JEE Main 2020 Paper

Date : 3rd September 2020

Time : 09 : 00 am - 12 : 00 pm

Subject : Maths

Q.1 The value of $(2.^1P_0 - 3.^2P_1 + 4.^3P_2 - \ldots$ up to 51^{th} term$) + (1! - 2! + 3! - \ldots$ up to 51^{th} term$)$ is equal to:

(1) 1-51(51)! (2) 1+(52)! (3) 1 (4) 1+ (51)!

Sol. **2**

$2.\ ^1P_0 = |\underline{2}$

$3.\ ^2P_1 = |\underline{3}$

$4.\ ^3P_2 = |\underline{4}$

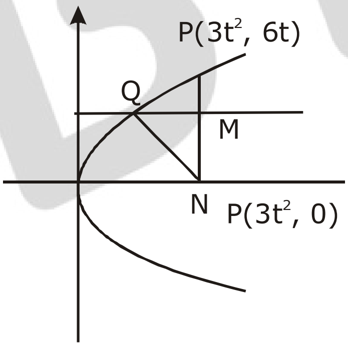
$(|\underline{2} - |\underline{3} + |\underline{4} - |\underline{5} + \ldots\ldots |\underline{52}) + (|\underline{1} - |\underline{2} + |\underline{3} - |\underline{4} \ldots\ldots + |\underline{51})$

$= |\underline{52} + 1$

Q.2 Let P be a point on the parabola, $y^2=12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then:

(1) PN=4 (2) $MQ=\frac{1}{3}$ (3) PN=3 (4) $MQ=\frac{1}{4}$

Sol. **4**



Q (h, 3t) lie on
Parabola
$9t^2 = 12h$

$h = \frac{3t^2}{4}$

$Q = \left(\frac{3t^2}{4}, 3t\right)$

Equation of NQ

$$y = \frac{3t}{\left(\frac{3t^2}{4} - 3t^2\right)} \quad (x - 3t^2)$$

$$y = \frac{-4t}{3t^2}\left(x - 3t^2\right)$$

put x = 0

$$y = \frac{-4}{3t}\left(-3t^2\right) = 4t$$

$$4t = \frac{4}{3} \Rightarrow t = \frac{1}{3}$$

$$PN = 6t = 6.\frac{1}{3} = 2$$

$$M = \left[\frac{1}{3}, 1\right], Q\left[\frac{1}{12}, 1\right]$$

$$MQ = \frac{1}{3} - \frac{1}{12} = \frac{1}{4}$$

Q.3 If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3+Bx^2+Cx+D$, then B+C is equal to:

(1) 1 (2) -1 (3) -3 (4) 9

Sol. **3**

$$\Rightarrow \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3+Bx^2+Cx+D$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_2$

$$\Rightarrow \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-1 & x-1 & x-1 \\ x-2 & 2(x-2) & 6(x-2) \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$$

$$\Rightarrow (x-1)(x-2)\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & 1 & 1 \\ 1 & 2 & 6 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$$

$\Rightarrow (x-1)(x-2)\{(x-2)(6-2)-(2x-3)(6-1)+(3x-4)(2-1)\} = Ax^3+Bx^2+Cx+D$

$\Rightarrow (x-1)(x-2)\{4x-8-10x+15+3x-4\} = Ax^3+Bx^2+Cx+D$

$\Rightarrow (x-1)(x-2)(-3x+3) = Ax^3+Bx^2+Cx+D$

$\Rightarrow -3(x-1)^2(x-2) = Ax^3+Bx^2+Cx+D$

$\Rightarrow -3x^3+12x^2-15x+6 = Ax^3+Bx^2+Cx+D$

comparing both side

$A=-3,\ B=12,\ C=-15$

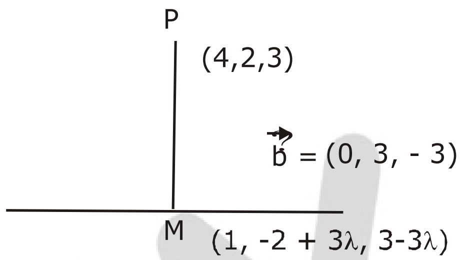
$B+C = 12-15 = -3$

Q.4 The foot of the perpendicular drawn from the point (4,2,3) to the line joining the points (1,-2,3) and (1,1,0) lies on the plane:

