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JEE Main Solved Paper 2018
Maths Jan 10 Shift 1

1. If the tangent at (1, 7) to the curve $x^2 = y-6$ touches the circle $x^2+y^2+16x+12y+c = 0$, then the value of c is :
a) 85
b) 95
c) 195
d) 185

Solution:
Equation of tangent at (1, 7) to $x^2 = y-6$ is $2x-y = -5$.
It touches circle $x^2+y^2+16x+12y+c = 0$.
Hence length of perpendicular from centre (-8, -6) to tangent equals radius of circle.

$$\therefore \quad \left|\frac{-16+6+5}{\sqrt{(2)^2+(-1)^2}}\right| = \sqrt{64+36-c}$$

$\sqrt{(100-c)} = \sqrt{5}$
So c = 95
Answer: (b)

2. If L_1 is the line of intersection of the planes $2x-2y+3z-2 = 0$, $x-y+z+1 = 0$ and L_2 is the line of intersection of the planes $x+2y-z-3 = 0$, $3x-y+2z-1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2 is :
a) $1/2\sqrt{2}$
b) $1/\sqrt{2}$
c) $1/4\sqrt{2}$
d) $1/3\sqrt{2}$

Solution:

$$\begin{vmatrix} \ell & m & n \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \ell + m \leftarrow \text{drs of line } L_1$$

$$\begin{vmatrix} \ell & m & n \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 3\ell - 5m - 7n \leftarrow \text{drs of line } L_2$$

$$\begin{vmatrix} \ell & m & n \\ 1 & 1 & 0 \\ 3 & 5 & -7 \end{vmatrix} = -7\ell - 7m - 8n \leftarrow \text{Normal plane containing line } L_1 \text{ and } L_2$$

For one point of line L_1

2x-2y+3z-2 = 0
x-y+z+1 = 0
Put x = 0, and solving above equations we get (0, 5, 4)
So the equation of the plane is -7(x-0)+7(y-5)-8(z-4) = 0
7x-7y+8z+3 = 0

$$\text{Distance} = \left|\frac{7\times0-7\times0+8\times0+3}{\sqrt{7^2+7^2+8^2}}\right|$$

$= 3/\sqrt{162}$
$= 1/3\sqrt{2}$
Answer: (d)

3. If α and $\beta \in C$ are the distinct roots of the equation $x^2-x+1 = 0$, then $\alpha^{101}+\beta^{107}$ is equal to:
a) 1
b) 2
c) -1
d) 0

Solution:
$x^2-x+1 = 0$
$x = 1\pm\sqrt{(1-4\times1\times1)}/2\times1$
$= (1\pm i\sqrt{3})/2$
$= (1+i\sqrt{3})/2\ ,\ (1-i\sqrt{3})/2$
$= -(-1-i\sqrt{3})/2,\ -(-1+i\sqrt{3})/2$
$= -\omega^2,\ -\omega$
$\alpha = -\omega^2$
$\beta = -\omega$
$\alpha^{101}+\beta^{107} = (-\omega^2)^{101}+(-\omega)^{107}$
$= -(\omega^{202}+\omega^{107})$
$= -[(\omega^3)^{67}\ \omega+(\omega^3)^{35}\ \omega^2]$
$= -[\omega+\omega^2]$
$= 1$
Answer: (a)

4. Tangents are drawn to the hyperbola $4x^2- y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of $\triangle PTQ$ is :
a) $60\sqrt{3}$
b) $36\sqrt{5}$
c) $45\sqrt{5}$
d) $54\sqrt{3}$

Solution:
Given equation of hyperbola is $4x^2-y^2 = 36$
$\Rightarrow (x^2/9)-(y^2/36) = 1$

$\Rightarrow a^2 = 9$
$b^2 = 36$
From T(0, 3) tangents are drawn to hyperbola at P and Q.
Hence equation of Chord of contact PQ is
$(x(0)/9) - (y(3)/36) = 1$
$\Rightarrow y = -12$
$\therefore (x^2/9)-(144/36) = 1$
$\Rightarrow x^2 = 45$
$\Rightarrow x = \pm 3\sqrt{5}$
Hence P = $(3\sqrt{5}, -12)$ and Q = $(-3\sqrt{5}, -12)$

$$\text{Hence A }(\Delta PQT)\text{ is } \frac{1}{2}\begin{vmatrix} 3\sqrt{5} & -12 & 1 \\ -3\sqrt{5} & -12 & 1 \\ 0 & 3 & 1 \end{vmatrix} = 45\sqrt{5}$$

Answer: (c)

5. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is :
a) 4
b) 9/2
c) 6
d) 7/2

