JEE Main 2020 Paper

Date : 5^{th} September 2020

Time : 09 : 00 am - 12 : 00 pm

Subject : Maths

Q.1 If the volume of a parallelopiped, whose coterminuous edges are given by the vectors $\vec{a} = \hat{i} + \hat{j} + n\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$ and $\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}$ $(n \geq 0)$, is 158 cu. units, then:

(1) $\vec{a}.\vec{c} = 17$ (2) $\vec{b}.\vec{c} = 10$ (3) $n = 9$ (4) $n = 7$

Sol. **2**

$$\begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix} = 158$$

$(12 + n^2) - (6+n) + n(2n-4) = 158$

$3n^2 - 5n + 6 - 158 = 0$

$3n^2 - 5n - 152 = 0$

$3n^2 - 24n + 19n - 152 = 0$

$(3n + 19)(n-8) = 0$

$\Rightarrow n = 8$

$\Rightarrow \vec{b}.\vec{c} = 10$

Q.2 A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:

(1) 63 (2) 54 (3) 38 (4) 36

Sol. 4

C $\rightarrow$ Coffee & T $\rightarrow$ Tea

n(C) = 73, n(T) = 65

n(coffee) = $\frac{73}{100}$

n(tea) = $\frac{65}{100}$

$n(T \cap C) = \frac{x}{100}$

$n(C \cup T) = n(C) + n(T) - x \leq 100$

$= 73 + 65 - x \leq 100$

$\Rightarrow x \geq 38$

$\Rightarrow 73 - x \geq 0 \Rightarrow x \leq 73$

$\Rightarrow 65 - x \geq 0 \Rightarrow x \leq 65$

$\boxed{38 \leq x \leq 65}$, $x = 36$

$x \leq \min(n(C), n(T)) \geq 38 \leq x \leq 65$

Q.3 The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2,4,10,12,14, then the absolute difference of the remaining two observations is:

(1) 1 (2) 4 (3) 3 (4) 2

Sol. **4**

$\text{Var}(x) = \sum \frac{x_i^2}{n} - (\bar{x})^2$

$16 = \frac{x_1^2 + x_2^2 + x_4^2 + x_5^2 + x_6^2 + x_7^2}{7} - 64$

$80 \times 7 = x_1^2 + x_2^2 + x_3^2 + ... + x_7^2$

Now, $x_6^2 + x_7^2 = 560 - (x_1^2 + ...x_5^2)$

$x_6^2 + x_7^2 = 560 - (4 + 16 + 100 + 144 + 196)$

$x_6^2 + x_7^2 = 100$ ......(1)

Now, $\frac{x_1 + x_2 + ... + x_7}{7} = 8$

$x_6 + x_7 = 14$ ......(2)

from (1) & (2)

$(x_6 + x_7)^2 - 2x_6 x_7 = 100$

$2x_6x_7 = 96 \Rightarrow x_6 x_7 = 48$ ......(3)

Now, $|x_6 - x_7| = \sqrt{(x_6 + x_7)^2 - 4x_6 x_7}$

$= \sqrt{196 - 192} = 2$

Q.4 If $2^{10}+2^9.3^1+2^8.3^2+.....+2.3^9+3^{10}=S-2^{11}$, then S is equal to:

(1) 3^{11} (2) $\frac{3^{11}}{2} + 2^{10}$ (3) 2.3^{11} (4) $3^{11} - 2^{12}$

Sol. **1**

let

$S' = 2^{10} + 2^9\, 3^1 + 2^8\, 3^2 + \text{------} + 2.3^9 + 3^{10}$

$\frac{3 \times S'}{2} = 2^9 \times 3^1 + 2^8 . 3^2 + \text{----} + 3^{10} + \frac{3^{11}}{2}$

