

JEE Main 2020 Paper



Date : 4th September 2020

Time : 02 : 00 pm - 05 : 00 pm

Subject : Maths

Q.1 Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, $Ax=b$ when the vector b on the right side is equal to b_1, b_2 and b_3 respectively. if

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, b_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \text{ and } b_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \text{ then the determinant of } A \text{ is}$$

equal to

- (1) 2 (2) $\frac{1}{2}$ (3) $\frac{3}{2}$ (4) 4

Sol.

(1)

Using $AX = B$

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}_{3 \times 3}$$

$$a_1 + a_2 + a_3 = 1 \quad 2a_2 + a_3 = 0$$

$$a_4 + a_5 + a_6 = 0 \quad 2a_5 + a_6 = 2$$

$$a_7 + a_8 + a_9 = 0 \quad 2a_8 + a_9 = 0$$

$$a_3 = 0, a_6 = 0, a_9 = 2$$

$$\therefore a_8 = -1, a_5 = 1, a_2 = 0 \Rightarrow a_1 = 1, a_4 = -1, a_7 = -1$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix}$$

$$|A| = 2(1) = 2$$

Q.2 If a and b are real numbers such that $(2 + \alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1 + i\sqrt{3}}{2}$ then $a+b$ is equal to:

- (1) 33 (2) 57 (3) 9 (4) 24

Sol.

(3)

$$(2 + \alpha)^4 = a + b\alpha$$

$$\left(2 + \frac{\sqrt{3}i - 1}{2}\right)^4 = a + b\alpha$$

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$$\left(\frac{3+\sqrt{3}i}{2}\right)^4 = 9\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^4$$

$$9\{e^{i\pi/6}\}^4 = 9e^{i2\pi/3} = 9\left(\frac{-1}{2} + \frac{\sqrt{3}i}{2}\right) = \frac{-9}{2} + \frac{9\sqrt{3}}{2}i$$

$$-\frac{9}{2} + \frac{9\sqrt{3}}{2}i = a + b\left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right)$$

$$= a - \frac{b}{2} + \frac{bi\sqrt{3}}{2}$$

$$\therefore \frac{b\sqrt{3}}{2} = \frac{9\sqrt{3}}{2} \Rightarrow b = 9$$

$$a = 0 \therefore a + b = 9$$

Q.3 The distance of the point (1, -2, 3) from the plane $x - y + z = 5$ measured parallel to the

line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is:

(1) $\frac{1}{7}$

(2) 7

(3) $\frac{7}{5}$

(4) 1

Sol.

(4)

Equation of line through (1, -2, 3) whose d.r.s. are (2, 3, -6)

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$$

any point on line $(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$

put in $(x - y + z = 5)$

$$2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5$$

$$-7\lambda = -1$$

$$\lambda = \frac{1}{7}$$

$$\text{distance} = \sqrt{(2\lambda)^2 + (3\lambda)^2 + (6\lambda)^2}$$

$$\sqrt{4\lambda^2 + 9\lambda^2 + 36\lambda^2} = 7\lambda = 1 \text{ unit}$$

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Q.4 Let $f: (0, \infty) \rightarrow (0, \infty)$ be a differentiable function such that $f(1) = e$ and

$$\lim_{t \rightarrow x} \frac{t^2 f^2(x) - x^2 f^2(t)}{t - x} = 0. \text{ If } f(x) = 1, \text{ then } x \text{ is equal to :}$$

- (1) e (2) $2e$ (3) $\frac{1}{e}$ (4) $\frac{1}{2e}$

Sol.

(3)

$$f(1) = e \quad \dots(1)$$

$$\lim_{t \rightarrow x} \frac{t^2 f^2(x) - x^2 f^2(t)}{t - x} = 0$$

L' Hospital

$$\Rightarrow \lim_{t \rightarrow x} (2t f^2(x) - 2x^2 f(t) \cdot f'(t)) = 0$$

$$\Rightarrow 2x f^2(x) - 2x^2 f(x) \cdot f'(x) = 0$$

$$\Rightarrow 2x f(x) \{f(x) - x f'(x)\} = 0$$

$$\Rightarrow \frac{f'(x)}{f(x)} = \frac{1}{x}$$

$$\ln f(x) = \ln x + \ln c$$

$$\Rightarrow f(x) = cx \quad \dots(2)$$

$$\text{if } x = 1$$

$$f(1) = c(1)$$

$$f(1) = c$$

From eq.(1) & (2)

$$c = e \quad \dots(3)$$

From eq.(3)

$$f(x) = ex$$

$$\Rightarrow y = ex \text{ or } y = cx$$

$$\therefore \text{ if } f(x) = 1 \Rightarrow x = \frac{1}{e}$$

Q.5 Contrapositive of the statement :

