## JEE Main 2020 Paper

Date $: 2^{\text {nd }}$ September 2020
Time : 02: 00 pm- 05: 00 pm
Subject : Maths
Q. 1 Let $f: R \rightarrow R$ be a function which satisfies $f(x+y)=f(x)+f(y) \forall x, y \in R$. If $f(1)=2$ and $g(n)=\sum_{k=1}^{(n-1)} f(k), n \in N$ then the value of $n$, for which $g(n)=20$, is:
(1) 9
(2) 5
(3) 4
(4) 20

Sol. (2)
$f(1)=2 ; f(x+y)=f(x)+f(y)$
$x=y=1 \Rightarrow f(2)=2+2=4$
$x=2, y=1 \Rightarrow f(3)=4+2=6$
$g(n)=f(1)+f(2)+$ .$+f(n-1)$
$=2+4+6+$ $+2(n-1)$
$=2 \Sigma(n-1)$
$=2 \frac{(n-1) \cdot n}{2}$
$=n^{2}-n$
Given $\mathrm{g}(\mathrm{n})=20 \quad \Rightarrow \mathrm{n}^{2}-\mathrm{n}=20$

$$
\begin{aligned}
& n^{2}-n-20=0 \\
& n=5
\end{aligned}
$$

Q. 2 If the sum of first 11 terms of an A.P., $a_{1}, a_{2}, a_{3}, \ldots$ is $0\left(a_{1} \neq 0\right)$ then the sum of the A.P., $a_{1}, a_{3}, a_{5}, \ldots, a_{23}$ is $k a_{1}$, where $k$ is equal to:
(1) $-\frac{121}{10}$
(2) $-\frac{72}{5}$
(3) $\frac{72}{5}$
(4) $\frac{121}{10}$

Sol. (2)
$\sum_{k=1}^{11} a_{k}=0 \Rightarrow 11 a+55 d=0$
$a+5 d=0$
Now $a_{1}+a_{3}+\ldots .+a_{23}=k a_{1}$
$12 a+d(2+4+6+\ldots . .+22)=k a$
$12 a+2 d .66=k a$
$12(a+11 d)=k a$
$12\left(a+11\left(-\frac{a}{5}\right)\right)=k a$
$12\left(1-\frac{11}{5}\right)=k$
$k=-\frac{72}{5}$

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Q. 3 Let $\mathrm{E}^{c}$ denote the complement of an event E . Let $\mathrm{E}_{1}, \mathrm{E}_{2}$ and $\mathrm{E}_{3}$ be any pairwise independent events with $P\left(E_{1}\right)>0$ and $P\left(E_{1} \cap E_{2} \cap E_{3}\right)=0$. Then $P\left(E_{2}^{C} \cap E_{3}^{c} / E_{1}\right)$ is equal to:
(1) $P\left(E_{3}^{C}\right)-P\left(E_{2}^{c}\right)$
(2) $P\left(E_{3}\right)-P\left(E_{2}^{C}\right)$
(3) $P\left(E_{3}^{C}\right)-P\left(E_{2}\right)$
(4) $P\left(E_{2}^{C}\right)+P\left(E_{3}\right)$

Sol. (3)

$$
\begin{aligned}
& P\left(E_{2}^{c} \cap E_{3}^{c} / E_{1}\right)=\frac{P\left(E_{2}^{c} \cap E_{3}^{c} \cap E_{1}\right)}{P\left(E_{1}\right)} \\
& =\frac{P\left(E_{1}\right)-P\left(E_{1} \cap E_{2}\right)-P\left(E_{1} \cap E_{3}\right)+P\left(E_{1} \cap E_{2} \cap E_{3}\right)}{P\left(E_{1}\right)} \\
& =\frac{P\left(E_{1}\right)-P\left(E_{1}\right) \cdot P\left(E_{2}\right)-P\left(E_{1}\right) \cdot P\left(E_{3}\right)+0}{P\left(E_{1}\right)}=1-P\left(E_{2}\right)-P\left(E_{3}\right) \\
& =P\left(E_{3}^{c}\right)-P\left(E_{2}\right)
\end{aligned}
$$