$\frac{-S'}{2} = 2^{10} - \frac{3^{11}}{2}$

$S' = 3^{11} - 2^{11}$

Now $S' = S - 2^{11}$

$S = 3^{11}$

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Q.5 If $3^{2\sin 2\alpha - 1}$,14 and $3^{4-2\sin 2\alpha}$are the first three terms of an A.P. for some α, then the sixth term of this A.P. is:

(1) 65 (2) 81 (3) 78 (4) 66

Sol. **4**

$28 = 3^{2\sin 2\alpha - 1} + 3^{4 - 2\sin 2\alpha}$

$28 = \frac{9^{\sin 2\alpha}}{3} + \frac{81}{9^{\sin 2\alpha}}$

Let $9^{\sin 2\alpha} = t$

$28 = \frac{t}{3} + \frac{81}{t}$

$t^2 - 84t + 243 = 0$

$t^2 - 81t - 3t + 243 = 0$

$t(t - 81) - 3(t - 81) = 0$

$(t - 81)(t - 3) = 0$

$t = 81, 3$

$9^{\sin 2\alpha} = 9^2$ or 3

$\sin 2\alpha = 1/2, 2$ (rejected)

First term $a = 3^{2\sin 2\alpha - 1}$

$a = 1$

Second term = 14

$\therefore$ Common difference d = 13

$T_6 = a + 5d$

$T_6 = 1 + 5 \times 13 = 66$

Q.6 If the common tangent to the parabolas, $y^2 = 4x$ and $x^2 = 4y$ also touches the circle, $x^2+y^2 = c^2$, then c is equal to:

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2\sqrt{2}}$

Sol. **3**

$y = mx + \frac{1}{m}$

$x^2 = 4\left(mx + \frac{1}{m}\right)$

$x^2 - 4mx - \frac{4}{m} = 0$

$D = 0$

$16m^2 + \frac{16}{m} = 0$

$16\left(\frac{m^3 + 1}{m}\right) = 0$

$m = -1$

$\Rightarrow y + x = -1$

Now, $\left|\frac{-1}{\sqrt{2}}\right| = |c|$

$c = \pm\frac{1}{\sqrt{2}}$

Q.7 If the minimum and the maximum values of the function $f:\left[\frac{\pi}{4},\frac{\pi}{2}\right]\to R$, defined by

$$f(\theta)=\begin{vmatrix}-\sin^2\theta & -1-\sin^2\theta & 1\\ -\cos^2\theta & -1-\cos^2\theta & 1\\ 12 & 10 & -2\end{vmatrix}$$ are m and M respectively, then the ordered pair (m,M) is equal to :

(1) (0,4) (2) (-4,0)

(3) (-4,4) (4) $\left(0,2\sqrt{2}\right)$

Sol. **2**

$$f(\theta)=\begin{vmatrix}-\sin^2\theta & -1-\sin^2\theta & 1\\ -\cos^2\theta & -1-\cos^2\theta & 1\\ 12 & 10 & -2\end{vmatrix}$$

$C_1 \to C_1 - C_2$, $C_3 \to C_3 + C_2$

$$\begin{vmatrix}1 & -1-\sin^2\theta & -\sin^2\theta\\ 1 & -1-\cos^2\theta & -\cos^2\theta\\ 2 & 10 & 8\end{vmatrix}$$

$C_2 \to C_2 - C_3$

$$\begin{vmatrix}1 & -1 & -\sin^2\theta\\ 1 & -1 & -\cos^2\theta\\ 2 & 2 & 8\end{vmatrix}$$

$1(2\cos^2\theta-8) + (8+2\cos^2\theta) - 4\sin^2\theta$

$f(\theta) = 4\cos 2\theta$

Q.8 Let $\lambda \in R$. The system of linear equations

$2x_1-4x_2+\lambda x_3=1$

$x_1-6x_2+x_3=2$

$\lambda x_1-10x_2+4x_3=3$

is inconsistent for:

(1) exactly two values of λ

(2) exactly one negative value of λ.

(3) every value of λ.

(4) exactly one positive value of λ.

