JEE Main 2020 Paper

Date : 4th September 2020

Time : 09 : 00 am - 12 : 00 pm

Subject : Maths

1. Let y=y(x) be the solution of the differential equation, $xy'-y=x^2(x\cos x+\sin x), x > 0$. if $y(\pi)=\pi$, then $y''\left(\frac{\pi}{2}\right)+y\left(\frac{\pi}{2}\right)$ is equal to

(1) $2+\frac{\pi}{2}+\frac{\pi^2}{4}$ (2) $2+\frac{\pi}{2}$ (3) $1+\frac{\pi}{2}$ (4) $1+\frac{\pi}{2}+\frac{\pi^2}{4}$

Sol. **(2)**

$xy' - y = x^2(x\cos x + \sin x) \quad x > 0,\ y(\pi) = \pi$

$y' - \frac{1}{x}y = x\{x\cos x + \sin x\}$

$\text{I.F.} = e^{-\int \frac{1}{x}dx} = e^{-\ell nx} = \frac{1}{x}$

$\therefore\ y.\frac{1}{x} = \int \frac{1}{x}.x(x\cos x+\sin x)dx$

$\frac{y}{x} = \int (x\cos x+\sin x)dx$

$\frac{y}{x} = \int \frac{d}{dx}(x\sin x)dx$

$\frac{y}{x} = x\sin x + C$

$\Rightarrow y = x^2 \sin x + Cx$

$x = \pi, y = \pi$

$\pi = \pi C \Rightarrow C = 1$

$y = x^2\sin x + x \Rightarrow y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4}+\frac{\pi}{2}$

$y' = 2x\sin x + x^2\cos x + 1$

$y'' = 2\sin x + 2x\cos x + 2x\cos x - x^2\sin x$

$y''\left(\frac{\pi}{2}\right) = 2-\frac{\pi^2}{4} \Rightarrow y\left(\frac{\pi}{2}\right)+y''\left(\frac{\pi}{2}\right) = 2+\frac{\pi}{2}$

2. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to:

(1) ${}^{51}C_7 - {}^{30}C_7$ (2) ${}^{51}C_7 + {}^{30}C_7$ (3) ${}^{50}C_7 - {}^{30}C_7$ (4) ${}^{50}C_6 - {}^{30}C_6$

Sol. **(1)**

$\sum_{r=0}^{20} {}^{50-r}C_6$

$= {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + ... + {}^{31}C_6 + {}^{30}C_6$
add and subtract ${}^{30}C_7$
Using
${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r \Rightarrow {}^{30}C_6 + {}^{30}C_7 = {}^{31}C_7$
${}^{31}C_6 + {}^{31}C_7 = {}^{32}C_7$
Similarly solving
${}^{51}C_7 - {}^{30}C_7$

3. Let [t] denote the greatest integer $\le$ t. Then the equation in x, $[x]^2+2[x+2]-7=0$ has :
(1) exactly four integral solutions.
(2) infinitely many solutions.
(3) no integral solution.
(4) exactly two solutions.

Sol. **(2)**
$[x]^2 + 2[x + 2] - 7 = 0$
$[x]^2 + 2[x] - 3 = 0$
let $[x] = y$
$y^2 + 3y - y - 3 = 0$
$(y - 1)(y + 3) = 0$
$[x] = 1$ or $[x] = -3$
$x \in [1,2)$ or $x \in [-3,-2)$

4. Let P(3,3) be a point on the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at P intersects the x-axis at (9,0) and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to :
(1) (9,3)
(2) $\left(\frac{9}{2}, 2\right)$
(3) $\left(\frac{9}{2}, 3\right)$
(4) $\left(\frac{3}{2}, 2\right)$