Q. 4 If the equation $\cos ^{4} \theta+\sin ^{4} \theta+\lambda=0$ has real solutions for $\theta$, then $\lambda$ lies in the interval:
(1) $\left(-\frac{1}{2},-\frac{1}{4}\right]$
(2) $\left[-1,-\frac{1}{2}\right]$
(3) $\left[-\frac{3}{2},-\frac{5}{4}\right]$
(4) $\left(-\frac{5}{4},-1\right)$

Sol. (2)
$\cos ^{4} \theta+\sin ^{4} \theta+\lambda=0$
$\lambda=-\left\{1-\frac{1}{2} \sin ^{2} 2 \theta\right\}$
$2(\lambda+1)=\sin ^{2} 2 \theta$
$0 \leq 2(\lambda+1) \leq 1$
$0 \leq \lambda+1 \leq \frac{1}{2}$
$-1 \leq \lambda \leq-\frac{1}{2}$
Q. 5 The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^{2}=8 x$, with one of its vertices on the vertex of this parabola, is:
(1) $128 \sqrt{3}$
(2) $192 \sqrt{3}$
(3) $64 \sqrt{3}$
(4) $256 \sqrt{3}$

Sol. (2)


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A: $\left(a \cos 30^{\circ}, a \sin 30^{\circ}\right)$
lies on parabola
$\frac{a^{2}}{4}=8 \cdot \frac{a \cdot \sqrt{3}}{2}$
$a=16 \sqrt{3}$
Area of equilateral $\Delta=\frac{\sqrt{3}}{4} a^{2}$
$\Delta=\frac{\sqrt{3}}{4} \cdot 16 \cdot 16.3$
$\Delta=192 \sqrt{3}$
Q. 6 The imaginary part of $(3+2 \sqrt{-54})^{1 / 2}-(3-2 \sqrt{-54})^{1 / 2}$ can be :
(1) $\sqrt{6}$
(2) $-2 \sqrt{6}$
(3) 6
(4) $-\sqrt{6}$

Sol. (2)

$$
\begin{aligned}
& (3+2 i \sqrt{54})^{1 / 2}-(3-2 i \sqrt{54})^{1 / 2} \\
& =\left(9+6 i^{2}+2.3 \mathrm{i} \sqrt{6}\right)^{1 / 2}-\left(9+6 \mathrm{i}^{2}-2.3 \mathrm{i} \sqrt{6}\right)^{1 / 2} \\
& =\left((3+\sqrt{6} \mathrm{i})^{2}\right)^{1 / 2}-\left((3-\sqrt{6} \mathrm{i})^{2}\right)^{1 / 2} \\
& = \pm(3+\sqrt{6} \mathrm{i}) \mp(3-\sqrt{6 \mathrm{i}})=-2 \sqrt{6} \mathrm{i}
\end{aligned}
$$

Q. 7 A plane passing through the point $(3,1,1)$ contains two lines whose direction ratios are $1,-2,2$ and $2,3,-1$ respectively. If this plane also passes through the point $(\alpha,-3,5)$, then $\alpha$ is equal to:
(1) -5
(2) 10
(3) 5
(4) -10

Sol. (3)
Required plane is

$$
\begin{aligned}
& \left|\begin{array}{ccc}
x-3 & y-1 & z-1 \\
1 & -2 & 2 \\
2 & 3 & -1
\end{array}\right|=0 \\
& \Rightarrow-4(x-3)+5(y-1)+7(z-1)=0 \\
& \Rightarrow 4 x-5 y-7 z=0 \\
& (\alpha,-3,5) \text { lies on } 4 x-5 y-7 z=0 \\
& \Rightarrow \alpha=5
\end{aligned}
$$

Q. 8 Let $A=\left\{X=(x, y, z)^{\top}: P X=0\right.$ and $\left.x^{2}+y^{2}+z^{2}=1\right\}$, where $P=\left[\begin{array}{ccc}1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1\end{array}\right]$, then the set $A$ :
(1) contains more than two elements
(2) is a singleton.
(3) contains exactly two elements
(4) is an empty set.

