JEE Main 2020 Paper

Date of Exam: **9th January 2020 (Shift 1)**
Time: **9:30 A.M. to 12:30 P.M.**
Subject: **Mathematics**

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1. If C be the centroid of the triangle having vertices $(3,-1),(1,3)$ and $(2,4)$. Let P be the point of intersection of the lines $x+3y-1=0$ and $3x-y+1=0$, then the line passing through the points C and P also passes through the point:
   a. $(-9,-7)$
   b. $(-9,-6)$
   c. $(7,6)$
   d. $(9,7)$

   Answer: (b)

   Solution:

   Coordinates of C are $\left(\frac{3+1+2}{3},\frac{-1+3+4}{3}\right)=(2,2)$

   Point of intersection of two lines

   $x+3y-1=0$ and $3x-y+1=0$

   is $P\left(\frac{-1}{5},\frac{2}{5}\right)$

   Equation of line CP is $8x-11y+6=0$

   Point $(-9,-6)$ lies on CP

2. The product $2^{\frac{1}{4}}\times 4^{\frac{1}{16}}\times 8^{\frac{1}{48}}\times 16^{\frac{1}{128}}\ldots$ to ∞ is equal to:
   a. $2^{\frac{1}{4}}$
   b. 2
   c. $2^{\frac{1}{2}}$
   d. 1

   Answer: (c)

   Solution:

   $2^{\frac{1}{4}}\times 4^{\frac{1}{16}}\times 8^{\frac{1}{48}}\ldots\infty = 2^{\frac{1}{4}}\times 2^{\frac{2}{16}}\times 2^{\frac{4}{48}}\ldots\infty$

   $\Rightarrow 2^{\frac{1}{4}}\times 2^{\frac{1}{8}}\times 2^{\frac{1}{16}}\ldots\infty = 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\infty}$

   $\Rightarrow 2^{\left(\frac{\frac{1}{4}}{1-\frac{1}{2}}\right)}=\sqrt{2}$

3. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness that melts at the rate of $50\text{ cm}^3/\text{min}$. When the thickness of ice is 5 cm, then the rate (in cm/min.) at which the thickness of ice decreases, is:

   a. $\frac{5}{6\pi}$
   b. $\frac{1}{54\pi}$
   c. $\frac{1}{36\pi}$
   d. $\frac{1}{18\pi}$

   Answer: (d)

   Solution:

   Let thickness of ice be x cm.

   Therefore, net radius of sphere $= (10 + x)$ cm

   Volume of sphere $V = \frac{4}{3}\pi(10 + x)^3$

   $\Rightarrow \frac{dV}{dt} = 4\pi(10 + x)^2 \frac{dx}{dt}$

   At $x = 5, \frac{dV}{dt} = 50\text{ cm}^3/\text{min}$

   $\Rightarrow 50 = 4\pi \times 225 \times \frac{dx}{dt}$

   $\frac{dx}{dt} = \frac{1}{18\pi}$ cm/min

4. Let f be any function continuous on $[a, b]$ and twice differentiable on (a, b). If for all $x \in (a, b)$, $f'(x) > 0$ and $f''(x) < 0$, then for any $c \in (a, b)$, $\frac{f(c)-f(a)}{f(b)-f(c)}$ is greater than:

   a. $\frac{b-c}{c-a}$
   b. 1
   c. $\frac{c-a}{b-c}$
   d. $\frac{b+a}{b-a}$

   Answer: (c)

   Solution:

   $\therefore c \in (a, b)$ and f is twice differentiable and continuous function on (a, b)

