JEE Main 2020 Paper

Date of Exam: **7th January 2020 (Shift 2)**
Time**: 2:30 P.M. to 5:30 P.M.**
Subject**: Mathematics**

---

1. If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$, for some $a \in R$ then the distance between the foci of the ellipse is :

   a. $2\sqrt{5}$
   b. $2\sqrt{7}$
   c. $2\sqrt{2}$
   d. 4

   Answer: (b)
   Solution:

   $3x + 4y = 12\sqrt{2}$ is tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$

   Equation of tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ is $y = mx + \sqrt{a^2m^2 + 9}$

   Now, $3x + 4y = 12\sqrt{2} \Rightarrow y = -\frac{3}{4}x + 3\sqrt{2}$

   $\Rightarrow m = -\frac{3}{4}$ and $\sqrt{a^2m^2 + 9} = 3\sqrt{2}$

   $\Rightarrow a^2\left(-\frac{3}{4}\right)^2 + 9 = 18$

   $\Rightarrow a^2 \times \frac{9}{16} = 9$

   $\Rightarrow a^2 = 16 \Rightarrow a = 4$

   $$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$$

   Distance between foci is $2ae = 2 \times 4 \times \frac{\sqrt{7}}{4} = 2\sqrt{7}$

2. Let A, B, C and D be four non-empty sets. The Contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$ then $A \subseteq C$" is :

   a. If $A \subseteq C$, then $B \subset A$ or $D \subset B$
   b. If $A \nsubseteq C$, then $A \subseteq B$ and $B \subseteq D$
   c. If $A \nsubseteq C$, then $A \nsubseteq B$ and $B \subseteq D$
   d. If $A \nsubseteq C$, then $A \nsubseteq B$ or $B \nsubseteq D$

   Answer: (d)
   Solution:
   Given statements: $A \subseteq B$ and $B \subseteq D$
   Let $A \subseteq B$ be p
   $B \subseteq D$ be q
   $A \subseteq C$ be r

Modified statement: $(p \wedge q) \Rightarrow r$
Contrapositive: $\sim r \Rightarrow \sim (p \wedge q)$
$\sim r \Rightarrow (\sim p \vee \sim q)$
$\therefore$ A $\nsubseteq$ C, then A $\nsubseteq$ B or B $\nsubseteq$ D

3. The coefficient of x^7 in the expression $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \cdots + x^{10}$ is :

a. 420
b. 330
c. 210
d. 120

Answer: (b)
Solution:

Coefficient of x^7 in $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \cdots + x^{10}$

Applying sum of terms of G.P. = $\dfrac{(1+x)^{10}\left(1-\left(\frac{x}{1+x}\right)^{11}\right)}{\left(1-\frac{x}{1+x}\right)} = (1+x)^{11} - x^{11}$

Coefficient of $x^7 \Longrightarrow {}^{11}C_7 = 330$

4. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to :

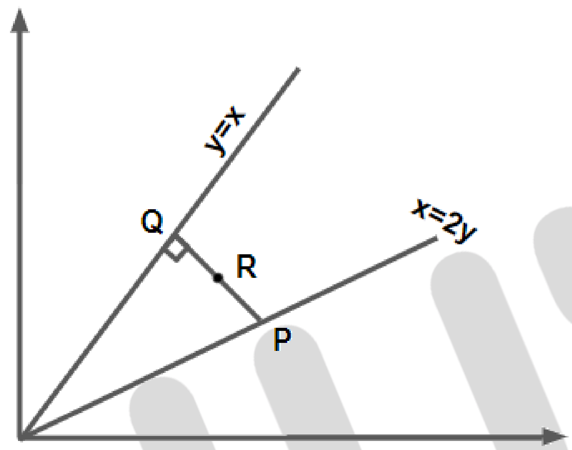
a. $\frac{17}{2}$
b. 4
c. $\frac{17}{4}$
d. $\frac{17}{8}$