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Sol. (3)
Clearly $|\mathrm{P}|=0$
$\Rightarrow P X=0$ has infinite solutions
The line concurrence passes through ( $0,0,0$ ) which is centre of sphere $x^{2}+y^{2}+z^{2}=1$
$\Rightarrow$ Diameter will intersect at two points
$\therefore$ two solutions (exactly) exist
Q. 9 The equation of the normal to the curve $y=(1+x)^{2 y}+\cos ^{2}\left(\sin ^{-1} x\right)$ at $x=0$ is:
(1) $y+4 x=2$
(2) $2 y+x=4$
(3) $x+4 y=8$
(4) $y=4 x+2$

## Sol. (3)

at $x=0 \Rightarrow y=1+\cos ^{2}(0)=2$
p: $(0,2)$
$y=(1+x)^{2 y}+\cos ^{2}\left(\sin ^{-1} x\right)$
$y=(1+x)^{2 y}+\cos ^{2}\left(\cos ^{-1} \sqrt{1-x^{2}}\right)$
$=(1+x)^{2 y}+\left(\cos \left(\cos ^{-1} \sqrt{1-x^{2}}\right)\right)^{2}$
$=(1+x)^{2 y}+\left(\sqrt{1-x^{2}}\right)^{2}$
$y=(1+x)^{2 y}+1-x^{2}$
differentiating with respect to ' $x$ '
Now $y^{\prime}=(1+x)^{2 y}\left\{\frac{2 y}{1+x}+\ln (1+x) \cdot 2 y^{\prime}\right\}-2 x$
$\left.y^{\prime}\right|_{(0,2)}=4-0$
$N_{0}: y-2=-\frac{1}{4}(x-0)$
$N_{0}: 4 y-8=-x$
$N_{0}: x+4 y=8$
Q. 10 Consider a region $R=\left\{(x, y) \in R^{2}: x^{2} \leq y \leq 2 x\right\}$. If a line $y=\alpha$ divides the area of region R into two equal parts, then which of the following is true.?
(1) $\alpha^{3}-6 \alpha^{2}+16=0$
(2) $3 \alpha^{2}-8 \alpha^{3 / 2}+8=0$
(3) $\alpha^{3}-6 \alpha^{3 / 2}-16$
(4) $3 \alpha^{2}-8 \alpha+8=0$

Sol. (2)


Area $=\int_{0}^{2}\left(2 x-x^{2}\right) d x=x^{2}-\left.\frac{x^{3}}{3}\right|_{0} ^{2}=\frac{4}{3}$

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$\therefore \int_{0}^{\alpha}\left(\sqrt{y}-\frac{y}{2}\right) d y=\frac{2}{3}$
$\Rightarrow \frac{2}{3} y^{3 / 2}-\left.\frac{y^{2}}{4}\right|_{0} ^{\alpha}=\frac{2}{3}$
$\Rightarrow 8 . \alpha^{3 / 2}-3 \alpha^{2}=8$
Q. 11 Let $\mathrm{f}:(-1, \infty) \rightarrow R$ be defined by $f(0)=1$ and $f(x)=\frac{1}{x} \log _{e}(1+x), x \neq 0$. Then the function f:
(1) increases in $(-1, \infty)$
(2) decreases in $(-1,0)$ and increases in $(0, \infty)$
(3) increase in $(-1,0)$ and decreases in $(0, \infty)$
(4) decreases in $(-1, \infty)$.

Sol. (4)
$f(x)=\frac{1}{x} \ln (1+x)$
$f^{\prime}=\frac{x-\frac{1}{1+x}-\operatorname{In}(1+x)}{x^{2}}$
$f^{\prime}=\frac{1-\frac{1}{1+x}-\ln (1+x)}{x^{2}}$
$f^{\prime}<0 \quad \forall x \in(-1, \infty)$
Q. 12 Which of the following is a tautology?
(1) $(p \rightarrow q) \wedge(q \rightarrow p)$
$(2)(\sim p) \wedge(p \vee q) \rightarrow q$
(3) $(q \rightarrow p) \vee \sim(p \rightarrow q)$
(4) $(\sim q) \vee(p \wedge q) \rightarrow q$