   $\therefore$ LMVT is applicable

   For $p \in (a, c)$, $\quad f'(p) = \frac{f(c)-f(a)}{c-a}$

   For $q \in (c, b)$, $\quad f'(q) = \frac{f(b)-f(c)}{b-c}$

   $\therefore f''(x) < 0 \Rightarrow f'(x)$ is decreasing

   $f'(p) > f'(q)$

$$\Rightarrow \frac{f(c)-f(a)}{c-a} > \frac{f(b)-f(c)}{b-c}$$

$\Rightarrow \frac{f(c)-f(a)}{f(b)-f(c)} > \frac{c-a}{b-c}$ (as $f'(x) > 0 \Rightarrow f(x)$ is increasing)

5. The value of $\cos^3\frac{\pi}{8}\cos\frac{3\pi}{8} + \sin^3\frac{\pi}{8}\sin\frac{3\pi}{8}$ is :

a. $\frac{1}{4}$
b. $\frac{1}{2\sqrt{2}}$
c. $\frac{1}{2}$
d. $\frac{1}{\sqrt{2}}$

Answer: (b)

Solution:

$\cos^3\frac{\pi}{8}\cos\frac{3\pi}{8} + \sin^3\frac{\pi}{8}\sin\frac{3\pi}{8} = \cos^3\frac{\pi}{8}\left[4\cos^3\frac{\pi}{8} - 3\cos\frac{\pi}{8}\right] + \sin^3\frac{\pi}{8}\left[3\sin\frac{\pi}{8} - 4\sin^3\frac{\pi}{8}\right]$

$= 4\left[\cos^6\frac{\pi}{8} - \sin^6\frac{\pi}{8}\right] + 3\left[\sin^4\frac{\pi}{8} - \cos^4\frac{\pi}{8}\right]$

$= 4\left[\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right]\left[\cos^4\frac{\pi}{8} + \sin^4\frac{\pi}{8} + \cos^2\frac{\pi}{8}\sin^2\frac{\pi}{8}\right] - 3\left[\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right]$

$= \left[\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right]\left[4\left(1 - \cos^2\frac{\pi}{8}\sin^2\frac{\pi}{8}\right) - 3\right]$

$= \cos\frac{\pi}{4}\left[1 - \sin^2\frac{\pi}{4}\right] = \frac{1}{2\sqrt{2}}$

6. The number of real roots of the equation, $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is:

a. 3
b. 4
c. 1
d. 2

Answer: (c)

Solution:

$e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$

$$\Rightarrow e^{2x} + e^x - 4 + \frac{1}{e^x} + \frac{1}{e^{2x}} = 0$$

$$\Rightarrow \left(e^{2x} + \frac{1}{e^{2x}}\right) + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$$

$$\Rightarrow \left(e^x + \frac{1}{e^x}\right)^2 - 2 + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$$

Let $e^x + \frac{1}{e^x} = u$

Then, $u^2 + u - 6 = 0$

$\Rightarrow u = 2, -3$

$u \neq -3$ as $u > 0$ ($\because e^x > 0$)

$\Rightarrow e^x + \frac{1}{e^x} = 2 \Rightarrow (e^x - 1)^2 = 0 \Rightarrow e^x = 1 \Rightarrow x = 0$

Hence, only one real solution is possible.

7. The value of $\int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx$ is equal to:

a. 2π b. 4π
c. $2\pi^2$ d. π^2

Answer: (d)

Solution:

Let $I = \int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx$ ... (1)

$I = \int_0^{2\pi} \frac{(2\pi - x)\sin^8(2\pi - x)}{\sin^8(2\pi - x) + \cos^8(2\pi - x)} dx$

$= \int_0^{2\pi} \frac{(2\pi - x)\sin^8 x}{\sin^8 x + \cos^8 x} dx$ ... (2)

Adding (1) & (2) , we get:

$\Rightarrow 2I = 2\pi \int_0^{2\pi} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx$

$I = \pi \int_0^{2\pi} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx$

$I = 4\pi \int_0^{\frac{\pi}{2}} \frac{\sin^8 x}{\sin^8 x + \cos^8 x} dx$ ... (3)

