E_2 : Sum of entries 7

7 1s 2 0's

Total $E_2 = \frac{9!}{7!2!} = 36$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix}$$

Per |A| to be 0, both zeroes should be in the same row/column.

$\therefore\ 3 \times 3 \times 2 = 18$ cases

$$P\left(\frac{E_1}{E_2}\right) = \frac{18}{36} = \frac{1}{2}$$

5. Three lines are given by

$$\vec{r} = \lambda\hat{i}, \lambda \in \mathbb{R}$$

$$\vec{r} = \mu\left(\hat{i} + \hat{j}\right), \mu \in \mathbb{R} \text{ and}$$

$$\vec{r} = v\left(\hat{i} + \hat{j} + \hat{k}\right), v \in \mathbb{R}.$$

Let the lines cut the plane $x + y + z = 1$ at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals __________

Solution:

$\vec{r} = \lambda\hat{i}, \lambda \in \mathbb{R}$

$\vec{r} = \mu\left(\hat{i} + \hat{j}\right), \mu \in \mathbb{R}$ cuts $x + y + z = 1$ at A, B, C graph

$\vec{r} = v\left(\hat{i} + \hat{j} + \hat{k}\right), v \in \mathbb{R}.$

1st line : $x = \lambda, y = 0, z = 0$

$$x + y + z = 1 \Rightarrow \lambda = 1 \quad A(1,0,0)$$

2nd line: $x = \mu \quad y = \mu \quad z = 0$

$$\therefore 2\mu = 1 \quad \mu = \frac{1}{2} \quad B\left(\frac{1}{2}, \frac{1}{2}, 0\right)$$

Parallels C: $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$$A = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$$

$$= \frac{1}{2}\left|\frac{\hat{i}}{6} + \frac{\hat{j}}{5} + \frac{\hat{k}}{6}\right| = \frac{\sqrt{3}}{12} \qquad (6\Delta) = \frac{3}{6}$$

6. Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set

$$\left\{\left|a + b\omega + c\omega^2\right|^2 : \text{a,b,c distinct non-zero integers}\right\}$$

equals ________

Solution:

$$\left|a + b\omega + c\omega^2\right|^2$$

$$= \left(a + b\omega + c\omega^2\right)\left(a + b\omega^2 + c\omega\right)$$

$$= \left(a^2 + b^2 + c^2 = ab - bc - ca\right) \qquad \left[\because a + b\omega + c\omega^2 = \overline{a} + \overline{b\omega} + \overline{c\omega^2} = a + b\omega^2 + c\omega\right]$$

$$\Rightarrow \frac{1}{2}\left((a-b)^2 + (b-c) + (c-a)^2\right)$$

$$= \frac{1}{2}(1 + 1 + 4) = 3$$