Sol. **2**

$$D = \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix}$$

$= 2(3\lambda + 2)(\lambda - 3)$
$D_1 = -2(\lambda - 3)$
$D_2 = -2(\lambda + 1)(\lambda - 3)$
$D_3 = -2(\lambda - 3)$
When $\lambda = 3$, then
$D = D_1 = D_2 = D_3 = 0$
$\Rightarrow$ Infinite many solution
When $\lambda = -2/3$ then D_1, D_2, D_3 none of them is zero so equations are inconsistent
$\therefore \lambda = -2/3$

Q.9 If the point P on the curve, $4x^2+5y^2=20$ is farthest from the point Q(0, -4), then PQ^2 is equal to:
(1) 48 (2) 29 (3) 21 (4) 36

Sol. **4**

Given ellipse is $\frac{x^2}{5} + \frac{y^2}{4} = 1$

Let point P is $\left(\sqrt{5}\cos\theta, 2\sin\theta\right)$

$(PQ)^2 = 5\cos^2\theta + 4(\sin\theta + 2)^2$
$(PQ)^2 = \cos^2\theta + 16\sin\theta + 20$
$(PQ)^2 = -\sin^2\theta + 16\sin\theta + 21$
$= 85 - (\sin\theta - 8)^2$
Will be maximum when $\sin\theta = 1$
$(PQ)^2_{max} = 85 - 49 = 36$

Q.10 The product of the roots of the equation $9x^2-18|x|+5=0$ is :

(1) $\frac{25}{81}$ (2) $\frac{5}{9}$

(3) $\frac{5}{27}$ (4) $\frac{25}{9}$

Sol. **1**
Let $|x| = t$
$9t^2 - 18t + 5 = 0$
$9t^2 - 15t - 3t + 5 = 0$
$(3t - 5)(3t - 1) = 0$

$|x| = \frac{5}{3}, \frac{1}{3}$

$\Rightarrow \quad x = \frac{5}{3}, \frac{-5}{3}, \frac{1}{3}, \frac{-1}{3}$

$\Rightarrow \quad P = \frac{25}{81}$

Q.11 If y = y(x) is the solution of the differential equation $\frac{5+e^x}{2+y}\cdot\frac{dy}{dx}+e^x=0$ satisfying y(0)=1, then a value of $y(\log_e 13)$ is:

(1) 1
(2) 0
(3) 2
(4) −1

Sol. **4**

$$\frac{5+e^x}{2+y}\cdot\frac{dy}{dx}=-e^x$$

$$\int\frac{dy}{2+y}=\int\frac{-e^x}{e^x+5}dx$$

$\ln(y+2)=-\ln(e^x+5)+C$
$(y+2)(e^x+5)=C$
$\because\ y(0)=1$
$\Rightarrow C=18$

$$y+2=\frac{18}{e^x+5}$$

at x = ln13

$$y+2=\frac{18}{13+5}=1$$

y = −1

Q.12 If S is the sum of the first 10 terms of the series $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)+\tan^{-1}\left(\frac{1}{21}\right)+\ldots\ldots$, then tan(S) is equal to :

(1) $\frac{5}{11}$
(2) $\frac{5}{6}$
(3) $-\frac{6}{5}$
(4) $\frac{10}{11}$

Sol. **2**

$$S=\tan^{-1}\left(\frac{1}{1+1\times 2}\right)+\tan^{-1}\left(\frac{1}{1+2\times 3}\right)+\ldots$$

$$T_r=\tan^{-1}\left(\frac{1}{1+r(r+1)}\right)$$

$T_r=\tan^{-1}(r+1)-\tan^{-1}r$
$T_1=\tan^{-1}2-\tan^{-1}1$
$T_2=\tan^{-1}3-\tan^{-1}2$
$T_3=\tan^{-1}4-\tan^{-1}3$
$T_{10}=\tan^{-1}11-\tan^{-1}10$
$\Rightarrow S=(\tan^{-1}2-\tan^{-1}1)+(\tan^{-1}3-\tan^{-1}2)+(\tan^{-1}4-\tan^{-1}3)+\ldots\ldots$
$\ldots.+(\tan^{-1}11-\tan^{-1}10)$