Solution:
Given $y^2 = 6x$
Differentiating
$2y(dy/dx) = 6$
$(dy/dx) = 3/y$
Given $9x^2 + by^2 = 16$
Differentiating
$18x+2by(dy/dx) = 0$
$(dy/dx) = -18x/2by$
$(dy/dx) = -9x/by$
Let the intersection point be (x_1, y_1)
$m_1 = 3/y_1$
$m_2 = -9x_1 /by_1$
Since the curves intersect at right angles, $m_1m_2 = -1$
$(3/y_1)(-9x_1 /by_1) = -1$
$27x_1 = by_1^2$ [Since (x_1, y_1) lies on $y^2 = 6x \Rightarrow y_1^2 = 6x_1$]
$27x_1 = b(6x_1)$
$b = 9/2$
Answer: (b)

6. If the system of linear equations :

x+ky+3z = 0
3x+ky-2z = 0
2x+4y-3z = 0
has a non-zero solution (x, y, z), then xz/y^2 is equal to :
a) -30
b) 30
c) -10
d) 10

Solution:
Given system of equations have non zero solutions.

$$\text{Non-zero solution} \Rightarrow \Delta = \begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$$

⇒ 1(-3k+8)-k(-9+4)+3(12-2k) = 0
⇒ -3k+8+5k+36-6k = 0
4k = 44
k = 44/4 = 11
Now the given equations become
x+11y+3z = 0
3x+11y-2z = 0
2x+4y-3z = 0
x/(-22-33) = y/(-(-2-9) = z/(11-33)
x/-55 = y/11 = x/-22
x/5 = y/-1 = z/2 = L (let)
$xz/y^2 = (5L)(2L)/(-L^2) = 10$
Answer: (d)

7. Let S = {x ∈R: x ≥ 0 and 2|√x-3|+√x(√x-6)+6 = 0}. Then S:
a) contains exactly two elements.
b) contains exactly four elements.
c) is an empty set.
d) contains exactly one element.

Solution:
2|√x-3|+√x(√x-6)+6 = 0
2√x-6+x-6√x+6 = 0 if √x >3
x-4√x = 0
⇒√x = 0 or 4
⇒√x = 4
Also 2(3-√x)+√x(√x-6)+6 = 0 if √x<3
6-2√x+x-6√x+6 = 0
x-8√x+12 = 0

$(\sqrt{x})^2-6\sqrt{x}-2\sqrt{x}+12 = 0$

$\therefore (\sqrt{x}-2)(\sqrt{x}-6) = 0$

$\therefore \sqrt{x} = 2, 6$

$\Rightarrow x = 4$

Answer: (a)

8. If sum of all the solutions of the equation 8cos x(cos ((π/6)+x). cos ((π/6)-(1/2))-(1/2)) = 1 in [0, π] is kπ, then k is equal to :

a) 8/9

b) 20/9

c) 2/3

d) 13/9

Solution:

8cos x(cos ((π/6)+x). cos ((π/6)-(1/2))-(1/2)) = 1

8.cos x[(cos (π/3)+ cos 2x-2)/2] = 1

4 cosx(cos 2x-(1/2)) = 1

4 cosx($2\cos^2$x-(3/2)) = 1

$8\cos^3x-6\cos x-1 = 0$

$2(4\cos^3x-3\cos x)-1 = 0$

2cos3x-1 = 0

cos 3x = 1/2 = cos(π/3)

3x = 2nπ ±π/3

x = (2nπ /3) ±(π/9)

= 7π/9, 5π/9, π/9 in [0, π]

k = 13/9

Answer: (d)

9. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is :

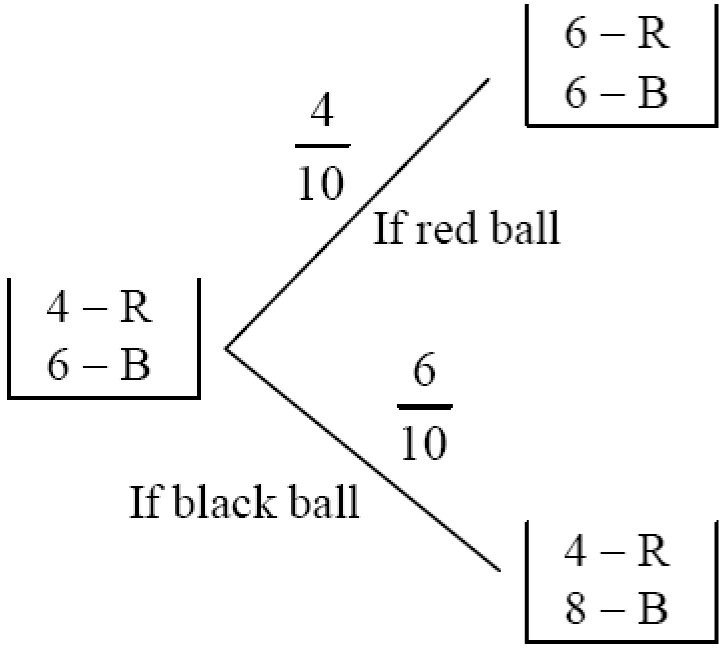
a) 1/5

b) 3/4

c) 3/10

d) 2/5

Solution:



From total Probability Theorem
P (R) = (4/10)×(1/2)+(6/10) × (4/12)
= (1/5)+(1/5)
= 2/5
Answer: (d)

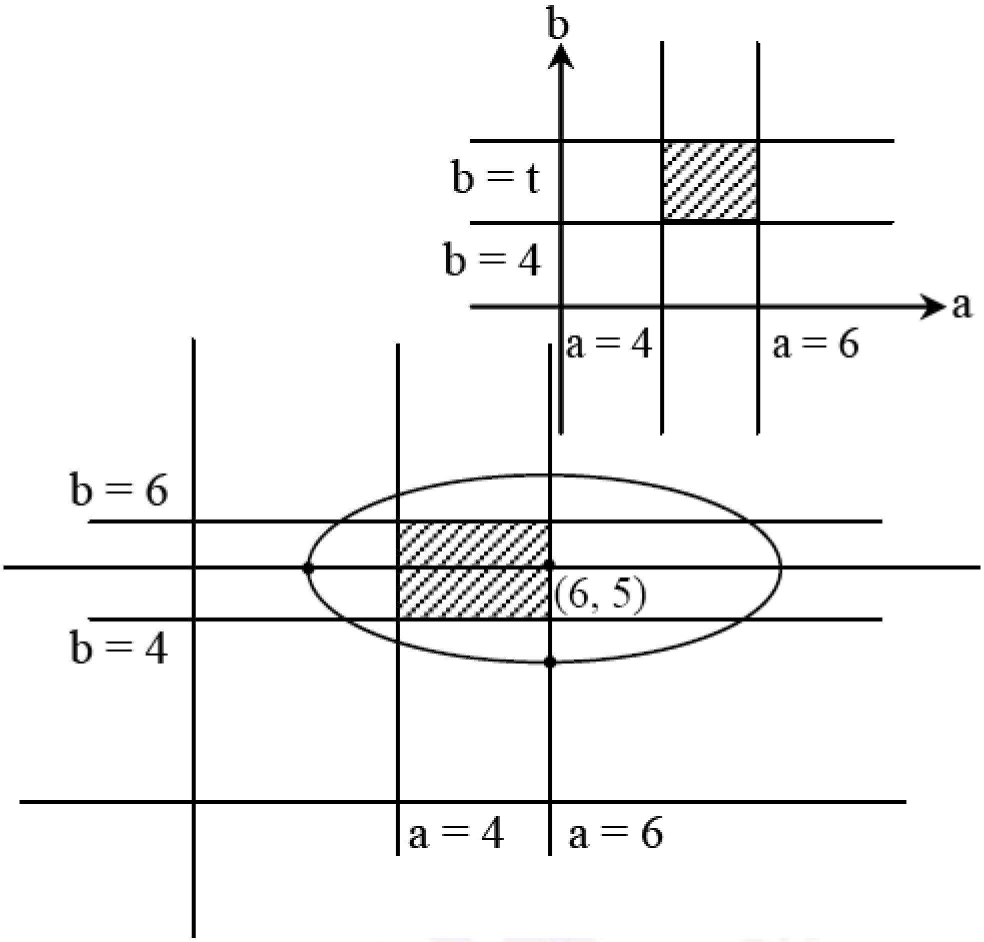
10. Let $f(x) = x^2+(1/x^2)$ and $g(x) = x-(1/x)$, $x\in R-\{-1,0,1\}$. If $h(x) = f(x)/g(x)$, then the local minimum value of h(x) is:
a) $-2\sqrt{2}$
b) $2\sqrt{2}$
c) 3
d) -3

Solution:
$f(x) = x^2+(1/x^2)$
$= (x-(1/x))^2+2$
$g(x) = x-(1/x)$
Put $x-(1/x) = t$
$h(x) = f(x)/g(x)$
Let $h(x) = k(t)$
$= (t^2+2)/t$
$= t+2/t$
Differentiating
$k'(t) = 1-2/t^2$
For minima or maxima, $1-2/t^2 = 0$
$\Rightarrow t^2 = 2$
$\Rightarrow t = \pm\sqrt{2}$