Sol. **(3)**
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ P(3,3)
$\frac{9}{a^2} - \frac{9}{b^2} = 1$ ....(1)
Equation of normal $\Rightarrow \frac{a^2x}{3} + \frac{b^2y}{3} = a^2e^2$
at x – axis $\Rightarrow$ y = 0
$\frac{a^2x}{3} = a^2e^2 \Rightarrow x = 3e^2 = 9$
$e^2 = 3$
$e = \sqrt{3}$
$e^2 = 1 + \frac{b^2}{a^2} = 3$
$b^2 = 2a^2$ ...(2)
put in equation 1
$\frac{9}{a^2} - \frac{9}{2a^2} = 1 \Rightarrow \frac{9}{2a^2} = 1 \Rightarrow a^2 = \frac{9}{2}$
$\therefore (a^2, e^2) = \left(\frac{9}{2}, 3\right)$

5. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (a>b) be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t)=\frac{5}{12}+t-t^2$, then a^2+b^2 is equal to

(1) 135 (2) 116 (3) 126 (4) 145

Sol. (3)

L.R = $\frac{2b^2}{a}$ = 10 ...(1)

$\phi(t) = \frac{5}{12} - \left(t-\frac{1}{2}\right)^2 + \frac{1}{4} = \frac{8}{12}-\left(t-\frac{1}{2}\right)^2$

$\therefore \phi(t)_{max} = \frac{2}{3} = e$

$e^2 = 1-\frac{b^2}{a^2} = \frac{4}{9} \Rightarrow \frac{b^2}{a^2}=\frac{5}{9}$

From (1)

$\frac{b^2}{a.a}=\frac{5}{9}$

$\frac{5}{a}=\frac{5}{9} \Rightarrow a = 9$

$\therefore b^2 = 45$

$a^2 + b^2 = 81 + 45 = 126$

6. Let $f(x)=\int\frac{\sqrt{x}}{(1+x)^2}dx\,(x\geq 0)$. Then f(3) − f(1) is eqaul to :

(1) $-\frac{\pi}{6}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (2) $\frac{\pi}{6}+\frac{1}{2}-\frac{\sqrt{3}}{4}$ (3) $-\frac{\pi}{12}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

Sol. (4)

$f(x) = \int\frac{\sqrt{x}}{(1+x)^2}dx$

Substituting $x = \tan^2 t$

$dx = 2\tan t \sec^2 t\, dt$

$f(x) = \int\frac{\tan t.\ 2\tan t\sec^2 t dt}{\sec^4 t}$

$= 2\int \sin^2 t\, dt$

$x = 3 \Rightarrow t = \frac{\pi}{3}$

$x = 1 \Rightarrow t = \frac{\pi}{4}$

$$\therefore f(3)-f(1)=\int_{\pi/4}^{\pi/3}(1-\cos 2t)\,dt \Rightarrow \left(t-\frac{1}{2}\sin 2t\right)_{\frac{\pi}{4}}^{\frac{\pi}{3}}=\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$$

7. If $1+(1-2^2.1)+(1-4^2.3)+(1-6^2.5)+......+(1-20^2.19)=\alpha-220\beta$, then an ordered pair (α,β) is equal to:

(1) (10,97) (2) (11,103) (3) (11,97) (4) (10,103)

Sol. **(2)** 1 + Sn

$T_n = 1-(2n)^2(2n-1)$
$= 1-4n^2(2n-1)$
$= 1-8n^3+4n^2$

$$S_n=\sum_{n=1}^{10}T_n = n-\sum 8n^3+\sum 4n^2$$

$$= n-8\times\frac{n^2(n+1)^2}{4}+\frac{4n(n+1)(2n+1)}{6}$$

$$S_{10}=10-2\times 100\times 121+\frac{2}{3}\times 10\times 11\times 21$$

$= 10-24200+1540$
$= 10-22660$

$\therefore$ Sum of series $= 11-220\times 103 = \alpha-220\beta$

$\alpha = 11,\ \beta = 103$

8. The integral $\int\left(\frac{x}{x\sin x+\cos x}\right)^2 dx$ is equal to

(where C is a constant of integration):