$S = \tan^{-1}11 - \tan^{-1}1$

$= \tan^{-1}\left(\frac{11-1}{1+11}\right)$

$\Rightarrow \tan S = \frac{10}{12} = \frac{5}{6}$

Q.13 The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+e^{\sin x}}dx$ is:

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$ (3) π (4) $\frac{3\pi}{2}$

Sol. 1

$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+e^{\sin x}}dx$

$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{e^{\sin x}}{1+e^{\sin x}}dx \quad \Rightarrow 2I = \pi$

$I = \frac{\pi}{2}$

Q.14 If (a, b, c) is the image of the point (1,2,-3) in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then a+b+c is

(1) 2 (2) 3 (3) -1 (4) 1

Sol. 1

$\overrightarrow{PM} \perp (2\hat{i} - 2\hat{j} - \hat{k})$

$\Rightarrow (2\lambda-2).2 + (1-2\lambda)(-2) + (3-\lambda)(-1) = 0$

$\Rightarrow 4\lambda - 4 + 4\lambda - 2 + \lambda - 3 = 0$

$\Rightarrow 9\lambda = 9 \Rightarrow \lambda = 1$

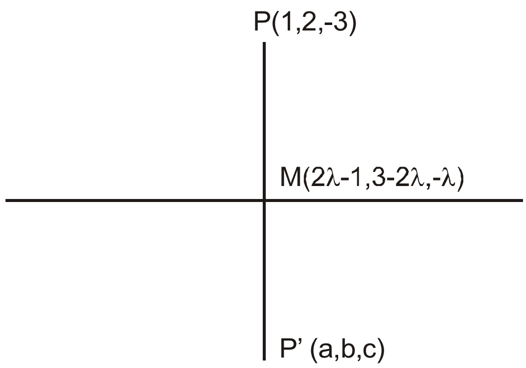
$\Rightarrow M(1,1,-1)$

Now, $P' = 2M - P$

$= 2(1,1,-1) - (1, 2, -3)$

$= (1, 0, 1)$

$a + b + c = 2$



Q.15 If the function $f(x) = \begin{cases} k_1(x-\pi)^2 - 1, & x \le \pi \\ k_2 \cos x, & x > \pi \end{cases}$ is twice differentiable, then the ordered pair (k_1, k_2) is equal to:

(1) (1,1) (2) (1,0) (3) $\left(\frac{1}{2}, -1\right)$ (4) $\left(\frac{1}{2}, 1\right)$

Sol. **4**

f(x) is continuous and differentiable

$f(\pi^-) = f(\pi) = f(\pi^+)$

$-1 = -K_2$

$\Rightarrow \quad K_2 = 1$

$$f'(x) = \begin{cases} 2k_1(x-\pi); & x \le \pi \\ -k_2 \sin x & ; x > \pi \end{cases}$$

$f'(\pi^-) = f'(\pi^+)$

$\Rightarrow 0 = 0$

So differentiable at x = 0

$$f''(x) = \begin{cases} 2k_1 & ; x \le \pi \\ -k_2 \cos x; & x > \pi \end{cases}$$

$f''(\pi^-) = f''(\pi^+)$

$2k_1 = k_2$

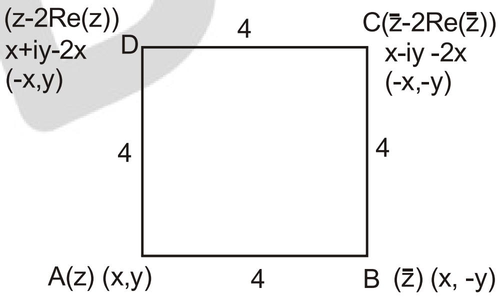
$k_1 = 1/2$

Q.16 If the four complex numbers $z, \bar{z}, \bar{z} - 2\mathrm{Re}(\bar{z})$ and $z - 2\mathrm{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then |z| is equal to:

(1) 2 (2) 4 (3) $4\sqrt{2}$ (4) $2\sqrt{2}$

Sol. **4**

Coordinates of A, B, C, D is



Let z = x + iy

$CA^2 = AB^2 + BC^2$

$2^2x^2 + 2^2y^2 = 32$

$x^2 + y^2 = 8$

$\sqrt{x^2 + y^2} = 2\sqrt{2}$

Q.17 If $\int\left(e^{2x}+2e^{x}-e^{-x}-1\right)e^{\left(e^{x}+e^{-x}\right)}dx=g(x)e^{\left(e^{x}+e^{-x}\right)}+c$, where c is a constant of integration, then g(0) is equal to :

(1) 2 (2) e (3) 1 (4) e^2

Sol. **1**

$$\int\left(e^{2x}+2e^{x}-e^{-x}-1\right)e^{\left(e^{x}+e^{-x}\right)}dx$$

$$\int\left(e^{2x}+e^{x}-1\right)e^{\left(e^{x}+e^{-x}\right)}dx+\int\left(e^{x}-e^{-x}\right)e^{\left(e^{x}+e^{-x}\right)}dx$$

$$\int\left(e^{x}+1-e^{-x}\right)e^{\left(e^{x}+e^{-x}+x\right)}dx+\int\left(e^{x}-e^{-x}\right)e^{\left(e^{x}+e^{-x}\right)}dx$$

$$e^{\left(e^{x}+e^{-x}+x\right)}+e^{e^{x}+e^{-x}}+c$$

$$\left(e^{e^{x}+e^{-x}}\right)\left[e^{x}+1\right]+c$$

$$\Downarrow$$

$$g(x)$$

$\Rightarrow$ g(0) = 2

Q.18 The negation of the Boolean expression $x\leftrightarrow\sim y$ is equivalent to :

(1) $(x\wedge y)\wedge(\sim x\vee\sim y)$

(2) $(x\wedge y)\vee(\sim x\wedge\sim y)$

(3) $(x\wedge\sim y)\vee(\sim x\wedge y)$

(4) $(\sim x\wedge y)\vee(\sim x\wedge\sim y)$

Sol. **2**

As we know

$$p\leftrightarrow q=(p\to q)\wedge(q\to p)$$

$$\sim(p\leftrightarrow q)=(p\wedge\sim q)\vee(\sim p\wedge q)$$

$\Rightarrow$ so, $\sim(x\leftrightarrow\sim y)=(x\wedge y)\vee(\sim x\wedge\sim y)$

Q.19 If α is positive root of the equation, p(x) $=x^2-x-2=0$, then $\lim\limits_{x\to\alpha^{+}}\frac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{3}{\sqrt{2}}$ (3) $\frac{3}{2}$ (4) $\frac{1}{\sqrt{2}}$

Sol. **2**

$f(x) = x^2 - x - 2 \left\langle {}^{2}_{-1} = \alpha \right.$

$$\lim_{x\to 2^+} \frac{\sqrt{1-\cos(x-2)(x+1)}}{x+\alpha-4}$$

$$\lim_{x\to 2+} \frac{\sqrt{1-\cos(x-2)(x+1)}}{(x-2)}$$

$$\lim_{h\to 0} \frac{\sqrt{1-\cos(\mathrm{h}\times(\mathrm{h}+3))}}{h}$$

$$\lim_{h\to 0} \sqrt{\frac{1-\cos(\mathrm{h}\ (\mathrm{h}+3))}{h^2\times(h+3)^2}\times(h+3)^2} \Rightarrow \sqrt{\frac{1}{2}\times 9} = \frac{3}{\sqrt{2}}$$

Q.20 If the co-ordinates of two points A and B are $\left(\sqrt{7},0\right)$ and $\left(-\sqrt{7},0\right)$ respectively and P is any point on the conic, $9x^2+16y^2=144$, then PA+PB is equal to :

(1) 6 (2) 16 (3) 9 (4) 8

Sol. **4**

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

$$e = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}$$

$$F_1\left(\sqrt{7},0\right), F_2\left(-\sqrt{7},0\right)$$

$PF_1 + PF_2 = 2a$

$PA + PB = 2 \times 4 = 8$

Q.21 The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m + \frac{1}{x^2}\right)^{22}$ is 1540, is ....

