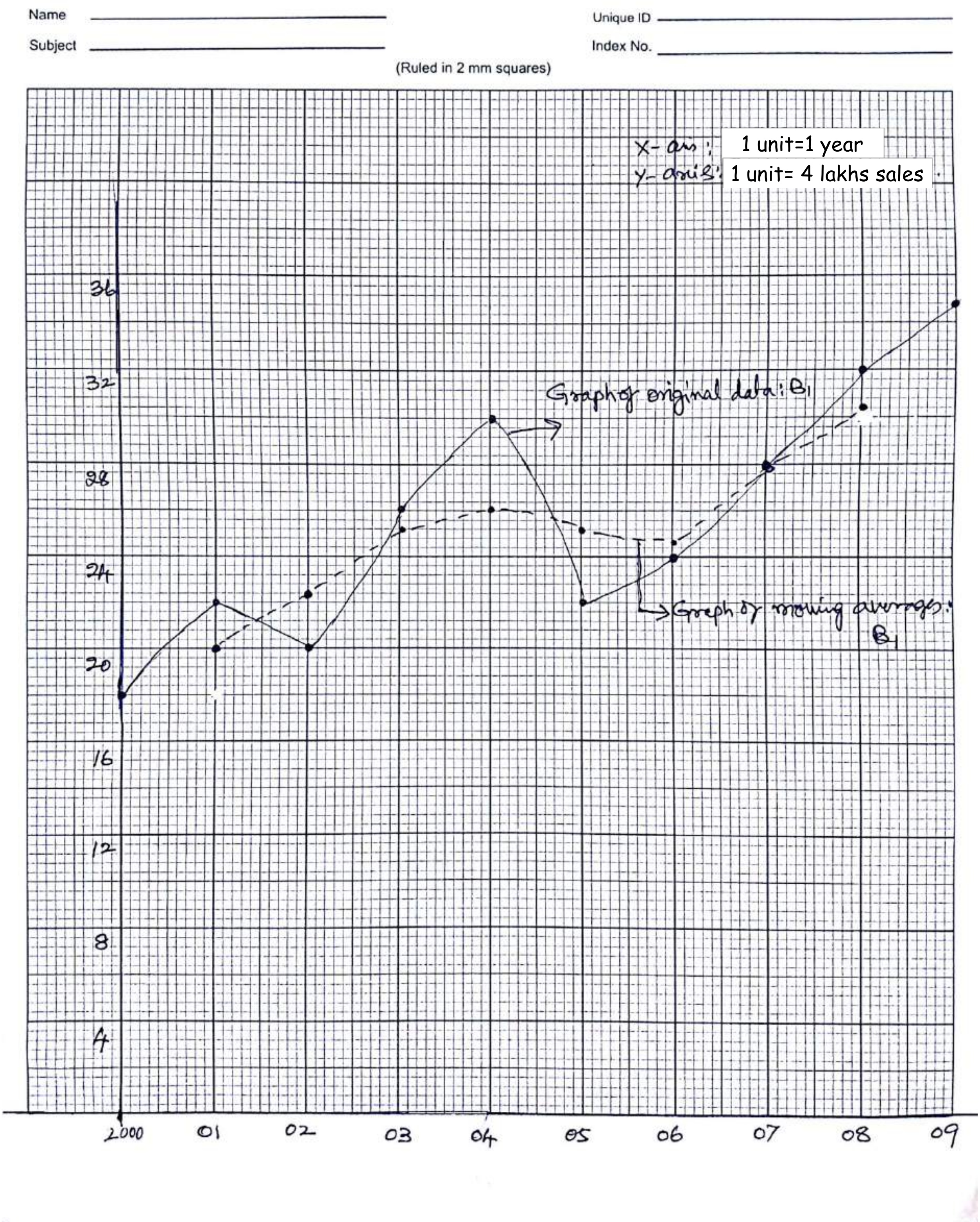
$365x + 246y = 4880 - 357$

$365x + 246y = 4523$ ----- (2)

Solving (1) & (2) $x = 7, y = 8$

(b)

Year	Annual sales	3 yearly moving total	3 yearly moving average
2000	18	--	--
2001	22	60	20
2002	20	68	22·7
2003	26	76	25·3
2004	30	78	26
2005	22	76	25·3
2006	24	74	24·7
2007	28	84	28
2008	32	95	31·7
2009	35	--	--



Note: For questions having more than one correct answer/solution, alternate correct answers/ solutions, apart from those given in the marking scheme, have also been accepted.

GENERAL COMMENTS

Topics found difficult by candidates

- Conic Section in general
- Integrals, definite Integrals and curve sketching
- Vectors and Interchange of vector equation to Cartesian form (vice versa) of plane and Straight-line equations of 3D Geometry and their applications
- Conditional Probability and their applications, Binomial Probability distribution
- Complex numbers and Inverse circular functions
- Maxima and Minima.

Concepts in which candidates got confused

- Hyperbola and Ellipse: Their standard form and other relations
- Lagrange's Mean Value Theorem
- Product and sum rule of probability and concepts of dependent and independent events
- Definite Integrals and their properties
- Dot and Cross product of vectors
- Present value of annuity and Amount of annuity at the end of the period

Suggestions for candidates

- Avoid selective study. Study the entire syllabus thoroughly and revise from time to time.
- Concepts of Class XI must be revised and integrated with the Class XII syllabus.
- Concepts of each chapter/topic must be clear. Formulae of related topics must be learnt.
- Revise all topics and formulae involved and make a chapter wise or topic-wise list of these.
- Time management is important while attempting the paper. Practice solving papers within a stipulated time. Make wise choices from the options available in the question paper and manage time wisely.