(1) x-y-2z=1 (2) x-2y+z=1 (3)2x+y-z=1 (4) x+2y-z=1

Sol. **3**

$\vec{r} = (1,-2,3)+\lambda(0,3,-3)$



$\overrightarrow{PM} \perp \vec{b}$

$\overrightarrow{PM}.\ \vec{b} = 0$

$(-3,\ 3\lambda-4,\ -3\lambda).\ (0,\ 3,\ -3) = 0$

$\Rightarrow 0 + 9\lambda - 12 + 9\lambda = 0 \Rightarrow \lambda = \frac{12}{18} = \frac{2}{3}$

m = (1, 0, 1) are on 2x + y - z = 1

Q.5 If $y^2+\log_e(\cos^2 x)=y,\ x \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, then

(1) $|y'(0)|+|y''(0)|=1$ (2) $y''(0)=0$
(3) $|y'(0)|+|y''(0)|=3$ (4) $|y''(0)|=2$

Sol. **4**

$2yy' + 2(-\tan x) = y'$ ....(1)

diff. w.r.t.x

$2yy'' + 2(y')^2 - 2\sec^2 x = y''$ ....(2)

Put x = 0 in given equation we get y = 0, 1

from (1) $x=0,\ y=0 \Rightarrow y'(0)=0$

$x=0,\ y=1, \Rightarrow y'(0)=0$

from (2) $x=0,\ y=0,\ y'(0)=0 \Rightarrow y''(0)=-2$

$x=0,\ y=1,\ y'(0)=0 \Rightarrow y''(0)=2$

$|y''(0)| = 2$

Q.6 $2\pi-\left(\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{65}\right)$ is equal to:

(1) $\frac{5\pi}{4}$ (2) $\frac{3\pi}{2}$ (3) $\frac{7\pi}{4}$ (4) $\frac{\pi}{2}$

Sol. **2**

$$=2\pi-\left[\tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\frac{16}{63}\right]$$

$$=2\pi-\tan^{-1}\left(\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}.\frac{5}{12}}\right)-\tan^{-1}\left(\frac{16}{63}\right)$$

$$=2\pi-\tan^{-1}\left(\frac{48+15}{36-20}\right)-\tan^{-1}\left(\frac{16}{63}\right)$$

$$=2\pi-\left[\tan^{-1}\left(\frac{63}{16}\right)+\cot^{-1}\left(\frac{63}{16}\right)\right]$$

$$=2\pi-\frac{\pi}{2}$$

$$=\frac{3\pi}{2}$$

Q.7 A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2+4y^2=12$, then this hyperbola does not pass through which of the following points ?

(1) $\left(\sqrt{\frac{3}{2}},\frac{1}{\sqrt{2}}\right)$ (2) $\left(1,-\frac{1}{\sqrt{2}}\right)$ (3) $\left(\frac{1}{\sqrt{2}},0\right)$ (4) $\left(-\sqrt{\frac{3}{2}},1\right)$

Sol. **1**

$$\frac{x^2}{4}+\frac{y^2}{3}=1$$

$$b_1^2=a_1^2\left(1-e_1^{\,2}\right)$$

$3=4(1-e_1^2)$

$$e_1=\frac{1}{2}$$

focus $=(\pm a_1e_1, 0)$
$=(\pm 1, 0)$

Length of transverse axis $2a_2 = \sqrt{2} \rightarrow a_2 = \frac{1}{\sqrt{2}}$

$a_2e_2 = 1$

$= e_2 = \sqrt{2}$

$b_2^2 = a_2^2\,(e_2^2 - 1)$

$b_2^2 = \frac{1}{2}(2-1) = \frac{1}{2}$

equation of Hyperbola

$x^2 - y^2 = \frac{1}{2}$

Q.8 For the frequency distribution:

Variate(x):	x_1	x_2	$x_3 \ldots\ldots x_{15}$
Frequency(f):	f_1	f_2	$f_3 \ldots\ldots f_{15}$

where $0<x_1<x_2<x_3<\ldots.<x_{15} \le 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be:

(1) 1 (2) 4 (3) 6 (4) 2

Sol. **3**

$\sigma^2 \le \frac{1}{4}(M-m)^2$

(M = upper bound of value of any random variable,
m = Lower bound of value of any random variable)