$k''(t) = 4/t^3$
At $t = \sqrt{2}$, $k''(t) = 2/\sqrt{2}$
At $t = -\sqrt{2}$, $k''(t) = -2/\sqrt{2}$
At $t = \sqrt{2}$. $k''(t)$ is positive.
So it is a minima.
$h(x) = t+2/t$
$= \sqrt{2}+2/\sqrt{2}$
$= 2\sqrt{2}$
Answer: (b)

11. Two sets A and B are as under :
$A = \{(a,b) \in R\times R: |a-5| < 1 \text{ and } |b-5| < 1\}$
$B = \{(a,b) \in R\times R: 4(a-6)^2+9(b-5)^2 \leq 36\}$. Then:
a) $A \cap B = \phi$ (an empty set)
b) neither $A \subset B$ nor $B \subset A$
c) $B \subset A$
d) $A \subset B$

Solution:
$A = \{(a,b) \in R\times R: |a-5| < 1 \text{ and } |b-5| < 1\}$
$\Rightarrow -1<a-5<1$
$4<a<6$
$4<b<6$
$B = \{(a,b) \in R\times R$
$4(a-6)^2+9(b-5)^2 \leq 36$
$((a-6)^2/9)+((b-5)^2/4) \leq 1$



Answer: (d)

12. The Boolean expression : ~(p ∨q) ∨(~p∧q) is equivalent to:
a) q
b) ~q
c) ~p
d) p

Solution:

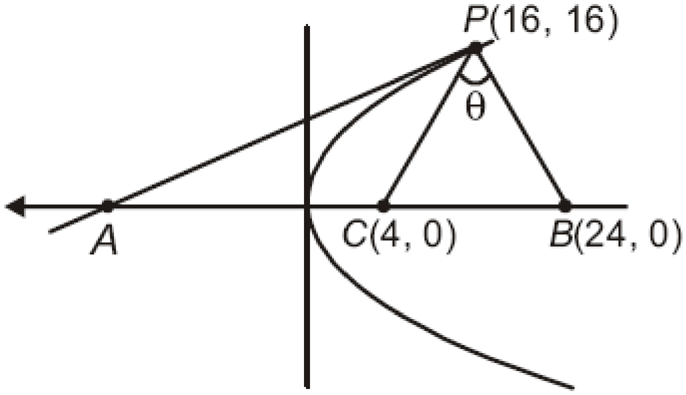
p (1)	q (2)	~p (3)	(p ∨q) (4)	~(p ∨q) (5)	~p∧q (6)	~(p ∨q) ∨(~p∧q)
T	T	F	T	F	F	F
T	F	F	T	F	F	F
F	T	T	T	F	T	T
F	F	T	F	T	F	T

Entries in column (3) and (7) are identical.
Answer: (c)

13. Tangent and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$ then a value of $\tan\theta$ is :

a) 3
b) 4/3
c) 1/2
d) 2

Solution:



Tangent and normal are drawn at P (16, 16) on $y^2 = 16x$
$y^2 = 16x$
Slope of tangent $dy/dx = 16/2y$
$= 8/y$
$(dy/dx)_{(16,16)} = 1/2$
Equation of the tangent is
$y-16 = (1/2)(x-16)$
$\Rightarrow 2y-32 = x-16$
$\Rightarrow x-2y+16 = 0$ ..(1)
Equation of normal
$y-16 = -2(x-16)$
$2x+y-48 = 0$ ..(2)
Tangent & normal intersect the axis of parabola
A(-16,0)
B(24,0)
Slope of AP×slope of PB
$\Rightarrow [(16-0)/(16+16)][(16-0)/(16-24)] = -1$
So AB is the diameter of circle with centre C(4,0)
$m_{PC} = (16-0)/(16-4) = 16/12 = 4/3$
$m_{PB} = (16-0)/(16-24) = 16/-8 = -2$
$\tan\theta = |m_1 - m_2/(1+m_1m_2)|$
$= |\ (4/3)+2)/(1+(4/3)\times 2)\ |$
$= 2$

Answer: (d)

14. If

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$$

then the ordered pair (A, B) is equal to :

a) (-4, 5)
b) (4, 5)
c) (-4, -5)
d) (-4, 3)

Solution:

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$$

$C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} 5x-4 & 2x & 2x \\ 5x-4 & x-4 & 2x \\ 5x-4 & 2x & x-4 \end{vmatrix} = (5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x-4 & 2x \\ 1 & 2x & x-4 \end{vmatrix}$$

$R_1 \rightarrow R_1 - R_2$

$$= (5x-4)\begin{vmatrix} 0 & x+4 & 0 \\ 1 & x-4 & 2x \\ 1 & 2x & x-4 \end{vmatrix}$$

= (5x-4)(-1)(x+4)(x-4-2x)
= -1(5x-4)(x+4)(-x-4)
= $(5x-4)(x+4)(x+4)^2$
= $(-4+5x)(x-(-4))^2$
Hence A = -4, B = 5
Answer: (a)