Sol. (2)

| p | q | $\sim \mathrm{p}$ | $\mathrm{p} \vee \mathrm{q}$ | $\sim \mathrm{p} \wedge(\mathrm{p} \vee \mathrm{q})$ | $\sim \mathrm{p} \wedge(\mathrm{p} \vee \mathrm{q}) \rightarrow \mathrm{q}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| T | T | F | T | F | T |
| T | F | F | T | F | T |
| F | T | T | T | T | T |
| F | F | T | F | F | T |

Q. 13 Let $f(x)$ be a quadratic polynomial such that $f(-1)+f(2)=0$. If one of the roots of $f(x)=0$ is 3, then its other roots lies in:
(1) $(0,1)$
(2) $(1,3)$
(3) $(-1,0)$
$(4)(-3,-1)$

## Sol. (3)

Let $f(x)=a(x-3)(x-\alpha)$
$f(-1)+f(2)=0$
$a[(-1-3)(-1-\alpha)+(2-3)(2-\alpha)]=0$
$a[4+4 \alpha-2+\alpha]=0$
$5 \alpha+2=0$
$\alpha=-\frac{2}{5}$

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Q. 14 Let $S$ be the sum of the first 9 terms of the series:
$\{x+k a\}+\left\{x^{2}+(k+2) a\right\}+\left\{x^{3}+(k+4) a\right\}+\left\{x^{4}+(k+6) a\right\}+\ldots$ where $a \neq 0$ and $a \neq 1$. If $S=\frac{x^{10}-x+45 a(x-1)}{x-1}$, then $k$ is equal to:
(1) 3
(2) -3
(3) 1
(4) -5

Sol. (2)
$S=\{x+k a\}+\left\{x^{2}+(k+2) a\right\}+\left\{x^{3}+(k+4) a\right\}$ up to 9 term
$S=\left(x+x^{2}+\ldots . x^{9}\right)+a\{k+(k+2)+(k+4)+\ldots .$. up to 9 term $\left.)\right\}$
$S=\frac{x\left(1-x^{9}\right)}{1-x}+a\{9 k+2.36\}$
$S=\frac{x^{10}-x}{x-1}+9 a k+72 a$
$S=\frac{x^{10}-x+45 a(x-1)}{x-1}=\frac{x^{10}-x+(9 k+72) a(x-1)}{x-1}$
$\Rightarrow 45=9 \mathrm{k}+72$
$9 k=-27$
$k=-3$
Q. 15 The set of all possible values of $\theta$ in the interval $(0, \pi)$ for which the points $(1,2)$ and $(\sin \theta, \cos \theta)$ lie on the same side of the line $x+y=1$ is:
(1) $\left(0, \frac{\pi}{4}\right)$
(2) $\left(0, \frac{\pi}{2}\right)$
(3) $\left(0, \frac{3 \pi}{4}\right)$
(4) $\left(\frac{\pi}{4}, \frac{3 \pi}{4}\right)$

Sol. (2)

$L_{11} \times L_{22}>0$
$\Rightarrow \sin \theta+\cos \theta>1$
$\Rightarrow \theta \in(0, \pi / 2)$
Q. 16 Let $n>2$ be an integer. Suppose that there are $n$ Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of $n$ is:
(1) 201
(2) 199
(3) 101
(4) 200

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Sol. (1)


Red line $=99$ blue line
${ }^{n} C_{2}-n=99 n$
$\frac{n(n-1)}{2}=100 n$
$n-1=200$
$n=201$
Q. 17 If a curve $y=f(x)$, passing through the point $(1,2)$ is the solution of the differential equation, $2 x^{2} d y=\left(2 x y+y^{2}\right) d x$, then $f\left(\frac{1}{2}\right)$ is equal to:
(1) $\frac{-1}{1+\log _{e} 2}$
(2) $1+\log _{\mathrm{e}} 2$
(3) $\frac{1}{1+\log _{\mathrm{e}} 2}$
(4) $\frac{1}{1-\log _{\mathrm{e}} 2}$