$\sigma^2 \le \frac{1}{4}(10-0)^2$

$\sigma^2 < 25$

$-5 < \sigma < 5$

$\sigma \ne 6$

Q.9 A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is:

(1) $\frac{1}{3}$ (2) $\frac{1}{4}$ (3) $\frac{1}{8}$ (4) $\frac{1}{9}$

Sol. **4**

Total Possibilities = (1, 3), (3, 1), (2, 2),
(2, 6), (6, 2) (4, 4)
(3, 5), (5, 3) (6, 6)

$(A \cap B) = (4,4), n(A \cap B) = 1$

Required probability $\equiv P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1}{9}$

Q.10 If the number of integral terms in the expansion of $\left(3^{\frac{1}{2}}+5^{\frac{1}{8}}\right)^{n}$ is exactly 33, then the least value of n is:

(1) 128 (2) 248 (3) 256 (4) 264

Sol. **3**

$T_{r+1} = {}^{n}C_{r}\left(3^{\frac{1}{2}}\right)^{n-r}\left(5^{\frac{1}{8}}\right)^{r}$

$\frac{n-r}{2} \rightarrow$ n - r = 0, 2, 4, 6, 8, ...

$\frac{r}{8} \rightarrow r = 0, 8, 16, 24$.................

common r = 0, 8, 16, 24..........

no. of integral term = 33.

L = 0 + (33 - 1) × 8 → L = 32 × 8

= 256

Q.11 $\int_{-\pi}^{\pi}|\pi-|x||\,dx$ is equal to:

(1) π^2 (2) $\frac{\pi^2}{2}$ (3) $\sqrt{2}\pi^2$ (4) $2\pi^2$

Sol. **1**

$\int_{-\pi}^{\pi}\left|\pi-|x|\right|dx$

even function

$2\int_{0}^{\pi}|\pi-x|\,dx$

$= 2\int_{0}^{\pi}(\pi-x)\,dx \Rightarrow 2\left[\pi x-\frac{x^2}{2}\right]_0^{\pi}$

$= 2\left[\frac{\pi^2}{2}\right] = \pi^2$

Q.12 Consider the two sets:

A={m ∈ R : both the roots of $x^2-(m+1)x+m+4=0$ are real} and B=[-3,5).

Which of the following is not true ?

(1) $A-B=(-\infty,-3)\cup(5,\infty)$ (2) $A\cap B=\{-3\}$

(3) B-A=(-3,5) (4) $A\cup B=R$

Sol. **1**

$D \geq 0$

$(m+1)^2-4(m+4)\geq 0$

$\Rightarrow m^2-2m-15\geq 0$

$(m - 5)(m + 3) \geq 0$
$m \in (-\infty, -3] \cup [5, \infty)$
$A = (-\infty, -3] \cup [5, \infty)$
$B = [-3, 5)$
$A - B = (-\infty, -3) \cup [5, \infty)$
$A \cup B = R$

Q.13 The proposition $p \to \sim(p \wedge \sim q)$ is equivalent to :

(1) $(\sim p) \vee (\sim q)$
(2) $(\sim p) \wedge q$
(3) q
(4) $(\sim p) \vee q$

Sol. **4**

$\sim(p \wedge \sim q) \to \sim p \vee q$
$p \to (\sim p \vee q)$
$\Rightarrow \sim p \vee (\sim p \vee q)$
$\Rightarrow \sim p \vee q$

Q.14 The function, $f(x) = (3x-7)x^{2/3}$, $x \in R$ is increasing for all x lying in:

(1) $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$
(2) $\left(-\infty, \frac{14}{15}\right)$
(3) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$
(4) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$

Sol. **3**

$$f'(x) = (3x - 7).\frac{2}{3x^{\frac{1}{3}}} + x^{\frac{2}{3}}.3$$

$$= \frac{6x - 14 + 9x}{3x^{\frac{1}{3}}}$$

$$= \frac{15x - 14}{3x^{\frac{1}{3}}}$$

+	-	+
	0	14/15

$f(x) > 0 \uparrow \Rightarrow x \in (-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

Q.15 If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is:

(1) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{7}$

Sol. **1**

a = 3

$$\frac{25}{2}[2a+24d] = \frac{15}{2}[2\times(a+25d)+14d]$$

$\Rightarrow$ 50a + 600d = 15 [2a + 50d + 14d]

$\Rightarrow$ 20a + 600d = 960d

$\Rightarrow$ 60 = 360d

d = $\frac{1}{6}$

Q.16 The solution curve of the differential equation, $(1+e^{-x})(1+y^2)\frac{dy}{dx} = y^2$, which passes through the point (0,1), is:

(1) $y^2 = 1 + y\log_e\left(\frac{1+e^{-x}}{2}\right)$ (2) $y^2 + 1 = y\left(\log_e\left(\frac{1+e^{-x}}{2}\right)+2\right)$

(3) $y^2 + 1 = y\left(\log_e\left(\frac{1+e^{x}}{2}\right)+2\right)$ (4) $y^2 = 1 + y\left(\log_e\left(\frac{1+e^{x}}{2}\right)\right)$

Sol. **4**

$$\int\left(\frac{1+y^2}{y^2}\right)dy = \int\left(\frac{1}{1+e^{-x}}\right)dx$$

$$\int\frac{1}{y^2}dy + \int dy = \int\left(\frac{e^x}{e^x+1}\right)dx$$

$$\Rightarrow \frac{-1}{y} + y = \ln|e^x+1| + c$$

x = 0, y = 1

$\Rightarrow$ -1 + 1 = ln2 + c $\Rightarrow$ c = -ln2

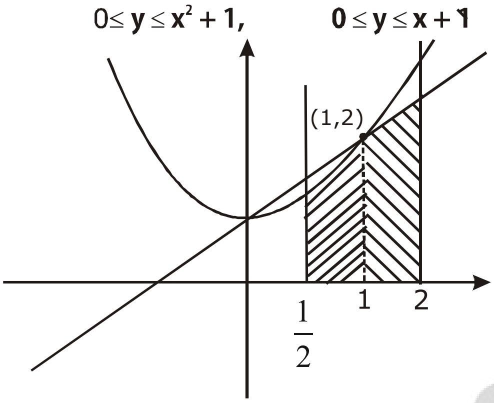
$$\Rightarrow \frac{-1}{y} + y = \ln|e^x+1| - \ln 2$$

$$\Rightarrow y^2 = 1 + y\left[\ln\left(\frac{e^x+1}{2}\right)\right]$$

Q.17 The area (in sq. units) of the region $\left\{(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, \frac{1}{2} \le x \le 2\right\}$ is

(1) $\frac{23}{16}$ (2) $\frac{79}{16}$ (3) $\frac{23}{6}$ (4) $\frac{79}{24}$

Sol. **4**



$$A = \int_{\frac{1}{2}}^{1} (x^2 + 1) dx + \int_{1}^{2} (x + 1) dx$$

$$\left(\frac{x^3}{3} + x\right)_{\frac{1}{2}}^{1} + \left(\frac{x^2}{2} + x\right)_{1}^{2}$$

$$= \left(\frac{1}{3} + 1\right) - \left(\frac{1}{24} + \frac{1}{2}\right) + \left((2 + 2) - \left(\frac{3}{2}\right)\right)$$

$$= \left(\frac{4}{3} - \frac{13}{24}\right) + \left(\frac{5}{2}\right)$$

$$= \left(\frac{32 - 13}{24}\right) + \left(\frac{5}{2}\right) = \frac{19 + 60}{24} = \frac{79}{24}$$

Q.18 If α and β are the roots of the equation $x^2+px+2=0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2+2qx+1=0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to :

(1) $\frac{9}{4}(9 + p^2)$ (2) $\frac{9}{4}(9 + q^2)$ (3) $\frac{9}{4}(9 - p^2)$ (4) $\frac{9}{4}(9 - q^2)$