15. The sum of the coefficients of all odd degree terms in the expansion of $(x+\sqrt{(x^3-1)})^5+(x-\sqrt{(x^3-1)})^5$, (x>) is :

a) 1
b) 2
c) -1
d) 0

Solution:

$(a+b)^n+(a-b)^n = 2({}^nC_0a^n + {}^nC_2a^{n-2}b^2 + {}^nC_4a^{n-4}b^4\ldots)$
$(x+\sqrt{(x^3-1)})^5+(x-\sqrt{(x^3-1)})^5$, (x>)

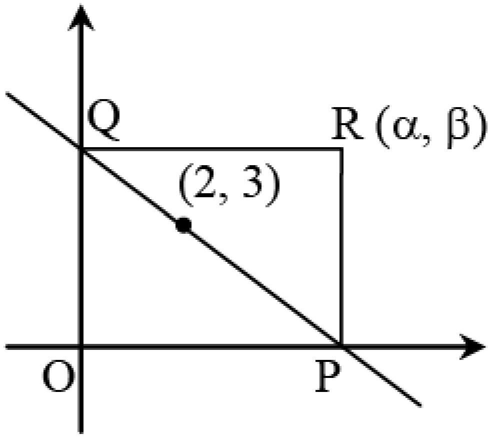
$= 2(^5C_0x^5+^5C_2x^3(\sqrt{(x^3-1)})^2+^5C_4x(\sqrt{(x^3-1)})^4)$
$= 2(x^5+10x^6-10x^3+5x^7-10x^4+5x)$
Sum of the coefficient of odd term is given by
2(1-10+5+5)
= 2
Answer: (b)

16. Let a_1, a_2, a_3,..a_{49} be in A.P such that $\Sigma_{k=0}^{12}\ a_{4k+1} = 416$ and $a_9+a_{43} = 66$. If $a_1^2+a_2^2+\ldots+a_{17}^2 = 140m$, then m is equal to:
a) 34
b) 33
c) 66
d) 68

Solution:
a_1, a_2, a_3,..a_{49} are in A.P
Let A be the first term and D is the common difference.
$a_9+a_{43} = 66$
A+8D+A+42D = 66
2A+50D = 66
A+25D = 33
$\therefore a_{26} = 33$ ..(i)
$\Sigma_{k=0}^{12}\ a_{4k+1} = 416$
$\Rightarrow a_1+a_5+a_9+\ldots+a_{49} = 416$
$\therefore 13A+312D = 416$
A+24D = 23
$\therefore a_{25} = 32$ ..(ii)
From (i) and (ii), $a_{26}-a_{25} = D = 1$
Also A+25D = 33
So A = 8
$\therefore\ a_1^2+a_2^2+\ldots+a_{17}^2 = 8^2+9^2+10^2+\ldots+24^2$
$\therefore \Sigma_{r=1}^{24} r^2 - \Sigma_{r=1}^{7} r^2 = 140m$
$\therefore (24\times25\times49/6)-(7\times 8\times15/6) = 140m$
4900-140 = 140m
$\Rightarrow$m = 34
Answer: (a)

17. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is :
a) 3x+2y = xy
b) 3x+2y = 6xy
c) 3x+2y = 6
d) 2x+3y = xy

Solution:



line y-3 = m(x-2)
y-3 = mx-2m
P = ((2m-3)/m, 0)
Q = (0, 3-2m)
Let R(α, β)
So α = ((2m-3)/m, 0) and β = 3-2m
∴ m = 3/(2-α) and m = (3-β)/2
∴ 3/(2-α) = (3-β)/2
⇒ 6 = 6-2β-3α+αβ
Locus of R (α,β) is 3x+2y = xy
Answer: (a)

18. The value of

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}\,dx \text{ is :}$$

a) 4π
b) π/4
c) π/8
d) π/2

Solution:

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x}\,dx \qquad \ldots\ldots\ldots (1)$$

$$= \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+\frac{1}{2^x}}\,dx \qquad \int_{-a}^{a} f(x)\,dx = \int_{-a}^{a} f(-x)\,dx$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x \times 2^x}{1+2^x}\,dx \qquad \ldots\ldots\ldots (2)$$

(1) + (2)

$$2I = \int_{-\pi/2}^{\pi/2} \sin^2 x\,dx \qquad \text{[Even Function]}$$

$$I = \int_{0}^{\pi/2} \sin^2 x\,dx$$

$$I = \frac{1}{2}\int_{0}^{\pi/2} (1-\cos 2x)\,dx = \pi/4$$

Answer: (b)