Answer: (d)
Solution:
P(machine being faulty) $= p = \frac{1}{4}$
$\therefore q = \frac{3}{4}$
P(at most two machines being faulty) = P(zero machine being faulty)+P(one machine being faulty)+P(two machines being faulty)
$= {}^5C_0 p^0 q^5 + {}^5C_1 p^1 q^4 + {}^5C_2 p^2 q^3$
$= q^5 + 5pq^4 + 10p^2q^3$
$= \left(\frac{3}{4}\right)^5 + 5 \times \frac{1}{4}\left(\frac{3}{4}\right)^4 + 10\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$
$= \left(\frac{3}{4}\right)^3 \left[\frac{9}{16} + \frac{15}{16} + \frac{10}{16}\right]$
$= \left(\frac{3}{4}\right)^3 \times \frac{34}{16} = \left(\frac{3}{4}\right)^3 \times \frac{17}{8}$
$\therefore k = \frac{17}{8}$

5. The locus of mid points of the perpendiculars drawn from points on the line $x = 2y$ to the line $x = y$ is :

a. $2x - 3y = 0$
b. $3x - 2y = 0$
c. $5x - 7y = 0$
d. $7x - 5y = 0$

Answer: (c)
Solution:



Let R be the midpoint of PQ
PQ is perpendicular on line $y = x$
$\therefore$ Equation of the line PQ can be written as $y = -x + c$
$y = -x + c$ intersects $y = x$ at Q: $\left(\frac{c}{2}, \frac{c}{2}\right)$
$y = -x + c$ intersects $x = 2y$ at P: $\left(\frac{2c}{3}, \frac{c}{3}\right)$
$\therefore$ Midpoint R: $\left(\frac{7c}{12}, \frac{5c}{12}\right)$
Locus of R : $x = \frac{7c}{12}$, $y = \frac{5c}{12}$
$\Rightarrow 5x - 7y = 0$

6. The value of α for which $4\alpha \int_{-1}^{2} e^{-\alpha|x|}\, dx = 5$, is :

a. $\log_e 2$
b. $\log_e \sqrt{2}$
c. $\log_e \left(\frac{4}{3}\right)$
d. $\log_e \left(\frac{3}{2}\right)$

Answer: (a)

Solution:

$$4\alpha \int_{-1}^{2} e^{-\alpha|x|} dx = 5$$

$$\Rightarrow 4\alpha\left[\int_{-1}^{0} e^{-\alpha|x|}dx + \int_{0}^{2} e^{-\alpha|x|}dx\right] = 5$$

$$\Rightarrow 4\alpha\left[\int_{-1}^{0} e^{\alpha x}\, dx + \int_{0}^{2} e^{-\alpha x}\, dx\right] = 5$$

$$\Rightarrow 4\alpha\left[\left(\frac{1 - e^{-\alpha}}{\alpha}\right) + \left(\frac{e^{-2\alpha} - 1}{-\alpha}\right)\right] = 5$$

$\Rightarrow 4[1 - e^{-2\alpha} - e^{-\alpha} + 1] = 5$

Let $e^{-\alpha} = t$

$\Rightarrow\ 4t^2 + 4t - 3 = 0$

$\Rightarrow\ t = \frac{1}{2} = e^{-\alpha} \Rightarrow \alpha = \log_e 2$

7. If the sum of the first 40 terms of the series, $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 +$ ..... is $(102)m$, then m is equal to :

a. 10
b. 25
c. 5
d. 20

Answer: (d)
Solution:

$S = \underline{3 + 4} + \underline{8 + 9} + 13 + 14 + \ \ldots\ldots\ 40$ terms

$S = 7 + 17 + 27 + 37 + \cdots \ldots\ldots .20$ terms

$S = \frac{20}{2}[14 + (19)10] = 20 \times 102$

$\therefore m = 20$

8. If $\frac{3+i\sin\theta}{4-i\cos\theta}$, $\theta \in [0,2\pi]$, is a real number, then the argument of $\sin\theta + i\cos\theta$ is :

a. $\pi - \tan^{-1}\left(\frac{4}{3}\right)$
b. $-\tan^{-1}\left(\frac{3}{4}\right)$
c. $\pi - \tan^{-1}\left(\frac{4}{3}\right)$
d. $\tan^{-1}\left(\frac{4}{3}\right)$

Answer: (a)
Solution:

Let $z = \frac{3+i\sin\theta}{4-i\cos\theta} \times \frac{4+i\cos\theta}{4+i\cos\theta}$

$$= \frac{12 - \sin\theta\cos\theta + i(4\sin\theta + 3\cos\theta)}{16 + \cos^2\theta}$$

z is real.

