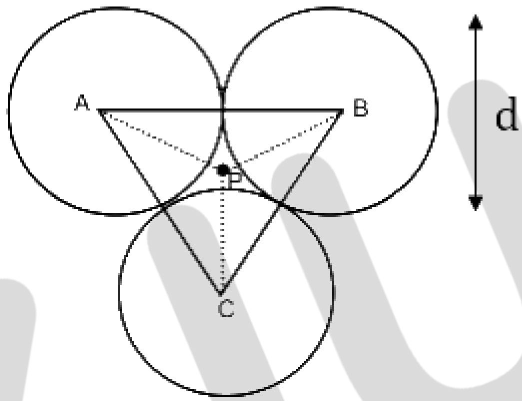
JEE Main 2020 Paper

Date of Exam: **9th January (Shift I)**
Time: **9:30 am – 12:30 pm**
Subject: **Physics**

1. Three identical solid spheres each have mass ‘m’ and diameter ‘d’ are touching each other as shown in the figure. Calculate ratio of moment of inertia about the axis perpendicular to plane of paper and passing through point P and B as shown in the figure. Given P is centroid of the triangle



a. $\frac{13}{23}$
b. $\frac{13}{15}$
c. $\frac{15}{13}$
d. $\frac{23}{13}$

Solution: (a)

Moment of Inertia of solid sphere $= \frac{2}{5}M\left(\frac{d}{2}\right)^2$

Distance of centroid $(Point\ P)$ from centre of sphere $=\left(\frac{2}{3}\times\frac{\sqrt{3}d}{2}\right)=\frac{d}{\sqrt{3}}$

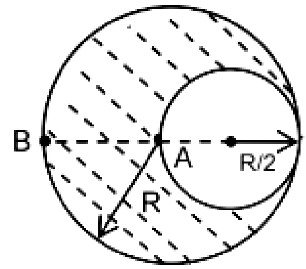
By Parallel axis theorem,

Moment of Inertia about $P = 3\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + M\left(\frac{d}{\sqrt{3}}\right)^2\right] = \frac{13}{10}M\,d^2$

Moment of Inertia about $B = 2\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + M(d)^2\right] + \frac{2}{5}M\left(\frac{d}{2}\right)^2 = \frac{23}{10}M\,d^2$

Now ratio $= \frac{13}{23}$

2. A solid sphere having a radius R and uniform charge density ρ. If a sphere of radius R/2 is carved out of it as shown in the figure. Find the ratio of the magnitude of electric field at point A and B



a. $\frac{17}{54}$
b. $\frac{18}{54}$
c. $\frac{18}{34}$
d. $\frac{21}{34}$

Solution: (c)

For solid sphere,

Field inside sphere, E = $\frac{\rho r}{3\epsilon_0}$ & field outside sphere, E = $\frac{\rho R^3}{3r^2\epsilon_0}$ where r is distance from centre and R is radius of sphere

Electric field at A due to sphere of radius R (sphere 1) is zero and therefore, net electric field will be because of sphere of radius $\frac{R}{2}$ (sphere 2) having charge density $(-\rho)$

$$E_A = \frac{-\rho R}{2(3\epsilon_0)}$$

$$|E_A| = \frac{\rho R}{6\varepsilon_0}$$

Similarly, Electric field at point $B = E_B = E_{1B} + E_{2B}$

E_{1B} = Electric Field due to solid sphere of radius $R = \frac{\rho R}{3\varepsilon_0}$

E_{2B} = Electric Field due to solid sphere of radius $\frac{R}{2}$ which having charge density $(-\rho)$

$$= -\frac{\rho\left(\frac{R}{2}\right)^3}{3\left(\frac{3R}{2}\right)^2 \varepsilon_0} = -\frac{\rho R}{54\varepsilon_0}$$

$$E_B = E_{1A} + E_{2A} = \frac{\rho R}{3\varepsilon_0} - \frac{\rho R}{54\varepsilon_0} = \frac{17\rho R}{54\varepsilon_0}$$

$$\frac{|E_A|}{|E_B|} = \frac{9}{17} = \frac{18}{34}$$

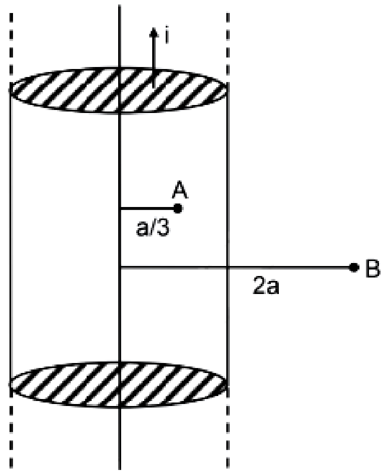
3. Consider an infinitely long current carrying cylindrical straight wire having radius 'a'. Then the ratio of magnetic field due to wire at distance a/3 and 2a, respectively from axis of wire is

a. 3/2
b. 2/3

c. 2

d. 1/2

Solution: (b)