'If a function f is differentiable at a , then it is also continuous at a ', is:

- (1) If a function f is not continuous at a , then it is not differentiable at a .
 (2) If a function f is continuous at a , then it is differentiable at a .
 (3) If a function f is continuous at a , then it is not differentiable at a .
 (4) If a function f is not continuous at a , then it is differentiable at a .

Sol.

(1)

Contrapositive of $p \rightarrow q = \sim q \rightarrow \sim p$

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Q.6 The minimum value of $2^{\sin x} + 2^{\cos x}$ is:

- (1) $2^{1-\sqrt{2}}$ (2) $2^{1-\frac{1}{\sqrt{2}}}$ (3) $2^{-1+\sqrt{2}}$ (4) $2^{-1+\frac{1}{\sqrt{2}}}$

Sol. (2)

Using A.M. \geq G.M.

$$y = 2^{\sin x} + 2^{\cos x}$$

$$\frac{2^{\sin x} + 2^{\cos x}}{2} \geq \sqrt{2^{\sin x + \cos x}}$$

$$2^{\sin x} + 2^{\cos x} \geq 2^1 \cdot 2^{\frac{\sin x + \cos x}{2}}$$

$$2^{\sin x} + 2^{\cos x} \geq 2^{\frac{2 + \sin x + \cos x}{2}}$$

$$\Rightarrow (2^{\sin x} + 2^{\cos x})_{\text{minimum}} = 2^{\frac{2-\sqrt{2}}{2}} = 2^{\frac{-1}{\sqrt{2}}+1}$$

Q.7 If the perpendicular bisector of the line segment joining the points P(1, 4) and Q(k, 3) has y-intercept equal to -4, then a value of k is:

- (1) -2 (2) $\sqrt{15}$ (3) $\sqrt{14}$ (4) -4

Sol. (4)

$$m_{PQ} = \frac{4-3}{1-k} \Rightarrow m_{\perp} = k-1$$

$$\text{midpoint of PQ} = \left(\frac{k+1}{2}, \frac{7}{2} \right)$$

equation of perpendicular bisector

$$y - \frac{7}{2} = (k-1) \left(x - \frac{k+1}{2} \right)$$

for y intercept put $x = 0$

$$y = \frac{7}{2} - \left(\frac{k^2-1}{2} \right) = -4$$

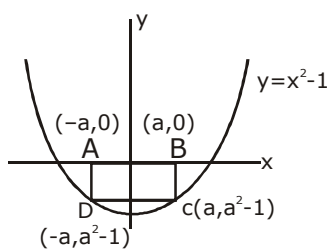
$$\frac{k^2-1}{2} = \frac{15}{2} \Rightarrow k = \pm 4$$

Q.8 The area (in sq. units) of the largest rectangle ABCD whose vertices A and B lie on the x-axis and vertices C and D lie on the parabola, $y = x^2 - 1$ below the x-axis, is:

- (1) $\frac{2}{3\sqrt{3}}$ (2) $\frac{4}{3}$ (3) $\frac{1}{3\sqrt{3}}$ (4) $\frac{4}{3\sqrt{3}}$

Sol. (4)

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$$\text{Area} = 2a(a^2 - 1)$$

$$A = 2a^3 - 2a$$

$$\frac{dA}{da} = 6a^2 - 2 = 0$$

$$\frac{d^2A}{da^2} = 12a, \text{ at } a = -\frac{1}{\sqrt{3}}$$

$$\frac{d^2A}{da^2} = -4\sqrt{3} < 0$$

So Area is maximum at $a = -\frac{1}{\sqrt{3}}$

$$A_{\max} = \frac{-2}{3\sqrt{3}} + \frac{2}{\sqrt{3}} = \frac{-2+6}{3\sqrt{3}} \text{ sq. units}$$

Q.9 The integral $\int_{\pi/6}^{\pi/3} \tan^3 x \cdot \sin^2 3x (2 \sec^2 x \cdot \sin^2 3x + 3 \tan x \cdot \sin 6x) dx$ is equal to:

(1) $\frac{9}{2}$

(2) $-\frac{1}{18}$

(3) $-\frac{1}{9}$

(4) $\frac{7}{18}$

Sol. (2)

$$I = \int_{\pi/6}^{\pi/3} 2 \tan^3 x \sec^2 x \sin^4 3x + 3 \tan^4 x \sin^2 3x \cdot 2 \sin 3x \cos 3x dx$$

$$= \frac{1}{2} \int_{\pi/6}^{\pi/3} 4 \tan^3 x \sec^2 x \sin^4 3x + 3 \cdot 4 \tan^4 x \sin^3 3x \cos 3x dx$$

$$= \frac{1}{2} \int_{\pi/6}^{\pi/3} \frac{d}{dx} (\tan^4 x \sin^4 3x) dx$$

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$$= \frac{1}{2} \left[\tan^4 x \sin^4 3x \right]_{\pi/6}^{\pi/3}$$

$$= \frac{1}{2} \left[9 \cdot (0) - \frac{1}{3} \cdot \frac{1}{3} (1) \right] = -\frac{1}{18}$$

Q.10 If the system of equations

$$x+y+z=2$$

$$2x+4y-z=6$$

$$3x+2y+\lambda z=\mu$$

has infinitely many solutions, then

(1) $\lambda - 2\mu = -5$

(2) $2\lambda + \mu = 14$

(3) $\lambda + 2\mu = 14$

(4) $2\lambda - \mu = 5$

Sol. (2)

$$D = 0 \quad \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0$$

$$(4\lambda + 2) - 1(2\lambda + 3) + 1(4 - 12) = 0$$

$$4\lambda + 2 - 2\lambda - 3 - 8 = 0$$

$$2\lambda = 9 \Rightarrow \lambda = \frac{9}{2}$$

$$D_x = \begin{vmatrix} 2 & 1 & 1 \\ 6 & 4 & -1 \\ \mu & 2 & 9/2 \end{vmatrix} = 0$$

$$\Rightarrow \mu = 5$$

Now check option

$$2\lambda + \mu = 14$$

Q.11 In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six. The game stops as soon as either of the players wins. The probability of A winning the game is:

(1) $\frac{5}{31}$

(2) $\frac{31}{61}$

(3) $\frac{30}{61}$

(4) $\frac{5}{6}$

Sol. 3

$$\text{sum total } 7 = \{(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)\}$$

$$P(\text{sum } 7) = \frac{6}{36}$$

$$\text{sum total } 6 = \{(1,5)(2,4)(3,3)(4,2)(5,1)\}$$

$$P(\text{sum } 6) = \frac{5}{36}$$

$$P(A_{\text{win}}) = P(6) + P(\bar{6}) \cdot P(\bar{7}) \cdot P(6) + \dots$$

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$$\begin{aligned}
 &= \frac{5}{36} + \frac{31}{36} \times \frac{30}{36} \times \frac{5}{36} + \dots \\
 &= \frac{\frac{5}{36}}{1 - \frac{31 \times 30}{36 \times 36}} = \frac{5 \times 36}{36 \times 36 - 31 \times 30} \\
 &= \frac{5 \times 36}{366} = \frac{30}{61}
 \end{aligned}$$

Q.12 If for some positive integer n , the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+5}$ are in the ratio 5:10:14, then the largest coefficient in this expansion is :

- (1) 792 (2) 252 (3) 462 (4) 330

Sol. 3

$$T_r : T_{r+1} : T_{r+2}$$

$${}^{n+5}C_{r-1} : {}^{n+5}C_r : {}^{n+5}C_{r+1} = 5:10:14$$

$$\frac{(n+5)!}{(r-1)!(n+6-r)!} : \frac{(n+5)!}{r!(n+5-r)!} = \frac{5}{10}$$

$$\frac{r}{n+6-r} = \frac{1}{2} \quad \frac{(r+1)!(n+4-r)!}{r!(n+5-r)!} = \frac{5}{7}$$

$$2r = n+6-r$$

$$3r = n+6 \quad \dots(1)$$

$$\frac{r+1}{n+5-r} = \frac{5}{7}$$

$$7r+7 = 5n+25-5r$$

$$12r = 5n+18 \quad \dots(2)$$

From eq.(1) & (2)

$$\therefore 4(n+6) = 5n+18$$

$$n = 6, r = 4$$

Then we can $(1+x)^{11}$

Largest coefficient in the expansion $(1+x)^{11} = {}^{11}C_6 = 462$

Q.13 The function $f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1} x, & |x| \leq 1 \\ \frac{1}{2}(|x| - 1), & |x| > 1 \end{cases}$ is :