$I = 4\pi \int_0^{\frac{\pi}{2}} \frac{\sin^8\left(\frac{\pi}{2} - x\right)}{\sin^8\left(\frac{\pi}{2} - x\right) + \cos^8\left(\frac{\pi}{2} - x\right)} dx = 4\pi \int_0^{\frac{\pi}{2}} \frac{\cos^8 x}{\sin^8 x + \cos^8 x} dx$ ... (4)

Adding (3) & (4) , we get:

$I = 2\pi \int_0^{\frac{\pi}{2}} 1\, dx = 2\pi \times \frac{\pi}{2} = \pi^2$

8. If for some α and β in R, the intersection of the following three planes

$x + 4y - 2z = 1$
$x + 7y - 5z = \beta$
$x + 5y + \alpha z = 5$

is a line in R^3, then $\alpha + \beta$ is equal to:

a. 0 b. 10

c. -10 d. 2

Answer: (b)

Solution:

The given planes intersect in a line

$\therefore D = D_x = D_y = D_z = 0$

$D = 0$

$\Rightarrow \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0 \Rightarrow 7\alpha + 25 - 4\alpha - 20 + 4 = 0$

$\Rightarrow \alpha = -3$

$D_z = 0$

$\Rightarrow \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0 \Rightarrow 35 - 5\beta - 20 + 4\beta - 2 = 0$

$\Rightarrow \beta = 13$

$\therefore \alpha + \beta = 10$

9. If e_1 and e_2 are the eccentricities of the ellipse, $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola, $\frac{x^2}{9} - \frac{y^2}{4} = 1$ respectively and (e_1, e_2) is a point on the ellipse, $15x^2 + 3y^2 = k$. Then k is equal to:

a. 14 b. 15

c. 17 d. 16

Answer: (d)

Solution:

$$e_1 = \sqrt{1 - \frac{4}{18}} = \frac{\sqrt{7}}{3} \ \& \ e_2 = \sqrt{1 + \frac{4}{9}} = \frac{\sqrt{13}}{3}$$

$\because (e_1, e_2)$ lies on the ellipse $15x^2 + 3y^2 = k$

$\therefore 15e_1^2 + 3e_2^2 = k$

$\Rightarrow 15 \times \frac{7}{9} + 3 \times \frac{13}{9} = k \Rightarrow k = 16$

10. If $f(x) = \begin{cases} \frac{\sin(a+2)x + \sin x}{x}, & x < 0 \\ b, & x = 0 \\ \frac{(x+3x^2)^{\frac{1}{3}} - x^{\frac{1}{3}}}{x^{\frac{4}{3}}}, & x > 0 \end{cases}$ is continuous at $x = 0$ then $a + 2b$ is equal to:

a. -2
b. 1
c. 0
d. -1

Answer: (c)

Solution:

$f(x)$ is continuous at $x = 0$

$\therefore \lim_{x\to 0^-} f(x) = b = \lim_{x\to 0^+} f(x)$

$$b = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{(h+3h^2)^{\frac{1}{3}} - h^{\frac{1}{3}}}{h^{\frac{4}{3}}}$$

$$\Rightarrow b = \lim_{h\to 0} \frac{h^{\frac{1}{3}}\left[(1+3h)^{\frac{1}{3}} - 1\right]}{h^{\frac{4}{3}}}$$

$$\Rightarrow b = \lim_{h\to 0} \frac{(1+3h)^{\frac{1}{3}} - 1}{h}$$

$$\Rightarrow b = \lim_{h\to 0} \frac{1}{3}(1+3h)^{-\frac{2}{3}} \times 3$$

or, $b = 1$

$$\lim_{x\to 0^-} f(x) = 1 \Rightarrow \lim_{h\to 0} \frac{\sin((a+2)(-h)) + \sin(-h)}{-h} = 1$$

$\Rightarrow a + 3 = 1 \Rightarrow a = -2$

$\Rightarrow a + 2b = 0$

11. If the matrices $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \text{adj } A$ and $C = 3A$, then $\frac{|\text{adj } B|}{|C|}$ is equal to:

a. 16
b. 2
c. 8
d. 72

Answer: (c)