19. Let g(x) = cos x^2, f(x) = √x and α, β (α< β) be the roots of the quadratic equation $18x^2$-9πx+π^2 = 0. Then the area in sq.units) bounded by the curve y = (gof)(x) and the lines x = α and x = β nd y =0 is:

a) (1/2)(√3-√2)
b) (1/2)(√2-1)
c) (1/2)(√3-1)
d) (1/2)(√3+1)

Solution:

$18x^2-9\pi x+\pi^2 = 0$
⇒ x = π/6, π/3 (so α = π/6, β= π/3)
y = (gof)(x)
= g(f(x))
= g(√x)
= cos x

$$A = \int_{\pi/6}^{\pi/3} \cos x \, dx = \sin x \Big|_{\pi/6}^{\pi/3}$$

= (√3/2)-(1/2)
= (√3-1)/2
Answer: (c)

20. For each t ∈ R, let [t] be the greatest integer less than or equal to t. Then

$$\lim_{x\to 0+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{15}{x}\right]\right)$$

a) is equal to 120
b) does not exist (in R)
c) is equal to 0
d) is equal to 15

Solution:

$$\lim_{x\to 0+} x\left(\frac{1}{x}-\left\{\frac{1}{x}\right\}+\frac{2}{x}-\left\{\frac{2}{x}\right\}+\ldots+\frac{15}{x}-\left\{\frac{15}{x}\right\}\right)$$

$$= \lim_{x\to 0+}\left(1+2+\ldots+15-x\left(\left\{\frac{1}{x}\right\}+\left\{\frac{2}{x}\right\}+\ldots+\left\{\frac{15}{x}\right\}\right)\right)$$

$$= 120-\lim_{x\to 0+} x\underbrace{\left(\left\{\frac{1}{x}\right\}+\left\{\frac{2}{x}\right\}+\ldots+\left\{\frac{15}{x}\right\}\right)}_{\text{finite}}$$

$$= 120$$

Answer: (a)

21. If $\sum^{9}_{i=1} (x_i-5) = 9$ and $\sum^{9}_{i=1} (x_i-5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, \ldots x_9$ is:
a) 2
b) 3
c) 9
d) 4

Solution:
Variance = $(1/n)\ \Sigma_{i=1}^{n}\ x_i^2-[(1/n)\ \Sigma_{i=1}^{n}\ x_i]^2$
= $(1/9)\times45-((1/9)\times9)^2$
= 5-1
= 4

S.D = √4 = 2
Answer: (a)

22. The integral

$$\int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x\right)^2} dx$$

is equal to:
a) $1/(1+\cot^3 x) + C$
b) $-1/(1+\cot^3 x) + C$
c) $1/3(1+\tan^3 x) + C$
d) $-1/3(1+\tan^3 x) + C$
where C is the constant of integration.

Solution:

$$\int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x\right)^2} dx$$

$$= \int \frac{\tan^2 x . \sec^4 x . \sec^2 x}{\left(\tan^5 x + \tan^2 x + \tan^3 x + 1\right)^2} dx \qquad \text{divide by } \cos^{10} x$$

$$= \int \frac{t^2\left(1+t^2\right)^2}{\left(t^5 + t^2 + t^3 + 1\right)^2} dt \qquad \tan x = t$$

$$= \int \frac{t^2\left(1+t^2\right)^2}{\left(t^3+1\right)^2\left(t^2+1\right)^2} dt$$

$$= \int \frac{t^2}{\left(t^3+1\right)^2} dt \qquad t^3 + 1 = y$$

$$= \int \frac{1}{y^2} . \frac{dy}{3} = \frac{1}{3}\left(-\frac{1}{y}\right) + c = -\frac{1}{3} . \left(\frac{1}{\tan^3 x + 1}\right) + c$$

Answer: (c)

23. Let S = {t∈R: $f(x) = |x-\pi|.(e^{|x|}-1)\sin|x|$ is not differentiabl at t}. Then the set S is equal to:
a) {π}
b) {0,π}
c) ϕ (an empty set)
d) {0}

Solution:

$f(x) = |x-\pi|(e^{|x|}-1)\sin|x|$

Obviously differentiable at x = 0

Check at $x = \pi$

$$\text{R.H.D.} = \lim_{h\to 0}\frac{f(\pi+h)-f(\pi)}{h}$$

$$= \lim_{h\to 0}\frac{|\pi+h-\pi|\left(e^{|\pi+h|}-1\right)\sin|\pi+h|-0}{h}$$

$$= \lim_{h\to 0}\frac{h.\left(e^{\pi+h}-1\right).\sin(\pi+h)}{h} = \lim_{h\to 0} -\sin h.\left(e^{\pi+h}-1\right) = 0$$

$$\text{L.H.D.} = \lim_{h\to 0}\frac{f(\pi-h)-f(\pi)}{-h}$$

$$= \lim_{h\to 0}\frac{|\pi-h-\pi|\left(e^{|\pi-h|}-1\right)\sin|\pi-h|}{-h} = \lim_{h\to 0}\frac{h.\left(e^{\pi-h}-1\right).\sin h}{-h} = 0$$

So, differentiable at $x = \pi$.