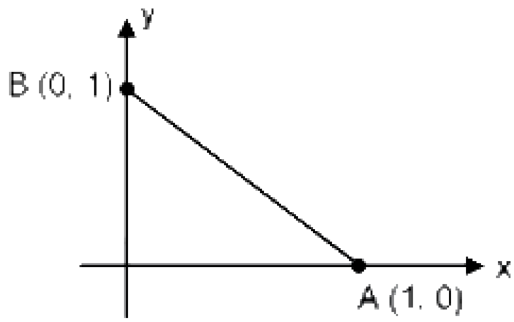


$$B_A = \frac{\mu_0 ir}{2\pi a^2} = \frac{\frac{\mu_0 ia}{3}}{2\pi a^2} = \frac{\mu_{0}i}{\pi a^2}\frac{a}{6} = \frac{\mu_{0}i}{6\pi a}$$

$$B_B = \frac{\mu_0 i(2a)}{2\pi(2a)^2} = \frac{\mu_0 i}{4\pi a}$$

$$\frac{B_A}{B_B} = \frac{4}{6} = \frac{2}{3}$$

4. Particle moves from point A to point B along the line shown in figure under the action of force $\vec{F} = -x\,\hat{\imath} + y\,\hat{\jmath}$. Determine the work done on the particle by $\vec{F}$ in moving the particle from point A to point B (all quantities are in SI units)



a. 1 J

b. 1/2 J

c. 2 J

d. 3/2 J

Solution: (a)

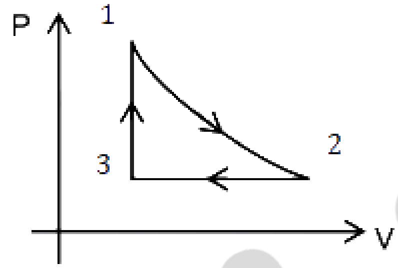
$$d\,\vec{s} = (dx\,\hat{\imath} + dy\,\hat{\jmath})$$

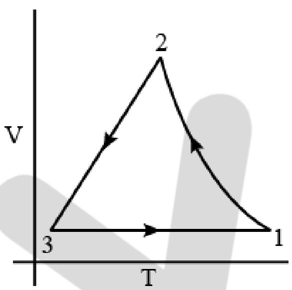
$$= (-x\,\hat{\imath} + y\,\hat{\jmath}).(dx\,\hat{\imath} + dy\,\hat{\jmath})$$

$$= \int_1^0 -x\,dx + \int_0^1 y dy$$

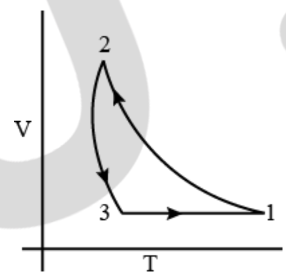
$$= -\frac{x^2}{2}\Big|_1^0 + \frac{y^2}{2}\Big|_0^1 = \frac{1}{2} + \frac{1}{2} = 1\,J$$

5. For the given $P - V$ graph of an ideal gas, chose the correct $V - T$ graph. Process BC is adiabatic. (Graphs are schematic and not to scale)

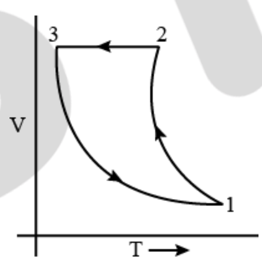




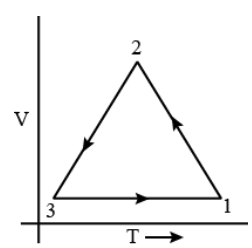
a.



b.



c.



d.

Solution: (a)

For process $3 - 1$; Volume is constant;

For process $1 - 2$, PV^{γ} = Constant & $PV = nRT$, therefore $TV^{\gamma-1}$ = Constant ; therefore as V increases T decreases and also relation is non linear, so curve will not be a straight line.

For process $2 - 3$; pressure is constant , therefore $V = kT$

From above, correct answer is option a.