Sol. (3)

$$
\begin{aligned}
& 2 \frac{d y}{d x}=2 \frac{y}{x}+\left(\frac{y}{x}\right)^{2} \rightarrow H D E \\
& \because y=v x \\
& 2\left(v+x \frac{d v}{d x}\right)=2 v+v^{2} \\
& 2 \frac{d v}{v^{2}}=\frac{d x}{x} \\
& -\frac{2}{v}=\ln x+c \\
& -\frac{2 x}{y}=\ln x+c \\
& \downarrow(1,2) \\
& c=-1 \\
& c: \ln x+\frac{2 x}{y}=1 \\
& \text { For } f(1 / 2) \Rightarrow \ln \left(\frac{1}{2}\right)+\frac{2}{2 y}=1 \\
& y=\frac{1}{1+\ln 2}
\end{aligned}
$$

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Q. 18 For some $\theta \in\left(0, \frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^{2}-y^{2} \sec ^{2} \theta=10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^{2} \sec ^{2} \theta+y^{2}=5$, then the length of the latus rectum of the ellipse, is:
(1) $\frac{4 \sqrt{5}}{3}$
(2) $\frac{2 \sqrt{5}}{3}$
(3) $2 \sqrt{6}$
(4) $\sqrt{30}$

Sol. (1)
$H: x^{2}-y^{2} \sec ^{2} \theta=10$
$E: x^{2} \sec ^{2} \theta+y^{2}=5$
$\sqrt{1+\frac{10 \cos ^{2} \theta}{10}}=\sqrt{5} \sqrt{1-\frac{5 \cos ^{2} \theta}{5}}$
$1+\cos ^{2} \theta=5-5 \cos ^{2} \theta$
$6 \cos ^{2} \theta=4$
$\cos \theta= \pm \sqrt{\frac{2}{3}}$
$I(L R)$ of ellipse $=\frac{2.5 \cos ^{2} \theta}{\sqrt{5}}$
$=2 \sqrt{5} \cdot \frac{2}{3}=\frac{4 \sqrt{5}}{3}$
Q. $19 \lim _{x \rightarrow 0}\left(\tan \left(\frac{\pi}{4}+x\right)\right)^{1 / x}$ is equal to:
(1) e
(2) $e^{2}(3) 2$
(4) 1

Sol. (2)
$\lim _{x \rightarrow 0}\left(\tan \left(\frac{\pi}{4}+x\right)\right)^{1 / x}\left(1^{\infty}\right)=e^{L}$
$\log L=\lim _{x \rightarrow 0} \frac{\tan \left(\frac{\pi}{4}+x\right)-1}{x}$
$\log L=\lim _{x \rightarrow 0} \frac{\frac{1+\tan x}{1-\tan x}-1}{x}$
$\log L=\lim _{x \rightarrow 0} 2\left(\frac{\tan x}{x}\right) \cdot\left(\frac{1}{1-\tan x}\right)$
$\log \mathrm{L}=+2$
Ans. $\mathrm{e}^{2}$

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Q. 20 Let $a, b, c \in R$ be all non-zero and satisfy $a^{3}+b^{3}+c^{3}=2$. If the matrix

$$
A=\left(\begin{array}{lll}
a & b & c \\
b & c & a \\
c & a & b
\end{array}\right)
$$

satisfies $A^{\top} A=I$, then a value of $a b c$ can be:
(1) $\frac{2}{3}$
(2) 3
(3) $-\frac{1}{3}$
(4) $\frac{1}{3}$

Sol. (4)
$a^{3}+b^{3}+c^{3}=2$
$A^{\top} A=I$
$\left[\begin{array}{lll}a & b & c \\ b & c & a \\ c & a & b\end{array}\right]\left[\begin{array}{lll}a & b & c \\ b & c & a \\ c & a & b\end{array}\right]=\left[\begin{array}{lll}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right]$
$=a^{2}+b^{2}+c^{2}=1$
$\& a b+b c+c a=0$
Now $(a+b+c)^{2}=\sum a^{2}+2 \sum a b$
$\left(\sum \mathrm{a}\right)^{2}=1+0 \Rightarrow\left(\sum \mathrm{a}\right)^{2}=1 \Rightarrow \sum \mathrm{a}= \pm 1$
Now $\sum a^{3}-3 a b c=\left(\sum a\right)\left(\sum a^{2}-\sum a b\right)$
$2-3 a b c= \pm 1(1-0)$
$2-3 a b c= \pm 1$