Answer: (c)

24. Let y = y(x) be the solution of the differential equation sin x(dy/dx)+y cosc = 4x, $x\in(0,\pi)$. If $y(\pi/2) = 0$, then $y(\pi/6)$ is equal to:

a) $(-8/9)\pi^2$

b) $(-4/9)\pi^2$

c) $(4/9\sqrt{3})\pi^2$

d) $(-8/9\sqrt{3})\pi^2$

Solution:

(dy/dx)+y cotx = 4x/sin x

$IF = e^{\int \cot x dx} = e^{\log|\sin x|} = \sin x$ as $x\in(0,\pi)$

$y \sin x = c+\int 4x\, dx$

$= c+2x^2$

As $y(\pi/2) = 0 \Rightarrow 0.\sin(\pi/2) = c+2(\pi/2)^2$

$\Rightarrow c = -(\pi^2/2)$

So $y \sin x = 2x^2-(\pi^2/2)$

Put $x = \pi/6$

$y \sin \pi/6 = 2(\pi/6)^2-(\pi^2/2)$

$y(1/2) = (\pi^2/18)- (\pi^2/2)$

$= (-8\pi^2/9)$

$= (-8/9)\pi^2$

Answer: (a)

25. Let u be a vector coplanar with the vectors a = 2i+3j-k and b = j+k. If u is perpendicular to vector a and u.b = 24, then $|u|^2$ is equal to:

a) 256
b) 84
c) 336
d) 315

Solution:

$$\vec{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$$

$$\text{As coplanar} \begin{vmatrix} u_1 & u_2 & u_3 \\ 2 & 3 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 0$$

$$4u_1 - 2u_2 + 2u_3 = 0$$

$$2u_1 - u_2 + u_3 = 0 \qquad \ldots\ldots.. (1)$$

$$\vec{u}.\vec{a} = 0$$

$$\Rightarrow \quad 2u_1 + 3u_2 - u_3 = 0 \qquad \ldots\ldots.. (2)$$

$$\vec{u}.\vec{b} = 0$$

$$\Rightarrow \quad u_2 + u_3 = 24 \qquad \ldots\ldots.. (3)$$

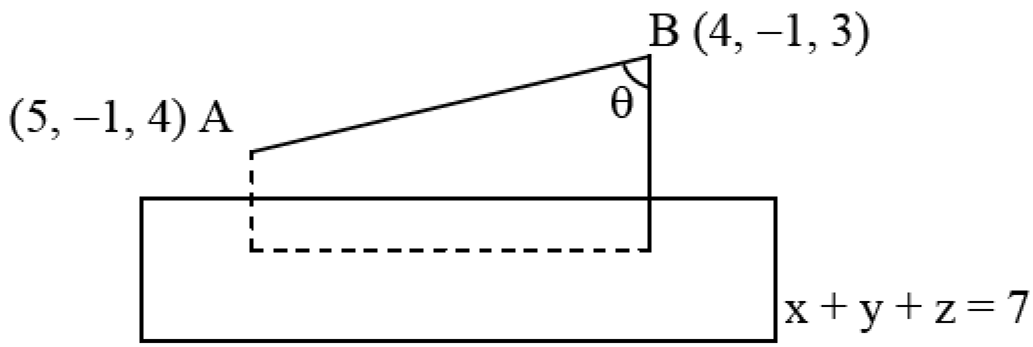
$$\text{Solving } u_1 = -4,\ u_2 = 8,\ u_3 = 16$$

$$|\vec{u}|^2 = 336$$

Answer: (c)

26. The length of the projection of the line segment joining the points (5, -1, 4) and (4, -1, 3) on the plane, x+y+z = 7 is :
a) 1/3
b) √(2/3)
c) 2/√3
d) 2/3