(1) $\tan x-\frac{x\sec x}{x\sin x+\cos x}+C$ (2) $\sec x-\frac{x\tan x}{x\sin x+\cos x}+C$

(3) $\sec x+\frac{x\tan x}{x\sin x+\cos x}+C$ (4) $\tan x+\frac{x\sec x}{x\sin x+\cos x}+C$

Sol. **(1)**

$$\int\left(\frac{x}{x\sin x+\cos x}\right)^2 dx$$

$$\int \underbrace{x\sec x}_{I}.\underbrace{\frac{x\cos x}{(x\sin x+\cos x)^2}}_{II}dx$$

Put $(x\sin x+\cos x)=t \Rightarrow x\cos x\,dx=dt$ in integration of II

$$=x\sec x\left(\frac{-1}{x\sin x+\cos x}\right)+\int\frac{\sec x+x\sec x\tan x}{(x\sin x+\cos x)}dx$$

$$= \frac{-x\sec x}{x\sin x+\cos x}+\int\frac{(\cos x + x\sin x)}{\cos^2 x(x\sin x+\cos x)}dx$$

$$= \frac{-x\sec x}{x\sin x+\cos x}+\tan x + C$$

9. Let f(x)= |x-2| and g(x) = f(f(x)) ,$x\in[0,4]$. Then $\int_0^3 (g(x)-f(x))dx$ is equal to:

(1) $\frac{1}{2}$ (2) 0 (3) 1 (4) $\frac{3}{2}$

Sol. **(3)**

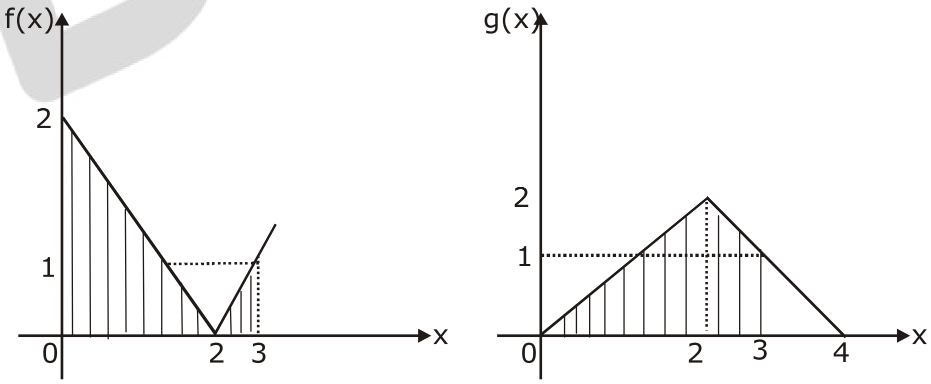
$$f(x) = |x-2| = \begin{cases} x-2, & x\ge 2 \\ 2-x, & x<2 \end{cases}$$

$$g(x) = ||x-2|-2| = \begin{cases} |x-4|, & x\ge 2 \\ |x|, & x<2 \end{cases}$$

$$= \begin{cases} 4-x, & x\in[2,4) \\ x-4, & x\ge 4 \\ x, & x\in[0,2) \end{cases}$$

$$\therefore \int_0^3 (g(x)-f(x))dx = \int_0^3 g(x) - \int_0^3 f(x)dx$$

$$= \left(\frac{1}{2}\times2\times2+1+\frac{1}{2}\times1\times1\right)-\left(\frac{1}{2}\times2\times2+\frac{1}{2}\times1\times1\right) = \frac{7}{2}-\frac{5}{2}=1$$



10. Let x_0 be the point of Local maxima of $f(x) = \vec{a}.(\vec{b} \times \vec{c})$, where $\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$. Then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ at $x=x_0$ is :

(1) -22 (2) -4 (3) -30 (4) 14

Sol. **(1)**

$$\vec{a}.(\vec{b} \times \vec{c}) = \begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix}$$