Solution:

$$A=\begin{bmatrix}1 & 1 & 2\\ 1 & 3 & 4\\ 1 & -1 & 3\end{bmatrix} \Rightarrow |A|=\begin{vmatrix}1 & 1 & 2\\ 1 & 3 & 4\\ 1 & -1 & 3\end{vmatrix}=13+1-8=6$$

$B=\text{adj}(A) \Rightarrow |\text{adj } B|=|\text{adj}(\text{adj } A)|=|A|^4=6^4$

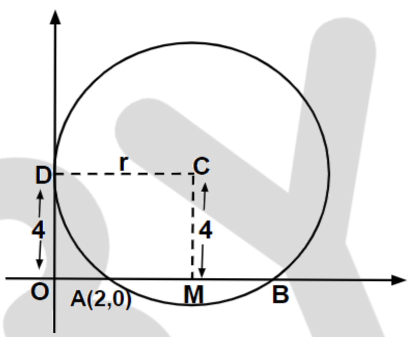
$|C|=|3A|=3^3|A|=3^3\times 6$

$$\frac{|\text{adj } B|}{|C|}=\frac{6^4}{3^3\times 6}=\frac{2^3\times 3^3}{3^3}=8$$

12. A circle touches the y-axis at the point (0,4) and passes through the point (2,0). Which of the following lines is not a tangent to the circle?
   a. $4x-3y+17=0$
   b. $3x+4y-6=0$
   c. $4x+3y-8=0$
   d. $3x-4y-24=0$

Answer: (c)

Solution:



$OD^2=OA\times OB \Rightarrow 16=2\times OB \Rightarrow OB=8$

$\therefore AB=6$

$\therefore AM=3, CM=4 \Rightarrow CA=5$

$\therefore OM=5$

Centre will be (5,4) and radius is 5

Now checking all the options

Option (c) is not a tangent.

$4x+3y-8=0$

$$\frac{20+12-8}{\sqrt{3^2+4^2}}=\frac{24}{5} \quad (p\neq r)$$

13. Let z be a complex number such that $\left|\frac{z-i}{z+2i}\right| = 1$ and $|z| = \frac{5}{2}$. Then the value of $|z+3i|$ is:

a. $\sqrt{10}$
b. $\frac{7}{2}$
c. $\frac{15}{4}$
d. $2\sqrt{3}$

Answer: (b)

Solution:

If $\left|\frac{z-i}{z+2i}\right| = 1$ & $|z| = \frac{5}{2}$

$\Rightarrow |z-i| = |z+2i|$

$\Rightarrow x^2 + (y-1)^2 = x^2 + (y+2)^2$

$\Rightarrow y - 1 = \pm(y+2)$

$\Rightarrow y - 1 = -y - 2$

$\Rightarrow y = -\frac{1}{2}$

$\Rightarrow |z| = \frac{5}{2} \Rightarrow x^2 + y^2 = \frac{25}{4}$

$\Rightarrow x^2 + \frac{1}{4} = \frac{25}{4}$

$\Rightarrow x = \pm\sqrt{6}$

$|z+3i| = \sqrt{x^2 + (y+3)^2}$

$\Rightarrow |z+3i| = \frac{7}{2}$

14. If $f'(x) = \tan^{-1}(\sec x + \tan x), -\frac{\pi}{2} < x < \frac{\pi}{2}$, and $f(0) = 0$, then $f(1)$ is equal to:

a. $\frac{\pi+1}{4}$
b. $\frac{\pi+2}{4}$
c. $\frac{1}{4}$
d. $\frac{\pi-1}{4}$

Answer: (a)

