

>99`A U]b`&\$%`5df]`A U\h g`DUdYf` k]h`Gc`i h]cbg

1. The sum of coefficients of even powers of x in $(x + \sqrt{x^3 - 1})^6 + x - \sqrt{(x^3 - 1)^6}$
- (A) 23
(B) 24
(C) 18
(D) 21

Solution:

$$\begin{aligned}(x + \sqrt{x^3 - 1})^6 + x - \sqrt{(x^3 - 1)^6} &= 2[6C_0x^6 + 6C_2x^4(x^3 - 1) + 6C_4x^2(x^3 - 1)^2 + 6C_6(x^3 - 1)^3] \\&= 2[x^6 + 15x^4(x^3 - 1) + 15x^2(x^6 - 2x^3 + 1) + (x^3 - 1)^3] \\&= 2[x^6 + 15x^7 - 15x^4 + 15x^8 - 30x^5 + 15x^2 + x^9 - 3x^6 + 3(x^3 - 1)] \\ \text{Terms with even power of } x &= 2[x^6 - 15x^4 + 15x^8 + 15x^2 + 15x^2 - 3x^6 - 1] \\ \text{Therefore Sum of coefficients} &= 2(1 - 15 + 15 + 15 - 3 - 1) = 24\end{aligned}$$

Answer: B

2. Let $\sin(\alpha - \beta) = 5/13$ and $\cos(\alpha + \beta) = 3/5$ where $\alpha, \beta \in (0, \pi/4)$ then $\tan 2\alpha$
- (A) 63/16
(B) 61/16
(C) 65/16
(D) 32/9

Solution:

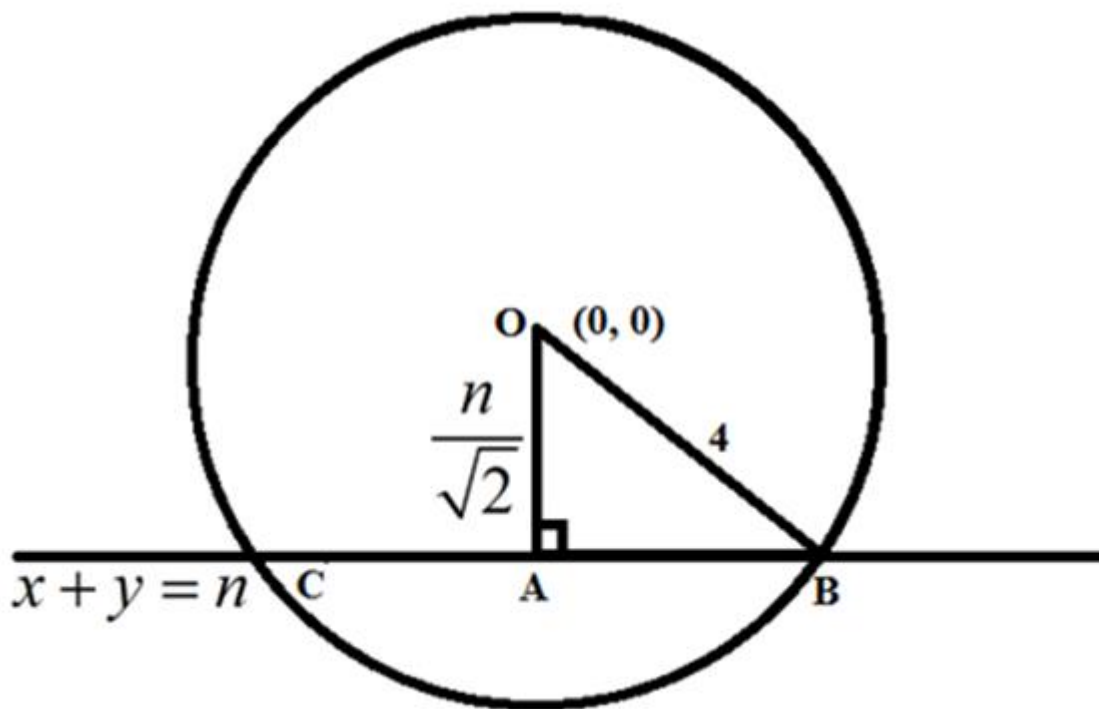
$$\begin{aligned}\sin(\alpha - \beta) &= 5/13 \\ \Rightarrow \cos(\alpha - \beta) &= 12/13 \\ \cos(\alpha + \beta) &= 3/5 \\ \sin(\alpha + \beta) &= 4/5 \\ \tan 2\alpha &= \tan[(\alpha + \beta) + (\alpha - \beta)] \\ &= (4/3) + (5/12) \div (1 - (4/3) \times (5/12)) \\ &= 63/16\end{aligned}$$