Sol. 13

$$T_{r+1} = {}^{22}C_r \left(x^m\right)^{22-r}\left(\frac{1}{x^2}\right)^r$$

$$= {}^{22}C_r \left(x\right)^{22m-mr-2r}$$

Given ${}^{22}C_r = 1540 = {}^{22}C_{19} \Rightarrow r=19$

$\because 22m - rm - 2r = 1$

$$\Rightarrow m = \frac{2r+1}{22-r}$$

$m = 13$(At $r=19$)

Q.22 Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is ....

Sol. **11**

P(at 2, 3 or 4) = $^4C_2\left(\frac{2}{6}\right)^2\left(\frac{4}{6}\right)^2 + {}^4C_3\left(\frac{2}{6}\right)^3\left(\frac{4}{6}\right)^1 + {}^4C_4\left(\frac{2}{6}\right)^4$

$= 6 \times \frac{1}{9}\times\frac{4}{9} + 4 \times \frac{1}{27}\times\frac{2}{3}+\frac{1}{81}$

$=\frac{33}{81}=\frac{11}{27} \Rightarrow nP \quad \Rightarrow 11$

Q.23 Let $f(x) = x.\left[\frac{x}{2}\right]$, for-10<x<10, where [t] denotes the greatest integer function. Then the number of points of discontinuity of f is equal to...........

Sol. **8**

$f(x) = x\left[\frac{x}{2}\right]$

$x \in (-10, 10)$

$\frac{x}{2} \in (-5, 5) \rightarrow 9$ integers

Check continuity at x = 0

$$\left.\begin{matrix} f(0)=0 \\ f(0^+)=0 \\ f(0^-)=0 \end{matrix}\right\}\text{Continous at } x = 0$$

Function will be discontinuous when

$\Rightarrow \quad \frac{x}{2} = \pm4, \pm3, \pm2, \pm1$

$-4, -3, -2, -1, 0, 1, 2, 3, 4$

$\Rightarrow$ 8 Point of discontinuity

Q.24 The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is

Sol. **240**

SS, Y, LL, A, B, U

ABCC type words

$$= \underbrace{^2C_1}_{\substack{\text{selection of}\\\text{two alike}\\\text{letters}}} \times \underbrace{^5C_2}_{\substack{\text{selection of}\\\text{two distinct}\\\text{letters}}} \times \underbrace{\frac{\lfloor 4}{\lfloor 2}}_{\substack{\text{arrangement of}\\\text{selected letters}}}$$

= 240

Q.25 If the line, 2x-y+3=0 is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines 4x-2y+α=0 and 6x-3y+β=0, respectively, then the sum of all possible values of α and β is

Sol. **30**

$L_1 : 2x - y + 3 = 0$

$L_2 : 4x - 2y + \alpha = 0$

$L_3 : 6x - 3y + \beta = 0$

$$\frac{\left|\frac{\alpha}{2} - 3\right|}{\sqrt{5}} = \frac{1}{\sqrt{5}} \qquad \Rightarrow \frac{\alpha}{2} - 3 = 1, -1$$

$$\Rightarrow \alpha = 8, 4$$

$$\frac{\left|\frac{\beta}{3} - 3\right|}{\sqrt{5}} = \frac{2}{\sqrt{5}} \qquad \Rightarrow \frac{\beta}{3} - 3 = 2, -2$$

$$\Rightarrow \beta = 15, 3$$