Solution:

$$f'(x) = \tan^{-1}(\sec x + \tan x) = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$$

$$f'(x)=\tan^{-1}\left(\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}\right)$$

$$f'(x)=\tan^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right]$$

$$f'(x)=\tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right]$$

$$f'(x)=\tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$$

$$f'(x)=\frac{\pi}{4}+\frac{x}{2}$$

$$\Rightarrow f(x)=\frac{\pi}{4}x+\frac{x^2}{4}+c$$

$$f(0)=0\Rightarrow c=0$$

$$f(1)=\frac{\pi}{4}+\frac{1}{4}=\frac{\pi+1}{4}$$

15. Negation of the statement: '$\sqrt{5}$ is an integer or 5 is irrational' is:
   a. $\sqrt{5}$ is irrational or 5 is an integer.
   b. $\sqrt{5}$ is not an integer or 5 is not irrational.
   c. $\sqrt{5}$ is an integer and 5 is irrational.
   d. $\sqrt{5}$ is not an integer and 5 is not irrational.

Answer: (d)

Solution:

p: $\sqrt{5}$ is an integer

q: 5 is an irrational number

Given statement : $p \vee q$

Required negation statement: $\sim(p \vee q) = \sim p \wedge \sim q$

'$\sqrt{5}$ is not an integer and 5 is not irrational'

16. If for all real triplets (a, b, c), $f(x) = a + bx + cx^2$; then $\int_0^1 f(x)dx$ is equal to:

a. $2\left(3f(1)+2f\left(\frac{1}{2}\right)\right)$
b. $\frac{1}{3}\left(f(0)+f\left(\frac{1}{2}\right)\right)$
c. $\frac{1}{2}\left(f(1)+3f\left(\frac{1}{2}\right)\right)$
d. $\frac{1}{6}\left(f(0)+f(1)+4f\left(\frac{1}{2}\right)\right)$

Answer: (d)

Solution:

$f(x)=a+bx+cx^2$

$f(0)=a, f(1)=a+b+c$

$f\left(\frac{1}{2}\right)=\frac{c}{4}+\frac{b}{2}+a$

$\int_0^1 f(x)dx=\int_0^1(a+bx+cx^2)dx \;=a+\frac{b}{2}+\frac{c}{3}$

$=\frac{1}{6}(6a+3b+2c)=\frac{1}{6}\left(a+(a+b+c)+(4a+2b+c)\right)$

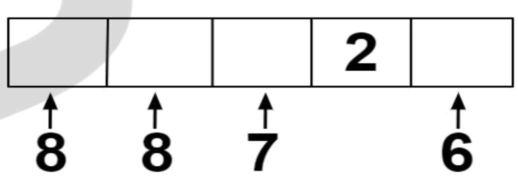
$=\frac{1}{6}\left(f(0)+f(1)+4f\left(\frac{1}{2}\right)\right)$

17. If the number of five digit numbers with distinct digits and 2 at the $10th$ place is $336k$, then k is equal to:

a. 8
b. 7
c. 4
d. 6

Answer: (a)

Solution:



Total numbers that can be formed are

$=8\times8\times7\times6$

$=8\times336$

$\therefore k=8$

18. Let the observations $x_i(1\le i\le 10)$ satisfy the equations, $\sum_{i=1}^{10}(x_i-5)=10$ and $\sum_{i=1}^{10}(x_i-5)^2=40$. If μ and λ are the mean and the variance of observations, $(x_1-3),(x_2-3)\ldots..(x_{10}-3)$, then the ordered pair (μ,λ) is equal to:

a. $(6, 3)$
b. $(3, 6)$
c. $(3, 3)$
d. $(6, 6)$

Answer: (c)

Solution:

$$\sum_{i=1}^{10}(x_i - 5) = 10 \Rightarrow \sum_{i=1}^{10} x_i - 50 = 10$$

$$\Rightarrow \sum_{i=1}^{10} x_i = 60$$

$$\mu = \frac{\sum_{i=1}^{10}(x_i - 3)}{10} = \frac{\sum_{i=1}^{10} x_i - 30}{10} = 3$$