$= x\{x^2 - 2\} + 2\{-2x + 7\} + 3\{4 - 7x\}$

$= x^3 - 2x - 4x + 14 + 12 - 21x$

$f(x) = x^3 - 27x + 26$

$f'(x) = 3x^2 - 27 = 0 \Rightarrow x = \pm 3$

$f''(x) = 6x$

at $x = 3$, $f''(3) = 6 \times 3 = 18 > 0$

at $x = -3$, $f''(-3) = 6 \times -3 = -18 < 0$

Max at $x_0 = -3$

$\therefore \vec{a} = (-3, -2, 3)$, $\vec{b} = (-2, -3, -1)$, $\vec{c} = (7, -2, -3)$

$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 6 + 6 - 3 - 14 + 6 + 3 - 21 + 4 - 9$

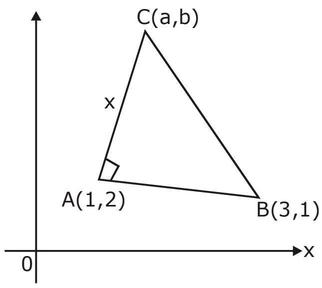
$= 25 - 47 = -22$

11. A triangle ABC lying in the first quadrant has two vertices as A(1,2) and B(3,1) If $\angle BAC = 90^0$, and ar(ΔABC) = $5\sqrt{5}$ s units, then the abscissa of the vertex C is :

(1) $1 + \sqrt{5}$ (2) $1 + 2\sqrt{5}$ (3) $2\sqrt{5} - 1$ (4) $2 + \sqrt{5}$

Sol. **(2)**

$AB = \sqrt{4 + 1} = \sqrt{5}$



$\frac{1}{2} \times \sqrt{5} \times x = 5\sqrt{5}$

$x = 10$

$m_{AB} = \frac{1}{-2}$

$m_{AC} = 2 = \tan\theta$

$\therefore \sin\theta = \frac{2}{\sqrt{5}}$, $\cos\theta = \frac{1}{\sqrt{5}}$

by parametric co-ordinates

$a = 1 + x\cos\theta = 1 + 10 \times \frac{1}{\sqrt{5}} = 1 + 2\sqrt{5}$

12. Let f be a twice differentiable function on (1,6). If f(2)=8, f′(2)=5, $f'(x) \geq 1$ and $f''(x) \geq 4$, for all $x \in (1,6)$, then:

(1) $f(5)+f'(5) \geq 28$
(2) $f'(5)+f''(5) \leq 20$
(3) $f(5) \leq 10$
(4) $f(5)+f'(5) \leq 26$

Sol. **(1)**

$f(2) = 8,\ f'(2) = 5,\ f'(x) \geq 1,\ f''(x) \geq 4$

$x \in (1,6)$

$$\int_2^5 f'(x) \geq \int_2^5 1\,dx$$

$f(5) - f(2) \geq 3$

$f(5) \geq 11$ ...(1)

also $\int_2^5 f''(x)\,dx \geq \int_2^5 4dx$

$f'(5) - f'(2) \geq 12$

$f'(5) - 5 \geq 12$

$f'(5) \geq 17$ ...(2)

adding (1) & (2)

we get,

$f(5) + f'(5) \geq 28$

13. Let α and β be the roots of $x^2-3x+p=0$ and γ and δ be the roots of $x^2-6x+q=0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression.Then ratio (2q+p): (2q-p) is:

(1) 33 :31 (2) 9 : 7 (3) 3 : 1 (4) 5 : 3

Sol. **(2)**

Roots of $x^2 - 3x + p = 0$ are α, β

and roots of $x^2 - 6x + q = 0$ are γ, δ

$\alpha + \beta = 3$

$\gamma + \delta = 6$

$\alpha = a$, $\beta = ar$, $\gamma = ar^2$, $\delta = ar^3$

$a(1 + r) = 3$ ...(1)