Answer: A

3. The line $x + y = n$, $n \in \mathbb{N}$, makes intercepts with $x^2 + y^2 = 16$. Then the sum of squares of all possible intercepts
- (A) 105/4
(B) 105
(C) 210
(D) 105/2

Solution:

$$\begin{aligned}x^2 + y^2 &= 16 \\ \text{Centre} &= (0, 0) \\ \text{OA} &= P \\ \text{Radius} &= 4 \\ P &= n/\sqrt{2}\end{aligned}$$



To make the intercepts $n/\sqrt{2} < 4$

$$\Rightarrow n < 4\sqrt{2}$$

Length of intercepts = $\sqrt{r^2 - p^2}$

$$AB = \sqrt{16 - n^2/2}$$

Length of chord BC = $2\sqrt{16 - n^2/2} = \sqrt{64 - 2n^2}$

Square of intercepts = $(64 - 2n^2)$, $n \in \mathbb{N}$

Possible value of $n = 1, 2, 3, 4, 5$

Sum of square of intercepts = $(64 - 2) + (64 - 8) + (64 - 18) + (64 - 32) + (64 - 50)$

$$= 62 + 56 + 46 + 32 + 14$$

$$= 210$$

Answer: C

4.

$$\int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} \cdot dx =$$

(A) $x + 2\sin x + \sin 2x + c$

(B) $x + 2\cos x + \sin 2x + c$

(C) $x - 2\sin x + \sin 2x + c$

(D) $x + 2\sin x - \sin 2x + c$

Solution:

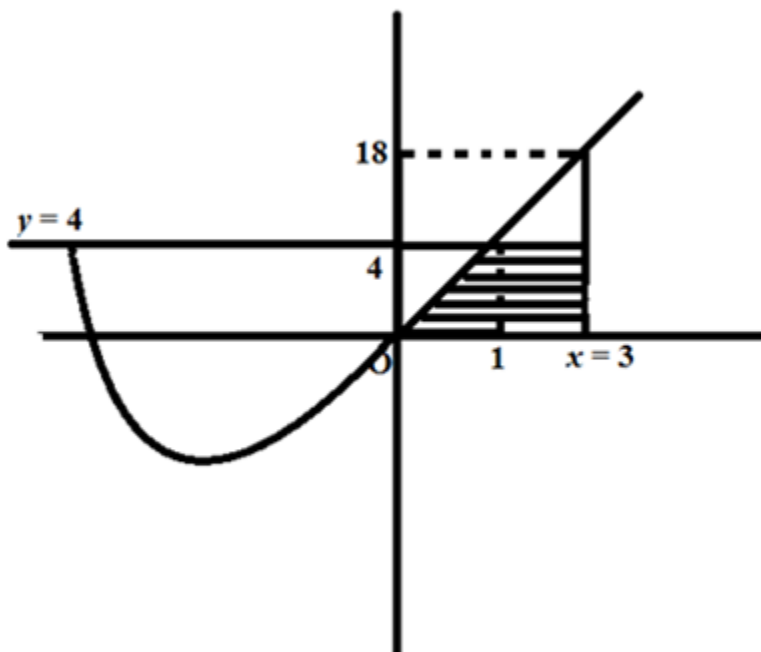
$$\begin{aligned}
 \int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} \cdot dx &= \int \frac{2 \sin \frac{5x}{2} \cdot \cos \frac{x}{2}}{2 \sin \frac{x}{2} \cdot \cos \frac{x}{2}} \cdot dx \\
 &= \int \frac{\sin 3x + \sin 2x}{\sin 2\left(\frac{x}{2}\right)} \cdot dx \\
 &= \int \frac{\sin 3x}{\sin x} \cdot dx + \int \frac{\sin 2x}{\sin x} \cdot dx \\
 &= \int \frac{3 \sin x - 4 \sin^3 x}{\sin x} \cdot dx + \int \frac{2 \sin x \cos x}{\sin x} dx \\
 &= 3 \int dx - 4 \int \sin^2 x \cdot dx + 2 \int \cos x dx \\
 &= 3x - 4 \int \frac{1 - \cos 2x}{2} \cdot dx + 2 \sin x \\
 &= 3x - 2x + \sin 2x + 2 \sin x + c \\
 &= x + 2 \sin x + \sin 2x + c
 \end{aligned}$$