Variance is unchanged, if a constant is added or subtracted from each observation

$$\lambda = Var(x_i - 3) = Var(x_i - 5) = \frac{\sum_{i=1}^{10}(x_i - 5)^2}{10} - \left(\frac{\sum_{i=1}^{10}(x_i - 5)}{10}\right)^2$$

$$= \frac{40}{10} - \left(\frac{10}{10}\right)^2 = 3$$

19. The integral $\int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}}$ is equal to: (where C is a constant of integration)

a. $-\left(\frac{x-3}{x+4}\right)^{-\frac{1}{7}} + C$
b. $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{\frac{3}{7}} + C$
c. $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$
d. $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{-\frac{13}{7}} + C$

Answer: (c)

Solution:

$$I = \int \frac{dx}{(x-3)^{\frac{6}{7}} \times (x+4)^{\frac{8}{7}}}$$

$$\Rightarrow I = \int \frac{(x+4)^{\frac{6}{7}} dx}{(x-3)^{\frac{6}{7}} \times (x+4)^2} = \int \left(\frac{x-3}{x+4}\right)^{-\frac{6}{7}} \times \frac{dx}{(x+4)^2}$$

Put $\frac{x-3}{x+4} = t \Rightarrow dt = 7\left(\frac{1}{(x+4)^2}\right) dx$

$$\Rightarrow I = \frac{\int t^{-\frac{6}{7}}}{7} dt = t^{\frac{1}{7}} + C = \left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$$

20. In a box, there are 20 cards out of which 10 are labelled as A and remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is:

a. $\frac{15}{16}$
b. $\frac{9}{16}$
c. $\frac{13}{16}$
d. $\frac{11}{16}$

Answer: (d)

Solution:

Here $P(A) = P(B) = \frac{1}{2}$

Then, these following cases are possible → $AA, BAA, ABA, ABBA, BBAA, BABA$

So, the required probability is $= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{11}{16}$

21. If the vectors $\vec{p} = (a+1)\hat{\imath} + a\hat{\jmath} + a\hat{k}$, $\vec{q} = a\hat{\imath} + (a+1)\hat{\jmath} + a\hat{k}$ and $\vec{r} = a\hat{\imath} + a\hat{\jmath} + (a+1)\hat{k}$ $(a \in R)$ are coplanar and $3(\vec{p}.\vec{q})^2 - \lambda|\vec{r} \times \vec{q}|^2 = 0$, then value of λ is _____ .

Answer: (1)

Solution:

As $\vec{p}, \vec{q}, \vec{r}$ are coplanar,

$$\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} 3a+1 & 3a+1 & 3a+1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$$(3a+1)\begin{vmatrix} 1 & 1 & 1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$(3a+1)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a & 0 & 1 \end{vmatrix} = 0$$

$3a + 1 = 0$

$\Rightarrow a = -\dfrac{1}{3}$

$\vec{p} = \frac{1}{3}(2\hat{\imath} - \hat{\jmath} - \hat{k}), \quad \vec{q} = \frac{1}{3}(-\hat{\imath} + 2\hat{\jmath} - \hat{k}), \quad \vec{r} = \frac{1}{3}(-\hat{\imath} - \hat{\jmath} + 2\hat{k})$

$\vec{r} \times \vec{q} = \frac{1}{9}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{vmatrix}$

$\vec{r} \times \vec{q} = \frac{1}{9}(-3\hat{\imath} - 3\hat{\jmath} - 3\hat{k}) = -\frac{1}{3}(\hat{\imath} + \hat{\jmath} + \hat{k})$

$|\vec{r} \times \vec{q}|^2 = \frac{1}{3}$

$\vec{p}.\vec{q} = \frac{1}{9}(-2 - 2 + 1) = -\frac{1}{3}$

$3(\vec{p}.\vec{q})^2 - \lambda|\vec{r} \times \vec{q}|^2 = 0$

$\Rightarrow \frac{1}{3} - \lambda \times \frac{1}{3} = 0 \Rightarrow \lambda = 1$

22. The projection of the line segment joining the points $(1, -1, 3)$ and $(2, -4, 11)$ on the line joining the points $(-1, 2, 3)$ and $(3, -2, 10)$ is ______ .