6. An electric dipole of moment $\vec{p} = (-\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) \times 10^{-29}$ Cm is at the origin (0,0,0). The electric field due to this dipole at $\vec{r} = \hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ is parallel to [Note that $\vec{r}\,.\vec{p} = 0$]

a. $\hat{\imath} - 3\hat{\jmath} - 2\hat{k}$
b. $-\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$
c. $+\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$
d. $-\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

Solution: (c)

The electric dipole of moment $\vec{p} = \text{q}.\,a$ where a is distance between charge.
Electric field $(\overrightarrow{E})$ at position $\vec{r}$ is given by $\frac{2K\,\vec{p}.r^{\to}}{|r|^4}$ along radial direction and $\frac{2K\,\vec{p}\times r^{\to}}{|r|^4}$ along tangential direction , where $\vec{r} = \hat{\imath} + 3\hat{\jmath} + 5\hat{k} - (0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}) = \hat{\imath} + 3\hat{\jmath} + 5\hat{k}$
Since already in question, $\vec{p}\,.\vec{r} = 0$, this means field is along tangential direction and dipole is also perpendicular to radius vector.
Since electric field and dipole are along same line, we can write $\vec{E} = \lambda\,(\vec{p})$ where λ is an arbitrary constant
From option, on putting $\lambda = -1\times 10^{29}$, we get, $\vec{E} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

7. A body A of mass m is revolving around a planet in a circular orbit of radius R. At the instant the particle B has velocity $\vec{V}$, another particle of mass $\frac{m}{2}$ moving at velocity of $\frac{\vec{V}}{2}$ collides perfectly inelastically with the first particle. Then, the combined body

a. Fall vertically downward towards the planet.
b. Continue to move in a circular orbit
c. Escape from the Planet's Gravitational field
d. Start moving in an elliptical orbit around the planet

Solution: (d)

By conservation of linear momentum and taking velocity inline for maximum momentum transfer in single direction.

$\frac{m}{2}\frac{V}{2} + mV = (m + \frac{m}{2})V_f$

$V_f = \frac{5V}{6}$, where V is orbital velocity

Escape velocity will be $\sqrt{2}V$ and at velocity less than escape velocity but greater than orbital velocity (V), the path will be elliptical. At orbital velocity (V), path will be circular. At velocity less than orbital velocity path will remain part of ellipse and it will either orbit in elliptical path whose length of semi major axis will be less than radius of circular orbit or start falling down and collide with the planet but it will not fall vertically down as path will remain part of ellipse. Hence the resultant mass will start moving in an elliptical orbit around the planet.

8. Two particles of equal mass m have respective initial velocities $\overrightarrow{u_1} = u\,\hat{\imath}$ and $\overrightarrow{u_2} = \frac{u}{2}\hat{\imath} + \frac{u}{2}\hat{\jmath}$. They collide completely inelastically. Find the loss in kinetic energy.

a. $\frac{3mu^2}{4}$

b. $\frac{\sqrt{2}mu^2}{\sqrt{3}}$

c. $\frac{mu^2}{3}$

d. $\frac{mu^2}{8}$

Solution: (d)

Let v_1 and v_2 be the final velocities after collision in x and y direction respectively.
Conserving linear momentum

$mu\hat{\imath} + m(\frac{u}{2}\hat{\imath} + \frac{u}{2}\hat{\jmath}) = 2m(v_1\hat{\imath} + v_2\hat{\jmath})$

By equating $\hat{\imath}$ and $\hat{\jmath}$

$v_1 = \frac{3u}{4}$ and $v_2 = \frac{u}{4}$

Initial K.E $= \frac{mv^2}{2} + \frac{m}{2} \times (\frac{u}{\sqrt{2}})^2 = \frac{3mu^2}{4}$

Final K.E $= \frac{2m}{2} \times (\frac{u\sqrt{10}}{4})^2 = \frac{5mu^2}{8}$

Change in KE =

$\frac{3mu^2}{4} - \frac{5mu^2}{8} = \frac{mu^2}{8}$

9. Three harmonic waves of same frequency (v) and intensity (I_0) having initial phase angles $0, \frac{\pi}{4}, -\frac{\pi}{4}$ rad respectively. When they are superimposed, the resultant intensity is close to

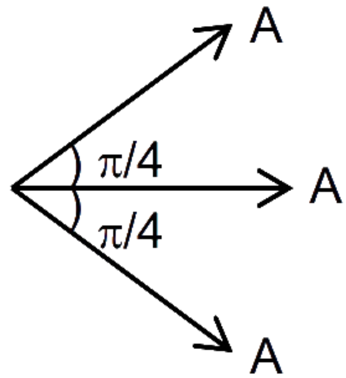
a. $5.8\, I_0$

b. I_0

c. $3\, I_0$

d. $0.2\, I_0$

Solution: (a)



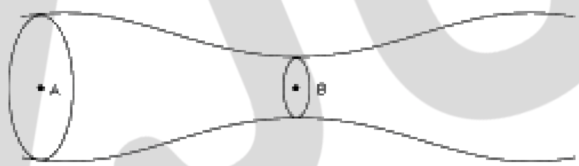
Amplitudes can be added using vector addition

$A_{resultant} = (\sqrt{2}+1)A$

Since, $I \propto A^2$, Where I is intensity.