$\therefore 4\sin\theta + 3\cos\theta = 0$

$\Rightarrow \tan\theta = -\frac{3}{4}$ $\quad$ [θ lies in 2^{nd} quadrant]

$\arg(\sin\theta + i\cos\theta) = \pi + \tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right) = \pi - \tan^{-1}\left(\frac{4}{3}\right)$

9. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i+j-2)}a_{ji}$, where $i, j = 1, 2, 3$. If the determinant of B is 81, then the determinant of A is :

a. $\frac{1}{9}$

b. $\frac{1}{81}$

c. $\frac{1}{3}$

d. 3

Answer: (c)
Solution:
$b_{ij} = (3)^{(i+j-2)}a_{ji}$

$$B = \begin{bmatrix} 3^0 a_{11} & 3a_{21} & 3^2 a_{31} \\ 3a_{12} & 3^2 a_{22} & 3^3 a_{32} \\ 3^2 a_{13} & 3^3 a_{23} & 3^4 a_{33} \end{bmatrix}$$

$$|B| = \begin{vmatrix} 3^0 a_{11} & 3a_{21} & 3^2 a_{31} \\ 3a_{12} & 3^2 a_{22} & 3^3 a_{32} \\ 3^2 a_{13} & 3^3 a_{23} & 3^4 a_{33} \end{vmatrix}$$

Taking 3^2 common each from C_3 and R_3

$$|B| = 81 \begin{vmatrix} a_{11} & 3a_{21} & a_{31} \\ 3a_{12} & 3^2 a_{22} & 3a_{32} \\ a_{13} & 3a_{23} & a_{33} \end{vmatrix}$$

Taking 3 common each from C_2 and R_2

$$|B| = 81(9) \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

Given $|B| = 81$

$\Rightarrow 81 = 81(9)|A| \Rightarrow |A| = \frac{1}{9}$

10. Let f(x) be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim_{x \to 0}\left(2 + \frac{f(x)}{x^3}\right) = 4$, then which one of the following is not true?

a. $f(1) - 4f(-1) = 4$
b. $x = 1$ is a point of maxima and $x = -1$ is a point of minimum of f.
c. f is an odd function.
d. $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f.

Answer: (d)
Solution:
Given $\lim_{x \to 0}\left(2 + \frac{f(x)}{x^3}\right) = 4$

$\lim_{x \to 0} \frac{f(x)}{x^3} = 2$

$\lim_{x\to 0}\frac{f(x)}{x^3}$ Limit exists and it is finite

$\therefore f(x) = ax^5 + bx^4 + cx^3$

$\Rightarrow \lim_{x\to 0}(ax^2 + bx + c) = 2$

$c = 2$

Also $f'(x) = 5ax^4 + 4bx^3 + 6x^2$

$f'(1) = 5a + 4b + 6 = 0$

$f'(-1) = 5a - 4b + 6 = 0$

$b = 0, \qquad a = -\frac{6}{5}$

$f(x) = -\frac{6}{5}x^5 + 2x^3 \qquad \Rightarrow f(x)$ is odd

$f'(x) = -6x^4 + 6x^2$

$f''(x) = -24x^3 + 12x$

$(f''(1) < 0) \qquad (f''(-1) > 0)$

At $x = -1$ there is local minima and at $x = 1$ there is local maxima.

And $f(1) - 4f(-1) = 4$

11. The number of ordered pairs (r, k) for which $6 \cdot {}^{35}C_r = (k^2 - 3) \cdot {}^{36}C_{r+1}$, where k is an integer, is :

a. 4
b. 6
c. 2
d. 3

Answer: (a)
Solution:

Using ${}^{36}C_{r+1} = \frac{36}{r+1} \times {}^{35}C_r$

$\frac{36}{r+1} \times {}^{35}C_r \times (k^2 - 3) = {}^{35}C_r \times 6$

$k^2 - 3 = \frac{r+1}{6}$

$k^2 = \frac{r+1}{6} + 3$

$k \in I$

$r \rightarrow$ Non-negative integer $0 \le r \le 35$

$r = 5 \Rightarrow k = \pm 2$

$r = 35 \Rightarrow k = \pm 3$

No. of ordered pairs $(r, k) = 4$

12. Let $a_1, a_2, a_3, \ldots$ be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to :

a. 171
b. $\frac{511}{3}$
c. -171
d. -513

Answer: (c)
Solution:
$a_1 + a_2 = 4 \Rightarrow a + ar = 4 \Rightarrow a(1 + r) = 4$
$a_3 + a_4 = 16 \Rightarrow ar^2 + ar^3 = 16 \Rightarrow ar^2(1 + r) = 16 \Rightarrow 4r^2 = 16$
$\Rightarrow r = \pm 2$
If $r = 2, a = \frac{4}{3}$ which is not possible as $a_1 < 0$
If $r = -2, a = -4$
$\sum_{i=1}^{9} a_i = \frac{a(r^9-1)}{r-1} = \frac{(-4)[(-2)^9-1]}{-3} = \frac{4}{3}(-512 - 1) = 4(-171)$
$\lambda = -171$

13. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. If $\lambda = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ and $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$, then the ordered pair $(\lambda, \vec{d})$ is equal to :

a. $\left(\frac{3}{2}, 3\vec{a} \times \vec{c}\right)$
b. $\left(-\frac{3}{2}, 3\vec{c} \times \vec{b}\right)$
c. $\left(-\frac{3}{2}, 3\vec{a} \times \vec{b}\right)$
d. $\left(\frac{3}{2}, 3\vec{b} \times \vec{c}\right)$

Answer: (c)
Solution:
Given $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ $\quad\lambda = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$
$\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{0}|^2$
$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$
$\lambda = -\frac{3}{2}$
Also $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$
$\Rightarrow \vec{d} = \vec{a} \times \vec{b} + \vec{b} \times (-\vec{a} - \vec{b}) + (-\vec{a} - \vec{b}) \times \vec{a}$
$\Rightarrow \vec{d} = \vec{a} \times \vec{b} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b}$
$\Rightarrow \vec{d} = 3(\vec{a} \times \vec{b}) = 3\vec{a} \times \vec{b}$

14. Let $y = y(x)$ be the solution curve of the differential equation, $(y^2 - x)\frac{dy}{dx} = 1$, satisfying $y(0) = 1$
This curve intersects the $x -$ axis at a point whose abscissa is :

a. $2 + e$
b. 2
c. $2 - e$
d. $-e$

Answer: (c)
Solution:
$(y^2 - x)\frac{dy}{dx} = 1$
$\frac{dx}{dy} + x = y^2$
$xe^y = \int y^2 e^y dy$
$x = y^2 - 2y + 2 + ce^{-y}$
Given $y(0) = 1$

$\Rightarrow c = -e$

$\therefore$ Solution is $x = y^2 - 2y + 2 - e^{-y+1}$

$\therefore$ The value of x where the curve cuts the $x -$ axis will be at $x = 2 - e$

15. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0,2\pi) - \{\pi\}$ which satisfy the equation, $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$, then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta \, d\theta$ is equal to :

a. $\frac{2\pi}{3}$

b. $\frac{\pi}{3}$

c. $\frac{\pi}{3} + \frac{1}{6}$

d. $\frac{\pi}{9}$

Answer: (b)

Solution:

$2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0, \theta \in [0,2\pi)$

$\Rightarrow 2\mathrm{cosec}^2\theta - 2 - 5\mathrm{cosec}\,\theta + 4 = 0$

$\Rightarrow 2\mathrm{cosec}^2\theta - 4\mathrm{cosec}\,\theta - \mathrm{cosec}\,\theta + 2 = 0$

$\Rightarrow \mathrm{cosec}\,\theta = 2$ or $\frac{1}{2}$ (Not possible)

As $\theta \in [0,2\pi)$,

$\theta_1 = \frac{\pi}{6}, \theta_2 = \frac{5\pi}{6}$

$$\Rightarrow \int_{\theta_1}^{\theta_2} \cos^2 3\theta \, d\theta = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{(1+\cos 6\theta)}{2} d\theta$$

$$= \frac{1}{2}\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) + \frac{\sin 6\theta}{12}\Bigg|_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$= \frac{\pi}{3}$$