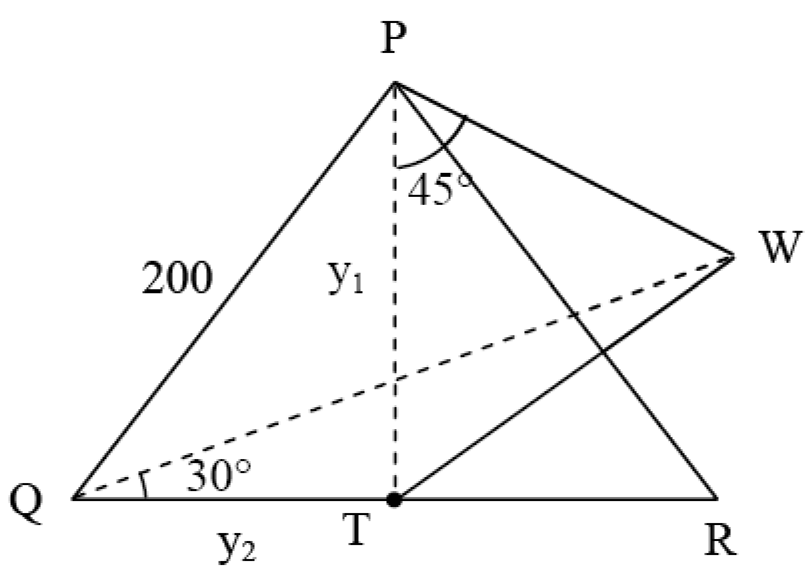
Solution:



Direction ratios of AB = (1,0,1)
Let θ be angle between line AB and normal of plane.
So $\cos\theta = (1\times1+0\times1+1\times1)/\sqrt{2}\sqrt{3}$
$= \sqrt{2}/\sqrt{3}$
So projection of line AB = |AB| sinθ
$= \sqrt{2}\times\sqrt{(1-2/3)}$
$= \sqrt{(2/3)}$
Answer: (b)

27. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45^0, 30^0 and 30^0, then the height of the tower (in m) is :
a) $100\sqrt{3}$
b) $50\sqrt{2}$
c) 100
d) 50

Solution:



Let TW = h

$\triangle$PTW, $\tan 45^0 = h/y_1$
$\Rightarrow h = y_1$
Similarly, $h/y_2 = \tan 30^0$
$\Rightarrow y_2 = \sqrt{3}h$
$\triangle$PQT, $PQ^2 = QT^2+PT^2$
$40000 = 3h^2+h^2$
$4h^2 = 40000$
$h = 100$
Answer: (c)

28. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is :
a) at least 500 but less than 750
b) at least 750 but less than 1000
c) at least 1000
d) less than 500

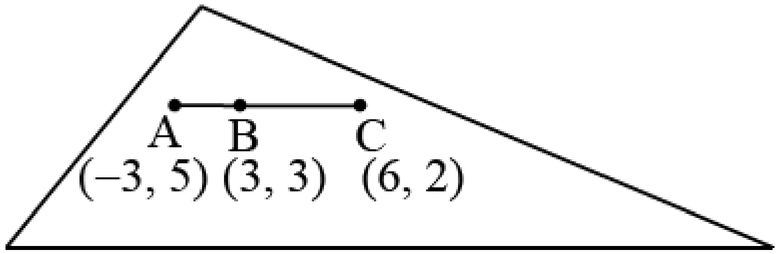
Solution:
We have 6 novels and 3 dictionaries. We can select 4 novels and 1 dictionary in
${}^6C_4 \times {}^3C_1 = 6!\times 3/(4!2!)$
$= 6\times 5\times 3/2$
= 45 ways.
Now 4 novels and 1 dictionary are to be arranged so that dictionary is always in middle. So remaining 4 novels can be arranged in 4! ways.
Hence total arrangements possible are
$45\times 24 = 1080$ ways
Answer: (c)

29. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series.
$1^2 + 2.2^2+3^2+2.4^2+5^2 + 2.6^2+..$
If $B-2A = 100\lambda$, then λ is equal to :
a) 464
b) 496
c) 232
d) 248

Solution:
$B = (1^2+3^2+5^2+\ldots+39^2)+2(2^2+4^2+\ldots+40^2)$
$= (1^2+2^2+3^2+\ldots+40^2)+(2^2+4^2+\ldots+40^2)$
$= (1^2+2^2+3^2+\ldots+40^2)+4(1^2+2^2+3^2+\ldots+20^2)$
$A = (1^2+2^2+3^2+\ldots+20^2)+4(1^2+2^2+3^2+\ldots+10^2)$
Using $1^2+2^2+..n^2 = n(n+1)(2n+1)/6$
$B-2A = 24800$
Hence $\lambda = 248$
Answer: (d)

30. Let the orthocentre and centroid of a triangle be A(-3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is:
a) $3\sqrt{(5/2)}$
b) $3\sqrt{5}/2$
c) $\sqrt{10}$
d) $2\sqrt{10}$

Solution:



As we know centroid divides line joining circumcentre and orthocentre internally 1:2.
So, C(6,2)
$AC = \sqrt{[(6+3)^3+(2-5)^2]}$
$= \sqrt{90}$
$= 3\sqrt{10}$
$r = AC/2$
$= 3\sqrt{10}/2$
$= 3\sqrt{(5/2)}$
Answer: (a)