Answer: A

5. The area bounded by the curve $y \leq x^2 + 3x$, $0 \leq y \leq 4$, $0 \leq x \leq 3$ is

- (A) 59/6
- (B) 57/4
- (C) 59/3
- (D) 57/6

Solution:



$$y = x^2 + 3x$$

$$\begin{aligned}
 y = 4 &\Rightarrow x^2 + 3x - 4 = 0 \\
 &\Rightarrow x^2 + 3x - 4 = 0 \\
 &\Rightarrow x = 1 \text{ or } x = -4
 \end{aligned}$$

$$\begin{aligned}
 \text{Area} &= \int_0^1 (x^2 + 3x) \cdot dx + \text{area of rectangle} \\
 &= \left[\frac{x^3}{3} + \frac{3x^2}{2} \right]_0^1 + 2(4)
 \end{aligned}$$

$$\begin{aligned}
 &= (1/3) + (3/2) + 8 \\
 &= 59/6
 \end{aligned}$$

Answer: A

6. "If you are born in India then you are citizen of India" contrapositive of this statement is
- (A) If you are born in India then you are not citizen of India
 - (B) If you are not citizen of India then you are not born in India
 - (C) If you are citizen of India then you are not born in India
 - (D) If you are citizen of India then you are born in India

Solution:

Statement: $p \Rightarrow q$

Contrapositive: $\sim p \Rightarrow \sim q$

Therefore answer is if you are not citizen of India then you are not born in India.

Answer: B

7.

$$A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \text{ and } A^{32} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

then α may be

- (A) 0
- (B) $\pi/32$
- (C) $\pi/64$
- (D) $\pi/16$

Solution:

$$A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$A^n = \begin{bmatrix} \cos n\alpha & -\sin n\alpha \\ \sin n\alpha & \cos n\alpha \end{bmatrix}$$

$$\Rightarrow \cos 32\alpha = 0 \text{ and } \sin 32\alpha = 1$$

$$\Rightarrow 32\alpha = \pi/2$$

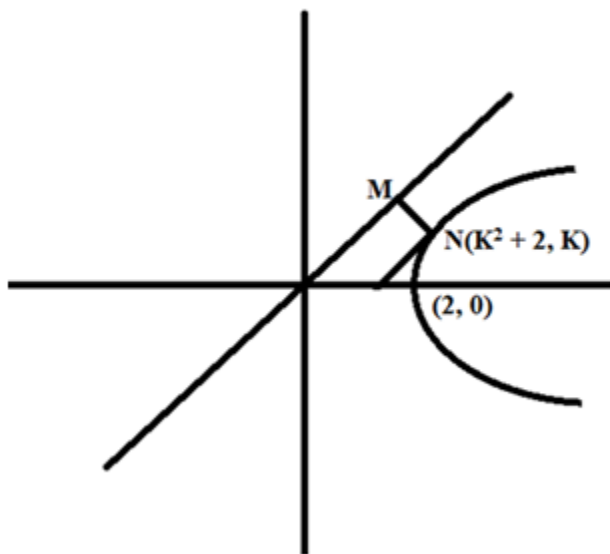
$$\alpha = \pi/64$$

Answer: C

8. Shortest distance between the curves $y^2 = x-2$ and $y = x$ is

- (A) greater than 4
- (B) less than 2
- (C) greater than 3
- (D) greater than 2

Solution:



$$y^2 = x-2$$

Differentiating we get,

$$2y (dy/dx) = 1$$

$$(dy/dx) = 1/2y$$

Slope of the line $y = x$ is 1.

Slope of the tangent to the curve = $1/2y$

$$1/2y = 1$$

$$\Rightarrow y = 1/2$$

$$\Rightarrow K = 1/2$$

$$\therefore N = (9/4, 1/2)$$

Minimum distance = MN = perpendicular distance from point N to line $y = x$

$$\left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right| = \left| \frac{1\left(\frac{9}{4}\right) - 1\left(\frac{1}{2}\right)}{\sqrt{1+1}} \right|$$

$$= 7/4\sqrt{2} \text{ which is less than } 2.$$

Answer: B

9.