16. Let α and β are the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k, k \geq 1$ then which one of the following statements is not true?

a. $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$
b. $p_5 = 11$
c. $p_5 = p_2 \cdot p_3$
d. $p_3 = p_5 - p_4$

Answer: (c)

Solution:

Given α, β are the roots of $x^2 - x - 1 = 0$

$\Rightarrow \alpha + \beta = 1 \;\&\; \alpha\beta = -1$

$\Rightarrow \alpha^2 = \alpha + 1 \;\&\; \beta^2 = \beta + 1$

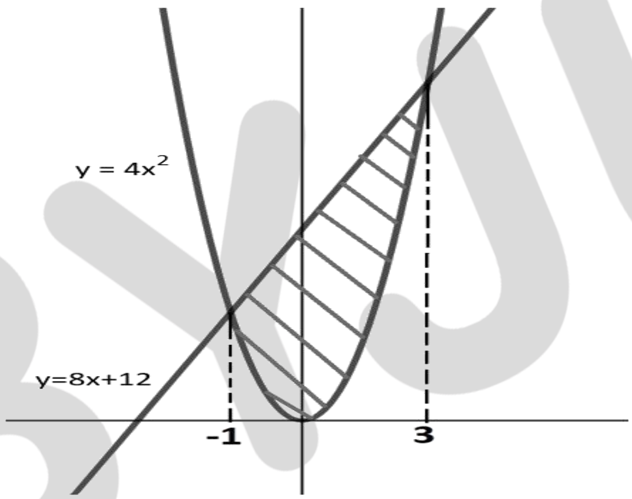
$p_k = \alpha^{k-2}\alpha^2 + \beta^{k-2}\beta^2$

$p_k = \alpha^{k-2}(\alpha+1) + \beta^{k-2}(\beta+1)$
$p_k = \alpha^{k-1} + \beta^{k-1} + \alpha^{k-2} + \beta^{k-2}$
$\Rightarrow p_k = p_{k-1} + p_{k-2}$
$\Rightarrow p_3 = p_2 + p_1 = 4$
$p_4 = p_3 + p_2 = 7$
$p_5 = p_4 + p_3 = 11$
$\therefore p_5 \neq p_2 \cdot p_3$ & $p_1 + p_2 + p_3 + p_4 + p_5 = 26$
& $p_3 = p_5 - p_4$

17. The area (in sq. units) of the region $\{(x, y) \epsilon R | 4x^2 \leq y \leq 8x + 12\}$ is :

a. $\frac{125}{3}$
b. $\frac{128}{3}$
c. $\frac{124}{3}$
d. $\frac{127}{3}$

Answer: (b)
Solution:



For point of intersection,
$4x^2 = 8x + 12$
$\Rightarrow x^2 - 2x - 3 = 0$
$\Rightarrow x = 3, -1$
Area bounded is given by
$A = \int_{-1}^{3} (8x + 12 - 4x^2) dx$
$$A = \left[\frac{8x^2}{2} + 12x - \frac{4x^3}{3}\right]_{-1}^{3}$$
$$A = (36 + 36 - 36) - \left(4 - 12 + \frac{4}{3}\right)$$
$A = 44 - \frac{4}{3} = \frac{128}{3}$

18. The value of c in Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, where $x \in [0,1]$ is :

a. $\frac{4-\sqrt{7}}{3}$

b. $\frac{2}{3}$

c. $\frac{\sqrt{7}-2}{3}$

d. $\frac{4-\sqrt{5}}{3}$

Answer: (a)
Solution:
LMVT is applicable on f(x) in [0,1], therefore it is continuous and differentiable in [0,1]
Now, $f(0) = 11, f(1) = 16$
$f'(x) = 3x^2 - 8x + 8$
$\therefore f'(c) = \frac{f(1)-f(0)}{1-0} = \frac{16-11}{1}$
$\Rightarrow 3c^2 - 8c + 8 = 5$
$\Rightarrow 3c^2 - 8c + 3 = 0$
$\Rightarrow c = \frac{8\pm2\sqrt{7}}{6} = \frac{4\pm\sqrt{7}}{3}$
As $c \in (0,1)$
We get, $c = \frac{4-\sqrt{7}}{3}$