Therefore, $I_{res} = (\sqrt{2}+1)^2 I_0 = 5.8\ I_0$ (approx.)

10. An ideal liquid (water) flowing through a tube of non-uniform cross-sectional area, where area at A and B are 40 cm^2 and 20 cm^2 respectively. If pressure difference between A & B is 700 N/m^2, then volume flow rate is (density of water = 1000 kgm^{-3})



a. 2720 cm^3/s
b. 2420 cm^3/s
c. 1810 cm^3/s
d. 3020 cm^3/s

Solution: (a)

Using equation of continuity

$V_A \times Area_A = V_B \times Area_B$

$40V_A = 20V_B$

$2V_A = V_B$

Using Bernoulli's equation

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$$P_A - P_B = \frac{1}{2}\rho(V_B^2 - V_A^2)$$

$$\Delta P = \frac{1}{2}1000\left(V_B^2 - \frac{V_B^2}{4}\right)$$

$$\Delta P = 500 \times \frac{3V_B^2}{4}$$

$$V_B = \sqrt{\frac{(\Delta P) \times 4}{1500}} = \sqrt{\frac{(700) \times 4}{1500}} = \sqrt{\frac{28}{15}}\,\text{m/s}$$

Volume flow rate $= V_B \times \text{Area}_B = (20 \times 100 \times \sqrt{\frac{28}{15}})\ \text{cm}^3/\text{s} = 2732.5\ \text{cm}^3/\text{s}$

So, answer comes nearly 2720 cm^3/s

11. A screw gauge advances by 3 mm on main scale in 6 rotations. There are 50 divisions on circular scale. Find least count of screw gauge?

a. 0.01 cm
b. 0.001 cm
c. 0.001 mm
d. 0.02 mm

Solution: (b)

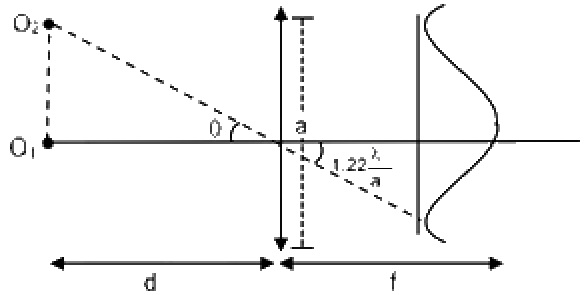
Pitch $= \frac{3}{6} = 0.5$ mm

Least count $= \frac{\text{Pitch}}{Number\ of\ divisions} = \frac{0.5\text{mm}}{50} = \frac{1}{100}\text{mm} = 0.01\ \text{mm} = 0.001\ \text{cm}$

12. A telescope of aperture diameter 5 m is used to observe the moon from the earth. Distance between the moon and earth is 4×10^5 km. The minimum distance between two points on the moon's surface which can be resolved using this telescope is close to (Wavelength of light is 5500 Å)

a. 60 m
b. 20 m
c. 600 m
d. 200 m

Solution: (a)



Minimum angle for clear resolution,

$$\theta = 1.22\frac{\lambda}{a}$$

$$distance = O_1O_2 = d\theta$$

$$= 1.22\frac{\lambda}{a}d$$

$$\text{Distance} = O_1O_2 = \frac{1.22\times5500\times10^{-10}\times4\times10^{8}}{5} = 53.68\ \text{m}$$

$\therefore$ Nearest option is 60 m

13. Radiation with wavelength 6561 Å falls on a metal surface to produce photoelectrons. The electrons are made to enter a uniform magnetic field of 3×10^{-4} T. If the radius of largest circular path followed by electron is 10 mm, the work function of metal is close to

a. 1.8 eV
b. 0.8 eV
c. 1.1 eV
d. 1.6 eV

Solution: (c) (Challenged question)

From photoelectric equation,

$$\frac{hc}{\lambda} = W + K.E_{max}$$

Where, $hc = 12400$ eV Å

$$\Rightarrow \frac{12400}{6561} = W + K.E_{max}$$

$$\Rightarrow 1.89\ \text{eV} = W + K.E_{max} ----- (1)$$

Radius of charged particle moving in a magnetic field is given by

$r = \frac{mv}{qB}$ and $\frac{1}{2}mv^2 = K.E_{max} = eV$

$$\Rightarrow r = \frac{\sqrt{\frac{2eV}{m}} \times m}{eB} = \frac{1}{B}\sqrt{\frac{2mV}{e}}$$

$$\Rightarrow 10^{-2} = \frac{1}{3 \times 10^{-4}}\sqrt{\frac{2 \times 9.1 \times 10^{-31} \times V}{1.6 \times 10^{-19}}}$$