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}} =$$

(A) $\sqrt{2}$

(B) 2

(C) 4

(D) $4\sqrt{2}$

Solution:

Rationalise the denominator

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin^2 x [\sqrt{2} + \sqrt{1 + \cos x}]}{2 - (1 + \cos x)} \\ = \lim_{x \rightarrow 0} \frac{\sin^2 x (\sqrt{2} + \sqrt{1 + \cos x})}{1 - \cos x} \\ = \lim_{x \rightarrow 0} \frac{\sin^2 x (\sqrt{2} + \sqrt{1 + \cos x})}{2 \sin^2 \frac{x}{2}} \\ = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \cdot \frac{1}{2} \cdot \frac{1}{\left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2 \times \frac{1}{4}} (\sqrt{2} + \sqrt{1 + \cos x}) \end{aligned}$$

$$= (4/2) \sqrt{2} + \sqrt{1+1}$$

$$= 4\sqrt{2}$$

Answer: D

10. How many 9 digit numbers can be formed by using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 such that odd numbers occur at even places.

(A) 160

(B) 175

(C) 180

(D) 220

Solution:

	Odd		Odd		Odd		Odd	
--	-----	--	-----	--	-----	--	-----	--

Even places = 4

Odd numbers = 3

Even numbers = 6

Therefore number of numbers = $4C_3 \times (3!/2!) \times (6!/4!2!) = 180$

11. Let $g(x) = \ln x$ and $f(x) = (1-x \cos x)/(1+x \cos x)$

$$\text{then } \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} g[f(x)] \cdot dx =$$

(A) $\ln 1$

(B) $\ln 2$

(C) $\ln e$

(D) $\ln 4$

Solution:

$$g(f(x)) = g\left[\frac{1-x \cos x}{1+x \cos x}\right]$$

$$= \ln\left(\frac{1-x \cos x}{1+x \cos x}\right)$$

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} g[f(x)]$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln\left(\frac{1-x \cos x}{1+x \cos x}\right) dx \quad \text{..(1)}$$

$$\text{We have } \int_a^b f(x) = \int_a^b f(a+b-x)$$

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln\left(\frac{1+x \cos x}{1-x \cos x}\right) dx \quad \text{..(2)}$$

Add (1) and (2)

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left[\ln \left(\frac{1+x \cos x}{1-x \cos x} \right) + \left(\frac{1-x \cos x}{1+x \cos x} \right) \right] dx$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln \left(\frac{1+x \cos x}{1-x \cos x} \cdot \frac{1-x \cos x}{1+x \cos x} \right) dx$$

Because $\log ab = \log a + \log b$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln 1 = 0$$

12. The sum of the series $2 \cdot {}^{20}C_0 + 5 \cdot {}^{20}C_1 + 8 \cdot {}^{20}C_2 + \dots + 62 \cdot {}^{20}C_{20}$ is equal to

- (A) $16 \cdot 2^{22}$
 (B) $8 \cdot 2^{22}$
 (C) $8 \cdot 2^{21}$
 (D) $16 \cdot 2^{21}$

Solution:

$$2 \cdot {}^{20}C_0 + 5 \cdot {}^{20}C_1 + 8 \cdot {}^{20}C_2 + \dots + 62 \cdot {}^{20}C_{20}$$

$$= \sum_{r=0}^{20} (3r + 2) {}^{20}C_r$$

$$= 3 \sum_{r=0}^{20} r \cdot {}^{20}C_r + 2 \cdot \sum_{r=0}^{20} {}^{20}C_r$$

$$= 3 \cdot 20 \sum_{r=1}^{20} {}^{19}C_{r-1} + 2 ({}^{20}C_0 + {}^{20}C_1 + \dots + {}^{20}C_{20})$$

$$= 60 \cdot 2^{19} + 2 \cdot 2^{20}$$

$$= 2^{21} (15 + 1)$$

$$= 2^{21} \cdot 16$$

Answer: D

13. Sum of natural numbers between 100 and 200 whose HCF with 91 should be more than 1.

- (A) 1121

- (B) 3210
(C) 3121
(D) 1520

Solution:

Given numbers: 101, 102, 103, ... 198, 199

$$91 = 13 \times 7$$

HCF of 91 and a number is more than 1 means the number should be either multiple of 7 or 13.