19. Let $y = y(x)$ be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x = \frac{1}{2}$, is equal to :

a. $-\frac{\sqrt{5}}{2}$

b. $\frac{\sqrt{5}}{2}$

c. $-\frac{\sqrt{5}}{4}$

d. $\frac{2}{\sqrt{5}}$

Answer: (a)
Solution:
$y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$
Differentiating w.r.t. x on both the sides, we get:
$y'\sqrt{1-x^2} + y \times \frac{1}{2\sqrt{1-x^2}} \times (-2x) = -\sqrt{1-y^2} - x \times \frac{1}{2\sqrt{1-y^2}} \times (-2y)y'$
$\Rightarrow y'\sqrt{1-x^2} - \frac{xy}{\sqrt{1-x^2}} + \sqrt{1-y^2} - \frac{xy}{\sqrt{1-y^2}}y' = 0$
$\Rightarrow y'\left[\sqrt{1-x^2} - \frac{xy}{\sqrt{1-y^2}}\right] = \frac{xy}{\sqrt{1-x^2}} - \sqrt{1-y^2}$
Putting $x = \frac{1}{2}, y = -\frac{1}{4}$
$\Rightarrow y'\left[\frac{\sqrt{3}}{2} + \frac{\frac{1}{8}}{\frac{\sqrt{15}}{4}}\right] = -\frac{\frac{1}{8}}{\frac{\sqrt{3}}{2}} - \frac{\sqrt{15}}{4}$

$\Rightarrow y'\left[\frac{\sqrt{3}}{2}+\frac{1}{2\sqrt{15}}\right]=-\frac{1}{4\sqrt{3}}-\frac{\sqrt{15}}{4}$

$\Rightarrow y'\left[\frac{\sqrt{45}+1}{2\sqrt{15}}\right]=-\frac{1+\sqrt{45}}{4\sqrt{3}}$

$\Rightarrow y'=-\frac{\sqrt{5}}{2}$

20. Let the tangents drawn from the origin to the circle, $x^2+y^2-8x-4y+16=0$ touch it at the points A and B. The $(AB)^2$ is equal to :

a. $\frac{32}{5}$

b. $\frac{64}{5}$

c. $\frac{52}{5}$

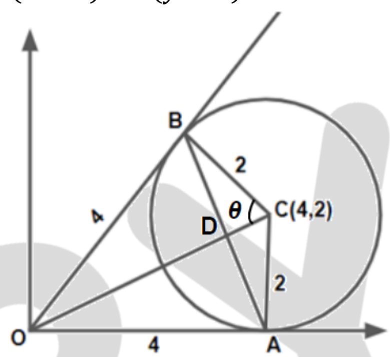
d. $\frac{56}{5}$

Answer: (b)

Solution:

$x^2+y^2-8x-4y+16=0$

$(x-4)^2+(y-2)^2=4 \ \Rightarrow$ Centre $(4,2)$, radius $=2$



$OA=4=OB$

In ΔOBC

$tan\,\theta=\frac{4}{2}=2\Rightarrow sin\,\theta=\frac{2}{\sqrt{5}}$

In ΔBDC

$sin\,\theta=\frac{BD}{2}\Rightarrow BD=\frac{4}{\sqrt{5}}$

Length of chord of contact $(AB)=\frac{8}{\sqrt{5}}$

Alternative

(l) length of tangent = 4 and (r) radius =2

$\Rightarrow$Length of chord of contact = $\frac{2lr}{\sqrt{(l^2+r^2)}}$

Square of length of chord of contact = $\frac{64}{5}$

21. If system of linear equations
$x + y + z = 6$
$x + 2y + 3z = 10$
$3x + 2y + \lambda z = \mu$
has more than two solutions, then $\mu - \lambda^2$ is equal to _______.

Answer: (13)
Solution:
The system of equations has more than 2 solutions
$\therefore\ D = D_3 = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix} = 0 \Rightarrow 2\lambda - 6 - \lambda + 9 + 2 - 6 = 0$$

$\Rightarrow \lambda = 1$

$$\begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 3 & 2 & \mu \end{vmatrix} = 0 \Rightarrow 2\mu - 20 - \mu + 30 - 24 = 0$$

$\Rightarrow \mu = 14$
So, $\mu - \lambda^2 = 13$

22. If the foot of perpendicular drawn from the point $(1, 0, 3)$ on a line passing through $(\alpha, 7, 1)$ is $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$, then α is equal to ________.