$ar^2(1 + r) = 6$ ...(2)

Divide (2) by (1)

$r^2 = 2$

$\alpha . \beta = p = a^2r$, $\gamma . \delta = q = a^2r^5$

$$\Rightarrow \frac{2q+p}{2q-p} = \frac{2r^4+1}{2r^4-1} = \frac{2.2^2+1}{2.2^2-1} = \frac{9}{7}$$

14. Let $u = \frac{2z+i}{z-ki}$, z = x +iy and k>0. If the curve represented by Re(u) +Im(u) =1 intersects the y-axis at the points P and Q where PQ =5, then the value of k is :

(1) 4 (2) 1/2 (3) 2 (4) 3/2

Sol. **(3)**

$u = \frac{2z+i}{z-ki}$, $z = x + iy$

$= \frac{2x+i(2y+1)}{x+i(y-k)} \times \frac{x-i(y-k)}{x-i(y-k)}$

$\Rightarrow \frac{2x^2+(2y+1)(y-k)+i\{2xy+x-2xy+2xk\}}{x^2+(y-k)^2}$

Re(u) + Img(u) = 1

$2x^2 + (2y+1)(y-k) + x + 2xk = x^2 + (y-k)^2$

at y - axis, x = 0

$(2y+1)(y-k) = (y-k)^2$

$2y^2 + y - 2yk - k = y^2 + k^2 - 2yk$

Roots of $y^2 + y - (k + k^2) = 0$ are y_1 and y_2

diff. of roots = 5

$\sqrt{1+4k+4k^2} = 5$

$4k^2 + 4k = 24$

$k^2 + k - 6 = 0$

$(k+3)(k-2) = 0$

$k = 2$

15. If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}, \left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, then which one of the following is not true?

(1) $a^2-d^2=0$ (2) $a^2-c^2=1$ (3) $0 \le a^2+b^2 \le 1$ (4) $a^2-b^2=\frac{1}{2}$

Sol. **(4)**

$$A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta-\sin^2\theta & 2i\sin\theta\cos\theta \\ 2i\cos\theta\sin\theta & \cos^2\theta-\sin^2\theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos2\theta & i\sin2\theta \\ i\sin2\theta & \cos2\theta \end{bmatrix}$$

$$A^3 = A^2 . A = \begin{bmatrix} \cos2\theta & i\sin2\theta \\ i\sin2\theta & \cos2\theta \end{bmatrix}\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

$= \begin{bmatrix} \cos 3\theta & i\sin 3\theta \\ i\sin 3\theta & \cos 3\theta \end{bmatrix}$

Similarly,

$A^5 = \begin{bmatrix} \cos 5\theta & i\sin 5\theta \\ i\sin 5\theta & \cos 5\theta \end{bmatrix}$

$a = d = \cos(5\theta)$
$b = c = i\sin(5\theta)$
$a^2 - b^2 = \cos^2(5\theta) + \sin^2 5\theta = 1$

16. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is:

(1) 3 (2) 9 (3) 7 (4) 5

Sol. 3

$\frac{5+7+10+12+14+15+x+y}{8} = 10$

$x + y = 17$ ....(1)

variance $= \frac{5^2+7^2+10^2+12^2+14^2+15^2+x^2+y^2}{8} - 100 = 13.5$

$= \frac{739+x^2+y^2}{8} - 100 = 13.5$

$x^2 + y^2 = 169$ ...(2)

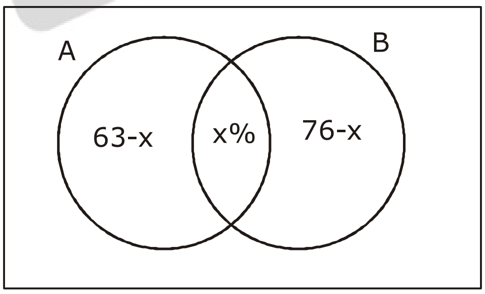
$\therefore x = 12, y = 5$

$|x - y| = 7$

17. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be:

(1) 37 (2) 29 (3) 65 (4) 55

Sol. (4)



$n(B) \le n(A \cup B) \le n(U)$

$\Rightarrow 76 \le 76 + 63 - x \le 100$

$\Rightarrow -63 \le -x \le -39$

$\Rightarrow 63 \ge x \ge 39$

18. Given the following two statements:

$(S_1):(q \vee p) \rightarrow (P \leftrightarrow \sim q)$ is a tautology

$(S_2):\sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy. Then:

(1) only (S_1) is correct.
(2) both (S_1) and (S_2) are correct.
(3) only (S_2) is correct
(4) both (S_1) and (S_2) are not correct.

Sol. **(4)**

$S_1 =$

p	q	~q	qvp	$p \leftrightarrow \sim q$	$(q \vee p) \rightarrow (p \leftrightarrow \sim q)$
T	T	F	T	F	F
T	F	T	T	T	T
F	T	F	T	T	T
F	F	T	F	F	T

S_1 is not correct

$S_2 =$

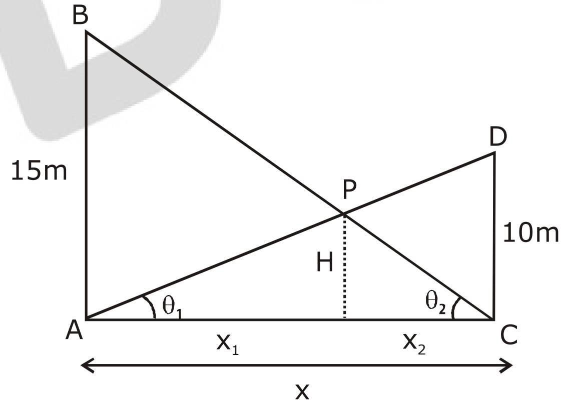
p	q	~q	~p	$\sim p \leftrightarrow q$	$\sim q \wedge (\sim p \leftrightarrow q)$
T	T	F	F	F	F
T	F	T	F	T	T
F	T	F	T	T	F
F	F	T	T	F	F

S_2 is false

19. Two vertical poles AB=15 m and CD=10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is:

(1) 5
(2) 20/3
(3) 10/3
(4) 6

Sol. **(4)**



Using similar triangle concept

$$\tan\theta_1 = \frac{10}{x} = \frac{H}{x_1}$$

$\Rightarrow x_1 = \frac{Hx}{10}$

$\tan\theta_2 = \frac{15}{x} = \frac{H}{x_2}$

$\Rightarrow x_2 = \frac{Hx}{15}$

$\because\ x_1 + x_2 = x$

$\Rightarrow \frac{Hx}{10} + \frac{Hx}{15} = x$

$\Rightarrow 15H + 10H = 150$

$\Rightarrow H = 6$ m

20. If $\left(a+\sqrt{2}\ b\ \cos x\right)\left(a-\sqrt{2}\ b\cos y\right) = a^2 - b^2$, where a>b>0, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is:

(1) $\frac{a+b}{a-b}$ (2) $\frac{a-2b}{a+2b}$ (3) $\frac{a-b}{a+b}$ (4) $\frac{2a+b}{2a-b}$

Sol. **(1)**

$\left(a+\sqrt{2}\ b\ \cos x\right)\left(a-\sqrt{2}\ b\cos y\right) = a^2 - b^2$

diff both sides w.r.t y

$-\sqrt{2}\, b \sin x . \frac{dx}{dy}\left(a-\sqrt{2}b\cos y\right) + \left(a+\sqrt{2}b\cos x\right)\left(\sqrt{2}\, b \sin y\right) = 0$

$x = y = \frac{\pi}{4} \Rightarrow \frac{-bdx}{dy}(a-b)+(a+b)(b) = 0$

$\frac{dx}{dy} = \frac{a+b}{a-b}$

21. Suppose a differentiable function f(x) satisfies the identity $f(x+y)=f(x)+f(y)+xy^2+x^2y$, for all real x and y. If $\lim_{x\to 0}\frac{f(x)}{x} = 1$, then f'(3) is equal to............