Therefore sum of the numbers = (Numbers divisible by 7) + (Numbers divisible by 13) –

(Numbers divisible by 91)

$$= (105+112+\dots+196 + (104+117+\dots+195)) - (182)$$

$$= (14/2)[105+196] + (8/2)[104+195] - 182$$

$$= 7(301) + 4(299) - 182$$

$$= 2107 + 1196 - 182$$

$$= 3121$$

Answer: C

14. If mean and variance of 7 variates are 8 and 16 respectively and five of them are 2, 4, 10, 12, 14 then the product of remaining two variates is

- (A) 49
(B) 48
(C) 45
(D) 40

Solution:

Let the remaining two variates be x and y

Given mean = 8

$$(x+y+2+4+10+12+14)/7 = 8$$

$$x+y+42 = 56$$

$$x+y = 14 \dots (1)$$

Given variance = 16

$$\Rightarrow [(x^2+y^2+4+16+100+144+196)/7] - 8^2 = 16$$

$$\Rightarrow (x^2+y^2+460)/7 = 80$$

$$\Rightarrow (x^2+y^2+460) = 560$$

$$\Rightarrow (x^2+y^2) = 100 \dots (2)$$

$$\Rightarrow (x+y)^2 - 2xy = 100$$

Substitute (1) in above equation

$$14^2 - 2xy = 100$$

$$196 - 2xy = 100$$

$$2xy = 96$$

$$xy = 48$$

Answer: B

15. If α and β are the roots of $x^2 - 2x + 2 = 0$ then the minimum value of n such that $(\alpha/\beta)^n = 1$

- (A) 4
(B) 3

- (C) 2
(D) 5

Solution:

$$x^2 - 2x + 2 = 0$$

$$x^2 - 2x + 1 + 1 = 0$$

$$\Rightarrow (x-1)^2 = -1$$

$$\Rightarrow (x-1) = \pm i$$

$$x = 1+i \text{ or } 1-i$$

$$\text{Let } \alpha = 1+i$$

$$\beta = 1-i$$

$$(\alpha/\beta) = (1+i)/(1-i)$$

$$= (1+i)^2/2$$

$$= (1+i^2+2i)/2$$

$$= (1-1+2i)/2$$

$$= i$$

$$(\alpha/\beta)^n = i^n = 1$$

Least value of n is 4.

Answer: A

16. Let $y = y(x)$ be the solution of the differential equation, $(x^2+1)^2 dy/dx + 2x(x^2+1)y = 1$ such that $y(0) = 0$. If $\sqrt{a} y(1) = \pi/32$, then the value of a is:

- (A) $1/2$
(B) $1/4$
(C) $1/16$
(D) 1

Solution:

$$(x^2+1)^2 dy/dx + 2x(x^2+1)y = 1$$

$$\frac{dy}{dx} + \frac{2x}{x^2+1} \cdot y = \frac{1}{(x^2+1)^2}$$

$$dy/dx + Py = Q$$

$$P = 2x/(x^2+1)$$

$$Q = 1/(x^2+1)^2$$

$$I.F = e^{\int \frac{2x}{1+x^2} dx}$$

$$= 1+x^2$$

The general solution of linear differential equation is

$$y.(I.F) = \int Q.(I.F)dx + C$$

$$y(1+x^2) = \int \frac{1}{1+x^2} dx + C$$

$$= \tan^{-1} x + C$$

$$\text{At } x = 0, y = 0$$

$$0(1) = \tan^{-1}0 + C$$

$$C = 0$$

$$y.(x^2+1) = \tan^{-1}x$$

$$\text{put } x = 1$$

$$y.(2) = \tan^{-1}1$$

$$y.(2) = \pi/4$$

$$y = \pi/8$$

$$(1/4)y = \pi/32$$

$$\text{i.e. } \sqrt{a} = 1/4$$

$$a = 1/16$$

Answer: C

17. If $f(x) = \log((1-x)/(1+x))$ then $f(2x/(1+x^2))$ is equal to

(A) $f(x)$

(B) $2f(x)$

(C) $-2f(x)$

(D) $[f(x)]^2$

Solution:

$$f(x) = \log\left(\frac{1-x}{1+x}\right)$$

$$f\left(\frac{2x}{1+x^2}\right) = \log\left[\frac{1 - \frac{2x}{1+x^2}}{1 + \frac{2x}{1+x^2}}\right]$$

$$= \log\left(\frac{1+x^2-2x}{1+x^2+2x}\right)$$

$$= \log\left(\frac{1-x}{1+x}\right)^2$$

$$= 2 \log\left(\frac{1-x}{1+x}\right)$$

$$= 2 f(x)$$

Answer: B

18. Given that $A \subset B$ then identify the correct statement.

(A) $P(A/B) = P(A)$

(B) $P(A/B) \leq P(A)$

(C) $P(A/B) \geq P(A)$

(D) $P(A/B) = P(A) - P(B)$

Solution:

$$P(A/B) = P(A \cap B)/P(B)$$

$$= P(A)/P(B) \geq P(A)$$

Answer: C

19. Find the value of 'c' for which the following equations have non-trivial solutions.

$$cx - y - z = 0$$

$$-cx + y - cz = 0$$

$$x + y - cz = 0$$

(A) $1/2$

(B) -1

(C) 2

(D) 0

Solution:

For non trivial solutions,

$$\begin{vmatrix} c & -1 & -1 \\ -c & 1 & -c \\ 1 & 1 & -c \end{vmatrix} = 0$$

$$\Rightarrow c(-c+c) + 1(c^2+c) - 1(-c-1) = 0$$

$$c^2+c+c+1 = 0$$

$$(c+1)^2 = 0$$

$$c+1 = 0$$

$$\therefore c = -1$$

Answer: B

20. Let

$$2y = \left[\cot^{-1} \left(\frac{\sqrt{3} \cos x + \sin x}{\cos x - \sqrt{3} \sin x} \right) \right]^2$$

Then dy/dx equals:

(A) $x - (\pi/6)$

(B) $x + (\pi/6)$

(C) $2x - (\pi/6)$

(D) $2x - (\pi/3)$

Solution:

$$2y = \left(\cot^{-1} \left(\frac{\sqrt{3} \cos x + \sin x}{\cos x - \sqrt{3} \sin x} \right) \right)^2 \quad x \in \left(0, \frac{\pi}{2} \right)$$

$$2y = \left(\cot^{-1} \left(\frac{\sqrt{3} + \tan x}{1 - \sqrt{3} \tan x} \right) \right)^2$$

$$2y = \left(\frac{\pi}{2} - \tan^{-1} \left(\tan \left(\frac{\pi}{3} + x \right) \right) \right)^2$$

$$2y = [(\pi/2) - (\pi/3) + x]^2$$

$$2y = [(\pi/6) - x]^2$$

Differentiating we get,

$$2 \, dy/dx = 2[(\pi/6) - x](-1)$$

$$dy/dx = x - (\pi/6)$$

Answer: A

21. Let S_1 is set of minima and S_2 is set of maxima for the curve $y = 9x^4 + 12x^3 - 36x^2 - 25$ then

- (A) $S_1 = \{-2, -1\}$ $S_2 = \{0\}$
- (B) $S_1 = \{-2, 1\}$ $S_2 = \{0\}$
- (C) $S_1 = \{-2, 1\}$ $S_2 = \{-1\}$
- (D) $S_1 = \{-2, 2\}$ $S_2 = \{0\}$

Solution:

$$y = 9x^4 + 12x^3 - 36x^2 - 25$$

$$dy/dx = 36x^3 + 36x^2 - 72x$$

$$= 36x(x^2 + x - 2)$$

$$= 36x(x^2 + 2x - x - 2)$$

$$= 36x(x+2)(x-1)$$

Critical points are 0, -2, 1

At $\{-2, 1\} \rightarrow$ points of minima

$\{0\} \rightarrow$ points of maxima.

Answer: B

22. Let $f: [0, 2] \rightarrow \mathbb{R}$ be a twice differentiable functions such that $f''(x) > 0$ for all $x \in (0, 2)$. If $\phi(x) = f(x) + f(2-x)$. then ϕ is:

- (A) Increasing in $(0, 1)$ and decreasing in $(1, 2)$
- (B) Decreasing in $(0, 1)$ and increasing in $(1, 2)$
- (C) Increasing in $(0, 2)$
- (D) Decreasing in $(0, 2)$

Solution:

Given $f'(x) > 0, x \in (0, 2)$

i.e $f(x)$ is an increasing function

$$\phi(x) = f(x) + f(2-x)$$

$$\phi'(x) = f'(x) - f'(2-x)$$

$\phi(x)$ is increasing $\Rightarrow \phi(x) > 0$

$$f'(x) - f'(2-x) > 0$$

$$f'(x) > f'(2-x)$$

$$x > 2-x$$

$$2x > 2$$

$$x > 1$$

i.e $x \in (1, 2)$

$\phi(x)$ is decreasing $\Rightarrow \phi(x) < 0$

$$f'(x) - f'(2-x) < 0$$

$$f'(x) < f'(2-x)$$

$$x < 2-x$$

$$2x < 2$$

$$x < 1$$

i.e $x \in (0, 1)$

Answer: B

23. Let vertices of the triangle ABC is A(0, 0), B(0, 1) and C(x, y) and perimeter is 4 then locus of 'C' is

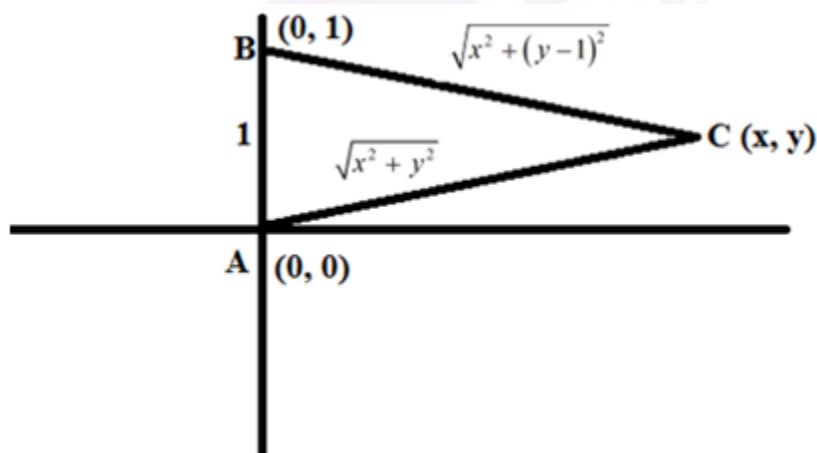
(A) $9x^2 + 8y^2 + 8y = 16$

(B) $8x^2 + 9y^2 + 9y = 16$

(C) $9x^2 + 8y^2 - 8y = 16$

(D) $8x^2 + 9y^2 - 9y = 16$

Solution:



Given $AB + AC + BC = 4$

$$1 + \sqrt{x^2 + (y-1)^2} + \sqrt{x^2 + y^2} = 4$$

$$x^2 + y^2 = 9 + x^2 + y^2 - 2y + 1 - 6\sqrt{x^2 + (y-1)^2}$$

$$3\sqrt{x^2 + (y-1)^2} = 5 - y$$

$$9(x^2 + (y-1)^2) = (5-y)^2$$

$$9x^2 + 8y^2 - 8y = 16$$

Answer: C

24. A point on the straight line, $3x + 5y = 15$ which is equidistant from the coordinate axes will lie only in

- (A) 1st, 2nd and 4th quadrants
- (B) 1st and 2nd quadrants
- (C) 4th quadrants
- (D) 1st quadrants

Solution:

$3x+5y = 15$ is the equation of the straight line.

If $x = y$

$$3x+5x = 15$$

$$8x = 15$$

$$x = 15/8$$

$$y = 15/8$$

P(15/8, 15/8) lies in 1st quadrant.

If $x = -y$

$$3x-5x = 15$$

$$-2x = 15$$

$$x = -15/2$$

$$y = 15/2$$

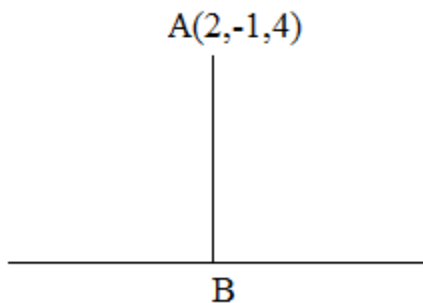
Q(-15/2, 15/2) lies in 2nd quadrant.

Answer: B

25. The perpendicular distance of point (2,-1,4) from the line $(x+3)/10 = (y-2)/-7 = z/1$ lies between

- (A) (2, 3)
- (B) (3, 4)
- (C) (4, 5)
- (D) (1, 2)

Solution:



Let A be (2, -1, 4).

Direction ratio of AB = $(10\lambda - 5, -7\lambda + 3, \lambda - 4)$

Drs of given line = (10, -7, 1)

$$10(10\lambda - 5) - 7(-7\lambda + 3) + (\lambda - 4) = 0$$

$$100\lambda - 50 + 49 - 21 + \lambda - 4 = 0$$

$$150\lambda = 75$$

$$\lambda = 1/2$$

Point B (2, -3/2, 1/2)

$$\text{Length AB} = 5/\sqrt{2}$$

$$= 3.53$$

Answer: B

26. If a plane passes through intersection of planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and also passes through the point (1, 1, 0). Then the equation of the plane is

(A) $x - y - z = 0$

(B) $2x - z = 0$

(C) $x + 2z - 1 = 0$

(D) $x - z - 1 = 0$

Solution:

Required equation of the plane is $(2x - y - 4) + \lambda(y + 2z - 4) = 0$

Given it pass through (1, 1, 0)

$$(2 - 1 - 4) + \lambda(1 + 0 - 4) = 0$$

$$\lambda = -1$$

$$(2x - y - 4) - (y + 2z - 4) = 0$$

$$\therefore 2x - 2y - 2z = 0$$

$$\therefore x - y - z = 0$$

Answer: A

27. The sum of the solutions of the equation $|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$, ($x > 0$) is equal to

(A) 4

(B) 10

(C) 9

(D) 12

Solution:

Given equation is $|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$, ($x > 0$)

Put $\sqrt{x} = t$

$$|t - 2| + t(t - 4) + 2 = 0$$

If $t \geq 2$

$$t - 2 + t^2 - 4t + 2 = 0$$

$$t^2 - 3t = 0$$

$$t(t - 3) = 0$$

$$t = 3 \text{ or } t = 0$$

$t = 0$ not possible

if $t = 3$, $x = 9$

If $t < 2$

$$2 - t + t^2 - 4t + 2 = 0$$

$$t^2 - 5t + 4 = 0$$

$$t = 1 \text{ or } t = 4$$

$t = 4$ not possible

so $t = 1$

$$\therefore x = 1$$

$$\text{Sum of solution} = 9 + 1 = 10$$

Answer: B

28. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points $(1, 2)$ and (a, b) are perpendicular to each other, then a^2 is equal to

(A) $2/17$

(B) $64/17$

(C) $128/17$

(D) $4/17$

Solution:

The equation of the ellipse $4x^2 + y^2 = 8$

$$dy/dx = -4x/y$$

The tangent at $(1, 2)$ and (a, b) are perpendicular

$$(-y/2)(-4a/b) = -1$$

$$b = -8a \text{ ..(i)}$$

(a, b) is on the ellipse.

$$4a^2 + b^2 = 8 \text{ [from eq(i)]}$$

$$4a^2 + 64a^2 = 8$$

$$\therefore a^2 = 8/68 = 2/17$$

Answer: A

29. Find the magnitude of projection of vector

$$2\hat{i} + 3\hat{j} + \hat{k}$$

on a vector which is perpendicular to the plane containing vectors

$$\hat{i} + \hat{j} + \hat{k} \text{ and } \hat{i} + 2\hat{j} + 3\hat{k}$$

(A) $\sqrt{3}/2$

(B) $\sqrt{3/2}$

(C) $3\sqrt{6}$

(D) $\sqrt{6}$

Solution:

The vector perpendicular to given vectors is $\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

$$\text{So } \vec{a} = \hat{i} - 2\hat{j} + \hat{k}$$

Projection of vector b in direction of vector a = $\frac{|\vec{a} \cdot \vec{b}|}{|\vec{a}|}$

$$= \frac{|(\hat{i} - 2\hat{j} + \hat{k}) \cdot (2\hat{i} + 3\hat{j} + \hat{k})|}{|\sqrt{6}|}$$

$$= |2 - 6 + 1|/\sqrt{6}$$

$$= 3/\sqrt{6}$$

$$= \sqrt{3/2}$$

Answer: B

30. If $\alpha = \cos^{-1}(3/5)$, $\beta = \tan^{-1}(1/3)$, where $0 < \alpha, \beta < \pi/2$, then $\alpha - \beta$ is equal to:

(A) $\tan^{-1}(9/5\sqrt{10})$

(B) $\sin^{-1}(9/5\sqrt{10})$

(C) $\tan^{-1}(9/15)$

(D) $\cos^{-1}(9/5\sqrt{10})$

Solution:

$$\alpha = \cos^{-1}(3/5) = \tan^{-1}(4/3)$$

$$\beta = \tan^{-1}(1/3)$$

$$\alpha - \beta = \tan^{-1}(4/3) - \tan^{-1}(1/3)$$

$$= \tan^{-1}\left(\frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{3} \times \frac{1}{3}}\right)$$

$$= \tan^{-1}\left(\frac{1}{1 + \frac{4}{9}}\right)$$

$$= \tan^{-1}(9/13)$$

$$= \sin^{-1} \frac{9}{\sqrt{13^2 + 9^2}}$$

$$= \sin^{-1}(9/\sqrt{250})$$

$$= \sin^{-1}(9/5\sqrt{10})$$

Answer: B