Q. 21 Let the position vectors of points ' $A$ ' and ' $B$ ' be $\hat{i}+\hat{j}+\hat{k}$ and $2 \hat{i}+\hat{j}+3 \hat{k}$, respectively. $A$ point ' $P^{\prime}$ ' divides the line segment $A B$ internally in the ratio $\lambda: 1(\lambda>0)$. If $O$ is the origin and $\overrightarrow{\mathrm{OB}} \cdot \overrightarrow{\mathrm{OP}}-3|\overrightarrow{\mathrm{OA}} \times \overrightarrow{\mathrm{OP}}|^{2}=6$, then $\lambda$ is equal to $\qquad$
Sol. 0.8
$\overrightarrow{\mathrm{OA}}=\langle 1,1,1\rangle, \overrightarrow{\mathrm{OB}}=\langle 2,1,3\rangle$

| $\lambda$ | $\vdots$ | 1 |
| :---: | :---: | :---: |
| A | p | B |

$\overrightarrow{\mathrm{OP}}=\left(\frac{2 \lambda+1}{\lambda+1}, 1, \frac{3 \lambda+1}{\lambda+1}\right)$
$\overrightarrow{\mathrm{OB}} \cdot \overrightarrow{\mathrm{OP}}=\frac{2(2 \lambda+1)}{\lambda+1}+1+\frac{3(3 \lambda+1)}{\lambda+1}$
$=\frac{14 \lambda+6}{\lambda+1}$

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$$
\begin{aligned}
& |\overrightarrow{\mathrm{OA}} \times \overrightarrow{\mathrm{OP}}|^{2}=|\overrightarrow{\mathrm{OA}}|^{2}|\overrightarrow{\mathrm{OP}}|^{2}-(\overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{OP}})^{2} \\
& 3 \cdot\left(\frac{(2 \lambda+1)^{2}+(\lambda+1)^{2}+(3 \lambda+1)^{2}}{(\lambda+1)^{2}}\right)-\left(\frac{2 \lambda+1+\lambda+1+3 \lambda+1}{\lambda+1}\right)^{2} \\
& =\frac{1}{(\lambda+1)^{2}}\left\{3\left(14 \lambda^{2}+12 \lambda+3\right)-(6 \lambda+3)^{2}\right\} \\
& =\frac{1}{(\lambda+1)^{2}}\left\{6 \lambda^{2}\right\} \\
& \text { Now } \frac{14 \lambda+6}{\lambda+1}-3\left(\frac{6 \lambda^{2}}{(\lambda+1)^{2}}\right)=6 \\
& (14 \lambda+6)(\lambda+1)-18 \lambda^{2}=6(\lambda+1)^{2} \\
& -4 \lambda^{2}+20 \lambda+6=6 \lambda^{2}+12 \lambda+6 \\
& 10 \lambda^{2}-8 \lambda=0 \\
& \lambda(10 \lambda-8)=0 \\
& \because \lambda>0 \\
& \lambda=.8
\end{aligned}
$$

Q. 22 Let [ $t$ ] denote the greatest integer less than or equal to $t$. Then the value of $\int_{1}^{2}|2 x-[3 x]| d x$ is $\qquad$
Sol. 1