- (1) both continuous and differentiable on $\mathbb{R} - \{-1\}$
 (2) continuous on $\mathbb{R} - \{-1\}$ and differentiable on $\mathbb{R} - \{-1, 1\}$
 (3) continuous on $\mathbb{R} - \{1\}$ and differentiable on $\mathbb{R} - \{-1, 1\}$
 (4) both continuous and differentiable on $\mathbb{R} - \{1\}$

Sol. (3)

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$$f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1} x & x \in [-1, 1] \\ \frac{1}{2}(x-1) & x > 1 \\ \frac{1}{2}(-x-1) & x < -1 \end{cases}$$

at $x = 1$

$$f(1) = \frac{\pi}{4} \quad f(1^+) = 0$$

\therefore discontinuous \Rightarrow non diff.

at $x = -1$

$$f(-1) = 0 \quad f(-1^-) = \frac{1}{2}\{+1-1\} = 0$$

cont. at $x = -1$

$$f'(x) = \begin{cases} \frac{1}{1+x^2} & x \in [-1, 1] \\ \frac{1}{2} & x > 1 \\ -\frac{1}{2} & x < -1 \end{cases}$$

at $x = -1$,

$$f'(-1^-) = -\frac{1}{2} \quad \text{and} \quad f'(-1^+) = \frac{1}{2}$$

So, at $x = -1$ it is non differentiable.

Q.14 The solution of the differential equation $\frac{dy}{dx} - \frac{y+3x}{\log_e(y+3x)} + 3 = 0$ is:
(where c is a constant of integration)

(1) $x - \log_e(y+3x) = C$

(2) $x - \frac{1}{2}(\log_e(y+3x))^2 = C$

(3) $x - 2\log_e(y+3x) = C$

(4) $y + 3x - \frac{1}{2}(\log_e x)^2 = C$

Sol. (2)

$$\frac{dy}{dx} - \frac{y+3x}{\ln(y+3x)} + 3 = 0$$

$$\Rightarrow \left(\frac{dy}{dx} + 3 \right) = \frac{y+3x}{\ln(y+3x)}$$

Let $\ln(y+3x) = t$

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$$\frac{1}{y+3x} \cdot \left(\frac{dy}{dx} + 3 \right) = \frac{dt}{dx}$$

$$\therefore (y+3x) \frac{dt}{dx} = \frac{y+3x}{t}$$

$$\Rightarrow t dt = dx$$

$$\frac{t^2}{2} = x + c$$

$$\frac{1}{2} (\ln(y+3x))^2 = x + c$$

Q.15 Let $\lambda \neq 0$ be in \mathbb{R} . If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to:

(1) 27

(2) 9

(3) 18

(4) 36

Sol.

(3)

$$x^2 - x + 2\lambda = 0 \quad (\alpha, \beta)$$

$$3x^2 - 10x + 27\lambda = 0 \quad (\alpha, \gamma)$$

$$3x^2 - 3x + 6\lambda = 0$$

$$\begin{array}{r} - \quad + \quad - \\ \hline -7x + 21\lambda = 0 \end{array}$$

$$\therefore \alpha = 3\lambda$$

Put in equation

$$9\lambda^2 - 3\lambda + 2\lambda = 0$$

$$9\lambda^2 - \lambda = 0 \Rightarrow \lambda = \frac{1}{9} \Rightarrow \alpha = \frac{1}{3}$$

$$\alpha \cdot \beta = \frac{2}{9} \Rightarrow \beta = \frac{2}{3}$$

$$\alpha \cdot \gamma = 1 \Rightarrow \gamma = 3$$

$$\therefore \frac{\beta \cdot \gamma}{\lambda} \Rightarrow \frac{\frac{2}{3} \cdot 3}{\frac{1}{9}} = 18$$

Q.16 The angle of elevation of a cloud C from a point P, 200 m above a still lake is 30° . If the angle of depression of the image of C in the lake from the point P is 60° , then PC (in m) is equal to :