Answer: (4)
Solution:
Given points $P(1, 0, 3)$ and $Q\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$

Direction ratios of line L: $\left(\alpha - \frac{5}{3}, 7 - \frac{7}{3}, 1 - \frac{17}{3}\right)$

$= \left(\frac{3\alpha - 5}{3}, \frac{14}{3}, -\frac{14}{3}\right)$

Direction ratios of PQ: $\left(-\frac{2}{3}, -\frac{7}{3}, -\frac{8}{3}\right)$

As line L is perpendicular to PQ

So, $\left(\frac{3\alpha-5}{3}\right)\left(-\frac{2}{3}\right) + \left(\frac{14}{3}\right)\left(-\frac{7}{3}\right) + \left(-\frac{14}{3}\right)\left(-\frac{8}{3}\right) = 0$

$\Rightarrow -6\alpha + 10 - 98 + 112 = 0 \Rightarrow 6\alpha = 24 \ \Rightarrow \alpha = 4$

23. If the function f defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by

$$f(x) = \begin{cases} \left(\frac{1}{x}\right)\log_e\left(\frac{1+3x}{1-2x}\right) & when \quad x \neq 0 \\ k \quad , & when \quad x = 0 \end{cases}$$

is continuous, the k is equal to _______.

Answer: (5)
Solution:

As $f(x)$ is continuous

$\Rightarrow \lim_{x\to 0} f(x) \quad = f(0) = k$

$\Rightarrow \lim_{x\to 0}\left(\frac{1}{x}\right)\log_e\left(\frac{1+3x}{1-2x}\right) = k$

$\Rightarrow \lim_{x\to 0}\frac{3\log(1+3x)}{3x} + \lim_{x\to 0}\frac{(-2)\log(1-2x)}{(-2x)} = k$

$\Rightarrow 3 + 2 = k \ \Rightarrow k = 5$

24. If the mean and variance of eight numbers $3, 7, 9, 12, 13, 20, x$ and y be 10 and 25 respectively then xy is equal to _______.

Answer: (54)
Solution:

Mean $= 10 \Rightarrow \frac{64+x+y}{8} = 10$

$\Rightarrow x + y = 16$

Variance $= \frac{\sum x_i^2}{n} - (\bar{x})^2$

$\Rightarrow 25 = \frac{3^2+7^2+9^2+12^2+13^2+20^2+x^2+y^2}{8} - 100$

$\Rightarrow 1000 = 852 + x^2 + y^2$

$\Rightarrow x^2 + y^2 = 148$

$\Rightarrow (x+y)^2 - 2xy = 148$

$\Rightarrow 256 - 2xy = 148$

So, $xy = 54$

25. Let $X = \{n \in \boldsymbol{N}: 1 \le n \le 50\}$. If $A = \{n \in X: n$ is a multiple of 2$\}$ and $B = \{n \in X: n$ is a multiple of 7$\}$, then the number of elements in the smallest subset of X containing both A and B is _______.

Answer: (29)
Solution:

$A = \{$x: x is multiple of 2$\} = \{2,4,6,8,\ldots\}$

$\mathrm{B} = \{$x: x is multiple of 7$\} = \{7,14,21,\ldots\}$

$\mathrm{X} = \{\mathrm{x}:\ 1 \le \mathrm{x} \le 50, \mathrm{x} \in \mathbf{N}\}$

Smallest subset of X which contains elements of both A and B is a set with multiples of 2 or 7 less than 50.

$\mathrm{P} = \{$x: x is a multiple of 2 less than or equal to 50$\}$

$\mathrm{Q} = \{$x: x is a multiple of 7 less than or equal to 50$\}$

$\mathrm{n}(\mathrm{P} \cup \mathrm{Q}) = \mathrm{n}(\mathrm{P}) + \mathrm{n}(\mathrm{Q}) - \mathrm{n}(\mathrm{P} \cap \mathrm{Q})$

$= 25 + 7 - 3$

$= 29$