$\Rightarrow V = 0.8\ V$

So, $K.E_{max} = 0.8\ eV$

Substituting in (1),

$1.89 = W + 0.8$

i.e. $W = 1.1\ eV$ (approx)

14. Kinetic energy of the particle is E and it's de–Broglie wavelength is λ. On increasing its K.E by ΔE, it's new de–Broglie wavelength becomes $\frac{\lambda}{2}$. Then ΔE is

a. 3E
b. E
c. 2E
d. 4E

Solution: (a)

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m(KE)}}$$

$$\Rightarrow \lambda \propto \frac{1}{\sqrt{KE}}$$

$$\frac{\lambda}{\lambda/2} = \sqrt{\frac{KE_f}{KE_i}}$$

$$4KE_i = KE_f$$

$$\Rightarrow \Delta E = KE_f - KE_i = 4KE_i - KE_i = 3KE_i = 3E$$

15. A quantity f is given by $f = \sqrt{\frac{hc^5}{G}}$ where c is speed of light, G is universal gravitational constant and h is the Planck's constant. Dimension of f is that of

a. area
b. energy
c. volume
d. Momentum

Solution: (b)

$$E = \frac{hc}{\lambda}$$

$$\Rightarrow\ hc = E\,\lambda$$
$$Since, [E] = [ML^2T^{-2}]$$

Therefore,

$$[hc] = [ML^3T^{-2}]$$
$$[c] = [LT^{-1}]$$
$$[G] = [[M^{-1}L^3T^{-2}]$$
$$\left[\sqrt{\frac{hc^5}{G}}\right] = [ML^2T^{-2}]$$

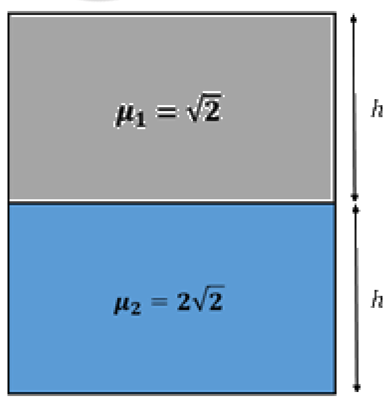
The above dimension is of energy.

16. A vessel of depth $2h$ is half filled with a liquid of refractive index $\sqrt{2}$ in upper half and with a liquid of refractive index $2\sqrt{2}$ in lower half. The liquids are immiscible. The apparent depth of inner surface of the bottom of the vessel will be

a. $\frac{3h\sqrt{2}}{4}$
b. $\frac{h}{\sqrt{2}}$
c. $\frac{h}{3\sqrt{2}}$
d. $\frac{h}{2(\sqrt{2}+1)}$

Solution: (a)

Assume, air is present outside container



Apparent height as seen from liquid 1 (having refractive index $\mu_1 = \sqrt{2}$) to liquid 2 (refractive index $\mu_2 = 2\sqrt{2}$)

D $= \frac{h\mu_1}{\mu_2} = \frac{h}{2}$

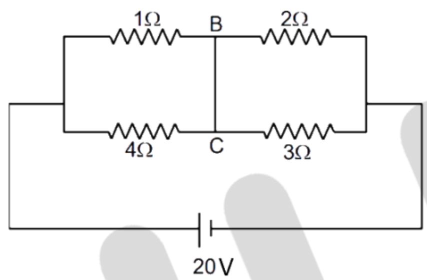
Now, actual height perceived from air, h $+ \frac{h}{2} = \frac{3h}{2}$

Therefore, apparent depth of bottom surface of the container (apparent depth as seen from air (having refractive index $\mu_0 = 1$) to liquid 1(having refractive index $\mu_1 = \sqrt{2}$)

$= \frac{3h}{2} \times \frac{\mu_0}{\mu_1}$

$= \frac{3h}{2} \times \frac{1}{\sqrt{2}} = \frac{3h}{2\sqrt{2}} = \frac{3\sqrt{2}h}{4}$

17. In the given circuit diagram, a wire is joining point B & C. Find the current in this wire



a. 0.4 A

b. 2 A

c. zero

d. 4 A

Solution: (b)

Since resistance 1 Ω and 4 Ω are in parallel

$\therefore R' = \frac{4\times1}{4+1} = \frac{4}{5}$

Similarly we can find equivalent resistance (R'') for resistances 2 Ω and 3 Ω

$\Rightarrow R'' = \frac{6}{5}$

And R' and R'' are in series

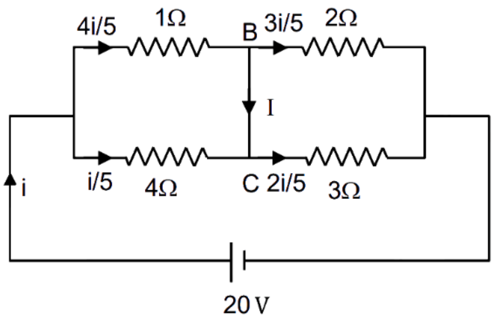
$\therefore\ R_{eff} = \frac{4}{5} + \frac{6}{5} = 2\ \Omega$

So total current flowing in the circuit 'i' can be given as

$$i = \frac{V}{R_{eff}} = \frac{20}{2} = 10\ A$$

Current will distribute in ratio opposite to resistance.