(1) $200\sqrt{3}$

(2) $400\sqrt{3}$

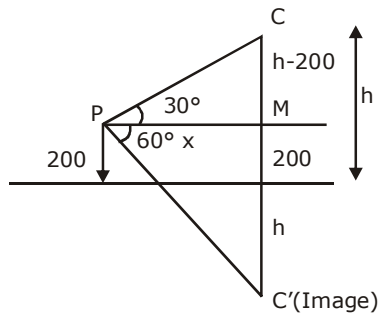
(3) 400

(4) 100

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Sol. (3)



$$\frac{h-200}{x} = \tan 30^\circ \quad \frac{h+200}{x} = \tan 60^\circ$$

$$\frac{h+200}{h-200} = 3$$

$$h+200 = 3h-600$$

$$2h = 800$$

$$h = 400$$

$$\therefore \frac{h-200}{PC} = \sin 30^\circ$$

$$PC = 400 \text{ m}$$

Q.17 Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^n Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i 's and exactly 6 of sets Y_i 's, then n is equal to :

(1) 15

(2) 30

(3) 50

(4) 45

Sol. (2)

$$\text{Number of elements set } T = \bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^n Y_i$$

$$\Rightarrow \frac{50 \times 10}{20} = \frac{n \times 5}{6}$$

$$\Rightarrow \frac{50}{2} \times \frac{6}{5} = n$$

$$\Rightarrow n = 30$$

Q.18 Let $x=4$ be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\frac{1}{2}$. If $P(1, \beta)$, $\beta > 0$ is a point on this ellipse, then the equation of the normal to it at P is :

(1) $8x-2y=5$

(2) $4x-2y=1$

(3) $7x-4y=1$

(4) $4x-3y=2$

Sol. (2)

$$e = \frac{1}{2}$$

$$x = \frac{a}{e} = 4$$

$$\Rightarrow a = 2$$

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$$e^2 = 1 - \frac{b^2}{a^2} \Rightarrow \frac{1}{4} = 1 - \frac{b^2}{4}$$

$$\frac{b^2}{4} = \frac{3}{4} \Rightarrow b^2 = 3$$

$$\therefore \text{Ellipse } \frac{x^2}{4} + \frac{y^2}{3} = 1$$

$P(1, \beta)$

$$x = 1 ; \frac{1}{4} + \frac{\beta^2}{3} = 1$$

$$\frac{\beta^2}{3} = \frac{3}{4} \Rightarrow \beta = \frac{3}{2}$$

$$\Rightarrow P\left(1, \frac{3}{2}\right)$$

$$\text{Equation of normal } \frac{a^2 x}{x_1} - \frac{b^2 y}{y_1} = a^2 - b^2$$

$$\frac{4x}{1} - \frac{3y}{\frac{3}{2}} = 4 - 3$$

$$4x - 2y = 1$$

Q.19 Let a_1, a_2, \dots, a_n be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + \dots + a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \leq n \leq 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to:

- (1) (2480, 248) (2) (2480, 249) (3) (2490, 249) (4) (2490, 248)

Sol. 4

$$a_1 = 1, a_n = 300, 15 \leq n \leq 50$$

$$300 = 1 + (n - 1)d$$

$$(n - 1) = \frac{299}{d}$$

$n - 1 = \text{integer}$, d has to be a factor of 299.

So, $d = 23$ or 13

if $d = 23$

$$\text{then } n - 1 = 13$$

$$n = 14 \text{ (reject)}$$

or $d = 13$

$$n - 1 = 23$$

$$n = 24$$

is possible ($15 \leq n < 50$)

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$$\text{or } S_{20} = \frac{20}{2} \{2 + 19.13\}$$

$$a_{20} = 1 + 19.13$$

$$= 10\{249\} = 2490$$

$$a_{20} = 248$$

$$(S_{20}, a_{20}) = (2490, 248)$$

Q.20 The circle passing through the intersection of the circles, $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 4y = 0$, having its centre on the line, $2x - 3y + 12 = 0$, also passes through the point:

- (1) $(-1, 3)$ (2) $(1, -3)$ (3) $(-3, 6)$ (4) $(-3, 1)$

Sol. (3)

$$S_1 + \lambda(S_1 - S_2) = 0$$

$$x^2 + y^2 - 6x + \lambda(4y - 6x) = 0$$

$$x^2 + y^2 - 6x(1 + \lambda) + 4\lambda y = 0$$

$$\text{Centre } (3(1 + \lambda), -2\lambda) \text{ put in } 2x - 3y + 12 = 0$$

$$6 + 6\lambda + 6\lambda + 12 = 0$$

$$12\lambda = -18$$

$$\lambda = -3/2$$

$$\therefore \text{Circle is } x^2 + y^2 + 3x - 6y = 0$$

Check options

Q.21 Let $\{x\}$ and $[x]$ denote the fractional part of x and the greatest integer $\leq x$ respectively of a real number x . If $\int_0^n \{x\} dx$, $\int_0^n [x] dx$ and $10(n^2 - n)$, ($n \in \mathbb{N}$, $n > 1$) are three consecutive terms of a G.P., then n is equal to _____