Answer: (8)

Solution:

$\overrightarrow{AB} = \hat{\imath} - 3\hat{\jmath} + 8\hat{k}$

$\overrightarrow{CD} = 4\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$

Projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$ is $= \frac{\overrightarrow{AB}.\overrightarrow{CD}}{|\overrightarrow{CD}|} = \frac{4+12+56}{\sqrt{4^2+4^2+7^2}} = \frac{72}{9} = 8$

23. The number of distinct solutions of the equation, $\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|$ in the interval $[0,2\pi]$, is ____________.

Answer: (8)

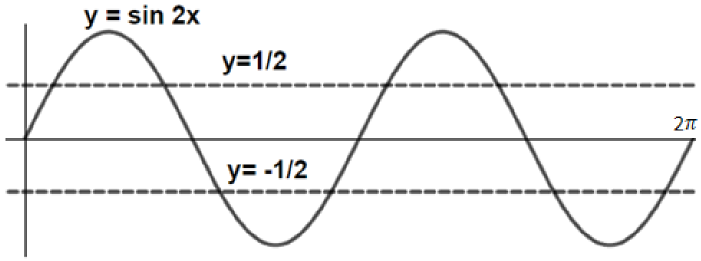
Solution:

$\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|, x \in [0,2\pi]$

$\Rightarrow \log_{\frac{1}{2}}|\sin x||\cos x| = 2$

$\Rightarrow |\sin x \cos x| = \frac{1}{4}$

$\therefore \sin 2x = \pm\frac{1}{2}$



$\therefore$ We have 8 solutions for $x \in [0,2\pi]$

24. If for $x \geq 0, y = y(x)$ is the solution of the differential equation $(1+x)dy = [(1+x)^2 + y - 3]dx,\ y(2) = 0$, then $y(3)$ is equal to ______ .

Answer: (3)

Solution:

$$(1+x)\frac{dy}{dx} = [(1+x)^2 + (y-3)]$$

$$\Rightarrow (1+x)\frac{dy}{dx} - y = (1+x)^2 - 3$$

$$\Rightarrow \frac{dy}{dx} - \frac{1}{(1+x)}y = 1 + x - \frac{3}{1+x}$$

$$\text{I.F.} = e^{-\int \frac{1}{1+x}dx} = \frac{1}{1+x}$$

$$y \times \frac{1}{1+x} = \int 1 - \frac{3}{(1+x)^2}dx$$

$$\frac{y}{1+x} = x + \frac{3}{1+x} + c$$

$$\Rightarrow y = x(1+x) + 3 + c(1+x)$$

At $x = 2, y = 0$, we get

$$0 = 6 + 3 + 3c$$

$$\Rightarrow c = -3$$

$\Rightarrow$ At $x = 3$,

$$y = x^2 - 2x = 9 - 6 = 3$$

$\Rightarrow y(3) = 3$

25. The coefficient of x^4 in the expansion of $(1 + x + x^2)^{10}$ is

Answer: (615)

Solution:

General term of the given expression is given by $\frac{10!}{p!q!r!}x^{q+2r}$

Here, $q + 2r = 4$

For $p = 6, q = 4, r = 0$, coefficient $= \frac{10!}{6!\times 4!} = 210$

For $p = 7, q = 2, r = 1$, coefficient $= \frac{10!}{7!\times 2!\times 1!} = 360$

For $p = 8, q = 0, r = 2$, coefficient $= \frac{10!}{8!\times 2!} = 45$

Therefore, sum $= 615$