So, distribution will be as



So current in the branch BC will be

$$I = \frac{4i}{5} - \frac{3i}{5} = \frac{i}{5} = \frac{10}{5} = 2\ A$$

18. Two plane electromagnetic waves are moving in vacuum in whose electric field vectors are given by $\vec{E}_1 = E_o\hat{j}\ \cos(kx - \omega t)$ and $\vec{E}_2 = E_o\hat{k}\cos(ky - \omega t)$. At t = 0 A charge q is at origin with velocity $\vec{v} = 0.8c\ \hat{j}$ (c is speed of light in vacuum). The instantaneous force on this charge (all data are in SI units)

a. $qE_o(0.4\ \hat{\imath} - 3\ \hat{\jmath} + 0.8\ \hat{k})$

b. $qE_o(0.8\ \hat{\imath} + \hat{\jmath} + 0.2\ \hat{k})$

c. $qE_o(0.8\ \hat{\imath} - \hat{\jmath} + 0.4\ \hat{k})$

d. $qE_o(-0.8\ \hat{\imath} + \hat{\jmath} + \hat{k})$

Solution: (b)

Given that the magnetic field vectors are:

$$\vec{E}_1 = E_o\hat{j}\cos(kx - \omega t)$$
$$\vec{E}_2 = E_o\hat{k}\cos(ky - \omega t)$$

Since, the variation of $\vec{E}_1$ & $\vec{E}_2$ is along x and y respectively. Therefore, direction of propagation of $\vec{E}_1$ & $\vec{E}_2$ will be along x and y respectively. Since, $\vec{E} \times \vec{B}$ gives direction of propagation & $\frac{E_0}{B_0} = c$ and variation of magnetic field will be same to magnetic field.

So, the magnetic field vectors of the electromagnetic wave are given by

$$\vec{B}_1 = \frac{E_o}{c}\hat{k}\cos(kx - \omega t)$$
$$\vec{B}_2 = \frac{E_o}{c}\hat{\imath}\cos(ky - \omega t)$$

Then force is

$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$

$= q(\vec{E}_1 + \vec{E}_2) + q(\vec{v} \times (\vec{B}_1 + \vec{B}_2)$

Now if we put the values of $\vec{E}_1, \vec{E}_2, \vec{B}_1 and\ \vec{B}_2$ we can get the net Lorentz force as

$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$

Putting values and solving we get

$\vec{F} = qE_o[\cos(kx - \omega t)\,\hat{j} + (\cos ky - \omega t)\,\hat{k} + 0.8\cos(kx - \omega t)\,\hat{\imath} - 0.8(\cos ky - \omega t)\,\hat{k}]$

$\vec{F} = qE_o[0.8\cos(kx - \omega t)\,\hat{\imath} + \cos(kx - \omega t)\,\hat{j} + 0.2(\cos ky - \omega t)\,\hat{k}]$

Now at $t = 0$ and $x = y = 0$ we get

$\vec{F} = qE(0.8\,\hat{\imath} + \hat{j} + 0.2\,\hat{k})$

19. Consider two ideal diatomic gases A and B at some temperature T. Molecules of the gas A are rigid, and have a mass m. Molecules of the gas B have an additional vibration mode and have a mass $\frac{m}{4}$. The ratio of molar specific heat at constant volume of gas A and B is

a. 7/9
b. 5/9
c. 3/5
d. 5/7

Solution: (d)

We know that,

Molar heat capacity at constant volume, $C_V = \frac{fR}{2}$ (Where f is degree of freedom)

Since, A is diatomic and rigid, degree of freedom for A is 5

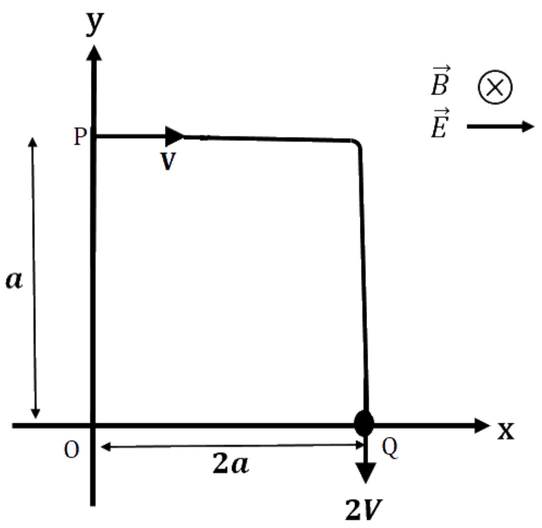
Therefore, Molar heat capacity of A at constant volume$(C_V)_A = \frac{5R}{2}$

Since, B is diatomic and has extra degree of freedom because of vibration; degree of freedom for B is $5 + 2\times 1 = 7$ (1 vibration for each atom).