Sol. $f(x + y) = f(x) + f(y) + xy^2 + x^2y$

$x = y = 0$

$f(0) = 2f(0) \Rightarrow f(0) = 0$

Now, $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{f(x)+f(h)+xh^2+x^2h-f(x)}{h}$ $(\text{take } y = h)$

$$f'(x) = \lim_{h\to 0}\frac{f(h)}{h} + \lim_{h\to 0}(x\,h) + x^2$$

$$f'(x) = 1 + 0 + x^2$$

$$f'(x) = 1 + x^2$$

$$f'(3) = 10$$

22. If the equation of a plane P, passing through the intersection of the planes, $x+4y-z+7=0$ and $3x+y+5z=8$ is $ax+by+6z=15$ for some $a, b \in R$, then the distance of the point (3,2,-1) from the plane P is.......... units.

Sol. $p_1 + \lambda p_2 = 0$

$(x + 4y - z + 7) + \lambda\,(3x + y + 5z - 8) = ax + by + 6z - 15$

$$\frac{1+3\lambda}{a} = \frac{4+\lambda}{b} = \frac{-1+5\lambda}{6} = \frac{7-8\lambda}{-15}$$

$\therefore\;\; 15 - 75\lambda = 42 - 48\lambda$

$-27 = 27\lambda$

$\lambda = -1$

$\therefore$ plane is $-2x + 3y - 6z + 15 = 0$

$$d = \left|\frac{-6+6+6+15}{\sqrt{4+9+36}}\right| = 3 \text{ units}$$

23. If the system of equations

$x-2y+3z=9$

$2x+y+z=b$

$x-7y+az=24$, has infinitely many solutions, then a-b is equal to.........

Sol. $D = 0$

$$\begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0$$

$1(a + 7) + 2(2a - 1) + 3(-14 - 1) = 0$

$a + 7 + 4a - 2 - 45 = 0$

$5a = 40$

$a = 8$

$$D_1 = \begin{vmatrix} 9 & -2 & 3 \\ b & 1 & 1 \\ 24 & -7 & 8 \end{vmatrix} = 0$$

$\Rightarrow 9(8 + 7) + 2(8b - 24) + 3(-7b - 24) = 0$

$\Rightarrow 135 + 16b - 48 - 21b - 72 = 0$

$15 = 5b \Rightarrow b = 3$

$a - b = 5$

24. Let $(2x^2+3x+4)^{10}=\sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to ....

Sol. **8**

Given $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$ ....(1)

Replace x by $\frac{2}{x}$ in above identity :

$$\frac{2^{10}(2x^2+3x+4)^{10}}{x^{20}}=\sum_{r=0}^{20}\frac{a_r 2^r}{x^r}$$

$$\Rightarrow 2^{10}\sum_{r=0}^{20} a_r x^r = \sum_{r=0}^{20} a_r 2^r x^{(20-r)} \text{ (from eq. (1))}$$

Now, comparing coefficient of x^7 from both sides
(take r =7 in L.H.S. & r = 13 in R.H.S)

$2^{10}a_7 = a_{13}\, 2^{13} \Rightarrow \frac{a_7}{a_{13}} = 2^3 = 8$

25. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is ....

Sol. **3**

Probability of hitting P(H) = $\frac{1}{10}$;

probability of missing P(M) = $\frac{9}{10}$

We have, 1 – (probability of all shots result in failure) > $\frac{1}{4}$

$= 1 - P(M)^n > \frac{1}{4}$

$= 1 - \left(\frac{9}{10}\right)^n > \frac{1}{4}$

$\left(\frac{9}{10}\right)^n < \frac{3}{4}$; $n \geq 3$