Sol. 21

$$\int_0^n \{x\} dx = n \int_0^1 x dx = n \left(\frac{x^2}{2} \right)_0^1 = \frac{n}{2}$$

$$\int_0^n [x] dx = \int_0^1 0 dx + \int_1^2 1 dx + \int_2^3 2 dx \dots + \int_{n-1}^n (n-1) dx$$

$$\Rightarrow 1 + 2 + \dots + n - 1 = \frac{n(n-1)}{2}$$

$$\Rightarrow \frac{n}{2}, \frac{n(n-1)}{2}, 10(n^2 - n) \rightarrow G.P$$

$$\Rightarrow \frac{n^2(n-1)^2}{4} = \frac{n}{2} \cdot 10 \cdot n(n-1)$$

$$\Rightarrow n - 1 = 20$$

$$\Rightarrow n = 21$$

JEE Main 2020 Paper



Q.22 A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is _____

Sol. 135

$${}^6C_4 \times 1 \times 3^2 = 15 \times 9 = 135$$

Q.23 If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, then the value of $|\hat{i} \times (\vec{a} \times \hat{i})|^2 + |\hat{j} \times (\vec{a} \times \hat{j})|^2 + |\hat{k} \times (\vec{a} \times \hat{k})|^2$ is equal to _____

Sol. 18

$$|\hat{i} \times (\vec{a} \times \hat{i})|^2 = |\vec{a} - (a\hat{i})\hat{i}|^2$$

$$= |\hat{j} + 2\hat{k}|^2 = 1 + 4 = 5$$

Similarly

$$|\hat{j} \times (\vec{a} \times \hat{j})|^2 = |2\hat{i} + 2\hat{k}|^2 = 4 + 4 = 8$$

$$|\hat{k} \times (\vec{a} \times \hat{k})|^2 = |2\hat{i} + \hat{j}|^2 = 4 + 1 = 5$$

$$\Rightarrow 5 + 8 + 5 = 18$$

Q.24 Let PQ be a diameter of the circle $x^2 + y^2 = 9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line, $x + y = 2$ respectively, then the maximum value of $\alpha\beta$ is _____

Sol. 7

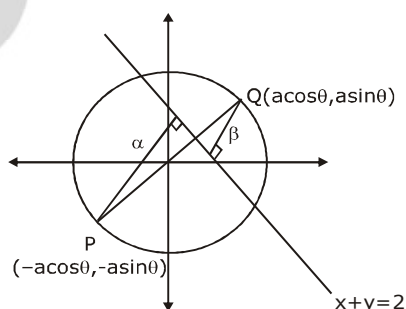
$$\alpha = \left| \frac{3\cos\theta + 3\sin\theta - 2}{\sqrt{2}} \right|$$

$$\beta = \left| \frac{+3\cos\theta + 3\sin\theta + 2}{\sqrt{2}} \right|$$

$$\alpha\beta = \left| \frac{(3\cos\theta + 3\sin\theta)^2 - 4}{2} \right|$$

$$\Rightarrow \alpha\beta = \left| \frac{9 + 9\sin 2\theta - 4}{2} \right| \Rightarrow \alpha\beta = \left| \frac{5 + 9\sin 2\theta}{2} \right|$$

$$\alpha\beta_{\max} = \frac{9+5}{2} = 7$$



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Q.25 If the variance of the following frequency distribution :

Class : 10-20 20-30 30-40

Frequency : 2 x 2

is 50, then x is equal to_____

Sol. 4

$$\text{L.H.S.} = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i} - \left(\frac{\sum f_i (x_i - \bar{x})}{\sum f_i} \right)^2$$

x_i	f_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
15	2	-10	100	200
25	x	0	0	0
35	2	10	100	200
<u>4+x</u>				<u>400</u>

$$\bar{x} = \frac{100 + 25x}{4 + x}$$

$$\bar{x} = 25$$

$$\therefore \frac{400}{4 + x} = 50$$

$$x = 4$$