Therefore, Molar heat capacity of B at constant volume$(C_V)_B = \frac{7R}{2}$

Ratio of molar specific heat of A and B= $\frac{(C_V)_A}{(C_V)_B} = \frac{5}{7}$

20. A charged particle of mass $'m'$ and charge $'q'$ is moving under the influence of uniform electric field $E\,\hat{\imath}$ and a uniform magnetic field B $\hat{k}$ follow a trajectory from P to Q as shown in figure. The velocities at P and Q are respectively $v\,\hat{\imath}$ and $-2v\,\hat{j}$. Then which of the following statements (A, B, C, D) are correct? (Trajectory shown is schematic and not to scale)



A. Magnitude of electric field $\vec{E} = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$

B. Rate of work done by electric field at P is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$

C. Rate of work done by both fields at Q is zero

D. The difference between the magnitude of angular momentum of the particle at P and Q is $2mva$

a. A, C and D are correct
b. A, B and C are correct
c. A, B, C and D are correct
d. B, C and D are correct

Solution: (b)

Considering statement A

Let, Net work done by magnetic field be W_B and net work done by electric field be W_E

By Work-Energy theorem

$$W_B + W_E = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2$$

$$\Rightarrow \qquad 0 + qE_o 2a = \frac{3}{2}mv^2$$

$$E_o = \frac{3}{4}\frac{mv^2}{qa}$$

So, statement A is correct

Now, considering statement B

Rate of work done at P = Power of electric force

$$= qE_o v$$

$$= \frac{3}{4}\frac{mv^3}{a}$$

So, statement B is correct

Now, considering statement C

At Q,

$\vec{E} \perp \vec{v}$ and already $\vec{B} \perp \vec{v}$

So, $\frac{dw}{dt} = 0$ for both forces as $\frac{dw}{dt} = q(\vec{E} + \vec{v} \times \vec{B}).\vec{v}$

So, statement C is correct

Now, considering statement D

Angular momentum should be defined about a point which is not given in question but let's find angular momentum about origin.

Change in magnitude of angular momentum of the particle at P and Q about origin

$\Delta\vec{L} = \Delta\overrightarrow{L_P} - \Delta\overrightarrow{L_q}$

$\overrightarrow{L_q} = m(2v)(2a)$

$\overrightarrow{L_p} = m(v)(a)$

Hence, $\Delta L = 3mva$

So, statement D is wrong

21. In a fluorescent lamp choke (a small transformer) 100 V of reversible voltage is produced when choke changes current in from 0.25 A to 0 A in 0.025 ms. The self-inductance of choke (in mH) is estimated to be

Solution: (10)

Fluorescent lamp choke will behave as an inductor

By using faraday law to write induced emf,

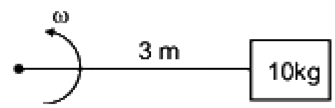
$$\in -L\frac{dI}{dt} = 0$$

$$\Rightarrow 100 = \frac{L(0.25)}{0.025} \times 10^3$$

$$L = 100 \times 10^{-4}\ \text{H}$$
$$= 10\ \text{mH}$$

22. A wire of length l = 0.3 m and area of cross section 10^{-2} cm^2 and breaking stress 4.8×10^7 N/m^2 is attached with block of mass 10 kg. Find the maximum possible value of angular velocity (rad/s) with which block can be moved in a circle with string fixed at one end.

Solution: (4)



Breaking stress

$$\sigma = \frac{T}{A}$$

$$T = m\omega^2 l$$

$$\Rightarrow \sigma = \frac{m\omega^2 l}{A}$$

$$\Rightarrow \omega^2 = \frac{\sigma A}{m l} = \frac{4.8\times10^7\times10^{-6}}{10\times0.3} = 16$$

$$\Rightarrow \omega = 4\, rad/s$$

23. The distance x covered by a particle in one dimension motion varies as with time t as $x^2 = at^2 + 2bt + c$, where a, b, c are constants. Acceleration of particle depend on x as x^{-n}, the value of n is

Solution: (3)