$$
\begin{aligned}
& \int_{1}^{2}|2 \mathrm{x}-[3 \mathrm{x}]| \mathrm{dx} \\
& 3 \mathrm{x}=\mathrm{t} \\
& =\frac{1}{3} \int_{3}^{6}\left|\frac{2 \mathrm{t}}{3}-[\mathrm{t}]\right| \mathrm{dt} \\
& =\frac{1}{9}\left[\int_{3}^{6}|2 \mathrm{t}-3[\mathrm{t}]|\right] \mathrm{dt} \\
& =\frac{1}{9}\left[\int_{3}^{4}|2 \mathrm{t}-9|+\int_{4}^{5}|2 \mathrm{t}-12|+\int_{5}^{6}|2 \mathrm{t}-15|\right] \mathrm{dt} \\
& =\frac{1}{9}\left[\int_{3}^{4}(9-2 \mathrm{t})+\int_{4}^{5}(12-2 \mathrm{t})+\int_{5}^{6}(15-2 \mathrm{t})\right] \mathrm{dt} \\
& =\frac{1}{9}\left[9.1+12.1+15.1-\left[4^{2}-3^{2}\right]-\left[5^{2}-4^{2}\right]-\left[6^{2}-5^{2}\right]\right] \\
& =\frac{1}{9}\left[36-\left[4^{2}-3^{2}+5^{2}-4^{2}+6^{2}-5^{2}\right]\right] \\
& =\frac{1}{9}[36-36+9]=1
\end{aligned}
$$

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Q. 23 If $\mathrm{y}=\sum_{\mathrm{k}=1}^{6} \mathrm{k} \mathrm{cos}^{-1}\left\{\frac{3}{5} \cos \mathrm{kx}-\frac{4}{5} \sin \mathrm{kx}\right\}$, then $\frac{\mathrm{dy}}{\mathrm{dx}}$ at $\mathrm{x}=0$ is $\qquad$
Sol. 91
$y=\sum_{k=1}^{6} k \cos ^{-1}\{\cos (k x+\theta)\}$
where $\tan \theta=\frac{4}{3}$
$\cos ^{-1}[\cos (k x+a)]=k x+a$ as $k x+a \in[0, \pi]$
$y=\cos ^{-1}\left(\cos (x+\theta)+2 \cos ^{-1}(\cos (2 x+\theta)) \ldots \ldots .+6 \cos ^{-1}(\cos (6 x+\theta))\right.$
$\therefore y=x+\theta+2(2 x+\theta)+$ $\qquad$ $\ldots . . . .6(6 x+\theta)$
$\frac{d y}{d x}=1.1+2.2+3.3+$ $\qquad$ $+6.6$
$\left.\frac{\mathrm{dy}}{\mathrm{dx}}\right|_{\text {at } \mathrm{x}=0}=1.1+2.2+3.3+$ $+6.6$
$=1.1+2.2+3.3+\ldots . .+6.6$
$=\sum 6^{2}=\frac{6.7 .13}{6}=91$
Q. 24 If the variance of the terms in an increasing A.P., $b_{1}, b_{2}, b_{3}, \ldots, b_{11}$ is 90 , then the common difference of this A.P. is $\qquad$
Sol. 3

$$
\operatorname{Var}(x)=\frac{\sum b i^{2}}{11}-\left(\frac{\sum b i}{11}\right)^{2}
$$

$90=\left\{\frac{11 a^{2}+385 d^{2}+110 a d}{11}\right\}-\left\{\frac{11 a+55 d}{11}\right\}^{2}$
$90=\left\{a^{2}+35 d^{2}+10 a d\right\}-\{a+5 d\}^{2}$
$90=a^{2}+35 d^{2}+10 a d-a^{2}-25 d^{2}-10 a d$
$90=10 d^{2}$
$9=d^{2}$
$d=3$

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Q. 25 For a positive integer $n,\left(1+\frac{1}{x}\right)^{n}$ is expanded in increasing powers of $x$. If three consecutive coefficients in this expansion are in the ratio, 2:5:12, then $n$ is equal to $\qquad$
Sol. 118
Let 3 consecutive coH are
${ }^{n} C_{1-1}:{ }^{n} C_{r}:{ }^{n} C_{r+1}: 2: 5: 12$
$\frac{{ }^{n} C_{-1}}{{ }^{n} C_{1}}=\frac{2}{5} \& \frac{{ }^{n} C_{r}}{{ }^{n} C_{r+1}}=\frac{5}{12}$
$\frac{r}{n-r+1}=\frac{2}{5} \quad \& \frac{r+1}{(n-r)}=\frac{5}{12}$
$7 r=2 n+2 \quad \& 17 r=5 n-12$
$\Rightarrow \frac{2 n+2}{7}=\frac{5 n-12}{17}$
$=34 n+34=35 n-84$
$\Rightarrow n=118$