Let, v be velocity, α be the acceleration then,

$$x^2 = at^2 + 2bt + c$$

$$2\, x\, v = 2\, a\, t + 2\, b$$

$$x\, v = a\, t + b \qquad \_\_\_\_\_\_(1)$$

$$\Rightarrow v = \frac{at + b}{x}$$

Now, differentiating equation (1),

$v^2 + \alpha x = a$

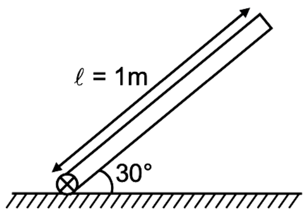
$\alpha x = a - \left(\frac{at+b}{x}\right)^2$

$\alpha = \frac{a(at^2 + 2bt + c) - (at+b)^2}{x^3}$

$\alpha = \frac{ac - b^2}{x^3}$

$$\alpha \propto x^{-3}$$

24. A rod of length 1 m pivoted at one end is released from rest when it makes 30° from the horizontal as shown in the figure below.



If ω of rod is $\sqrt{n}$ at the moment it hits the ground, then find n

Solution: (15)

By using conservation of energy,

$mg\frac{l}{2}\sin 30^\circ = \frac{1}{2}\frac{ml^2}{3}\omega^2$

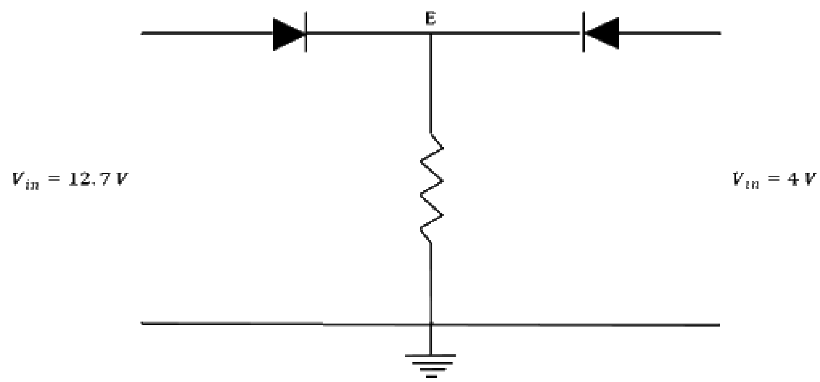
On solving

$\omega^2 = 15$

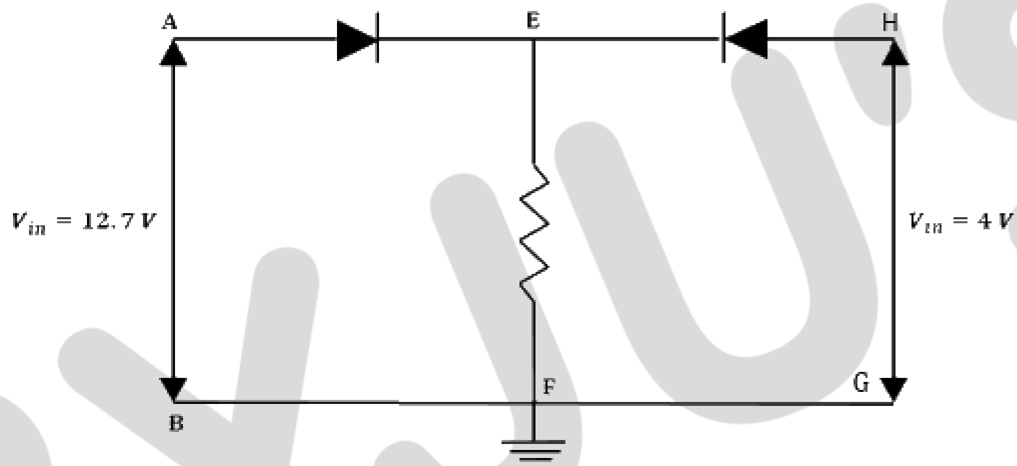
$\omega = \sqrt{15}$

Therefore, $n = 15$

25. In the given circuit both diodes are ideal having zero forward resistance and built-in potential of 0.7 V. Find the potential of point E in volts



Solution: (12)



We have to apply nodal analysis on both left and right side and check what can be voltage at E. For nodal analysis, voltage at B, F and G will be 0 volts and voltage at A will be 12.7 volt and voltage at H will be 4 volts.

If, we apply Nodal from right side, voltage at E will be 12 volt (diode between A and E will be forward biased). Now voltage at E is 12 volt and voltage at H is 4 volt and since, diode between E and H is reversed biased and any difference of voltage is possible across reverse biased. So, this is possible.

If, we apply Nodal from left side, voltage at E will be 3.3 volt (diode between E and H will be forward biased). Now voltage at E is 3.3 volt and voltage at A is 12 volt and since, diode between E and A is forward biased and in forward biased difference of voltage of 0.7 volt is allowable. So, this case is not possible. Therefore current will also not flow through GH.

Hence, $V_E = 12\ V$