Sol. **3**

$\alpha + \beta = -p, \alpha\beta = 2$

$$\frac{1}{\alpha} + \frac{1}{\beta} = -q, \ \frac{1}{\alpha\beta} = \frac{1}{2}$$

$$\frac{\alpha+\beta}{\alpha\beta} = -q \Rightarrow \frac{-p}{2} = -q$$

$\Rightarrow p = 2q$

$$\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right) = \alpha\beta + \frac{1}{\alpha\beta} + 2$$

$$= 2 + \frac{1}{2} + 2 = \frac{9}{2}$$

$$\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right) = \alpha\beta + \frac{1}{\alpha\beta} - \frac{\alpha}{\beta} - \frac{\beta}{\alpha}$$

$$= 2 + \frac{1}{2} - \left[\frac{\alpha^2+\beta^2}{\alpha\beta}\right]$$

$$= \frac{5}{2} - \left[\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}\right]$$

$$= \frac{5}{2} - \left[\frac{p^2-4}{2}\right]$$

$$= \frac{9-p^2}{2}$$

$$\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right) = \left(\frac{9-p^2}{2}\right)\left(\frac{9}{2}\right)$$

$$= \frac{9}{4}\left(9-p^2\right)$$

Q.19 The lines $\vec{r} = (\hat{i}-\hat{j}) + l(2\hat{i}+\hat{k})$ and $\vec{r} = (2\hat{i}-\hat{j}) + m(\hat{i}+\hat{j}-\hat{k})$

(1) do not intersect for any values of l and m
(2) intersect when l=1 and m=2
(3) intersect when l=2 and m=$\frac{1}{2}$
(4) intersect for all values of l and m

Sol. **1**

$\vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$ ...(1)

$\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$ ....(2)

Let two lines $\vec{r} = \vec{a} + t\vec{b}$

and $\vec{r} = \vec{c} + t\vec{d}$

shortest distance $(d) = \dfrac{\left|(\vec{a} - \vec{c}).(\vec{b} \times \vec{d})\right|}{\left|\vec{b} \times \vec{d}\right|}$

$$= \frac{\left|\{(\hat{i} - \hat{j}) - (2\hat{i} - \hat{j})\}.\{(2\hat{i} + \hat{k}) \times (\hat{i} + \hat{j} + \hat{k})\}\right|}{\left|(2\hat{i} + \hat{k}) \times (\hat{i} + \hat{j} + \hat{k})\right|}$$

$$= \frac{\left|(-\hat{i}).(-\hat{i} - \hat{j} + 2\hat{k})\right|}{\left|-\hat{i} - \hat{j} + 2\hat{k}\right|}$$

$$= \frac{1}{\sqrt{(-1)^2 + (-1)^2 + 2^2}}$$

$$= \frac{1}{\sqrt{6}}$$

Since shortest distance between these lines exist
Hence lines do not intersect.

Q.20 Let [t] denote the greatest integer $\leq$ t. if for some $\lambda \in R - \{0, 1\}$

$\lim\limits_{x \to 0} \left|\dfrac{1 - x + |x|}{\lambda - x + [x]}\right| = L$, then L is equal to:

(1) 0 (2) 2 (3) $\frac{1}{2}$ (4) 1

Sol. **2**

$$\lim_{x \to 0} \left|\frac{1 - x + |x|}{\lambda - x + [x]}\right| = L$$

RHL : $\lim\limits_{x \to 0^+} \left|\dfrac{1 - x + x}{\lambda - x + [x]}\right|$

RHL : $\lim_{h\to 0}\left|\dfrac{1-h+h}{\lambda-h+[h]}\right|$

$\lim_{h\to 0}\left|\dfrac{1}{\lambda-h+0}\right| = \left|\dfrac{1}{\lambda}\right|$, [h] = 0

LHL : $\lim_{x\to 0^-}\left|\dfrac{1-x+x}{\lambda-x+[x]}\right|$

LHL : $\lim_{h\to 0}\left|\dfrac{1+h-h}{\lambda+h+[-h]}\right|$

$= \left|\dfrac{1}{\lambda-1}\right|$,[-h] = -1

$\because\ |\lambda| = |\lambda - 1|$

$\lambda^2 = \lambda^2 - 2\lambda + 1 \Rightarrow \lambda = \dfrac{1}{2}$

L = 2

Q.21 If $\lim_{x\to 0}\left\{\dfrac{1}{x^8}\left(1-\cos\dfrac{x^2}{2}-\cos\dfrac{x^2}{4}+\cos\dfrac{x^2}{2}\cos\dfrac{x^2}{4}\right)\right\} = 2^{-k}$, then the vlaue of k is ....

Sol. **8**

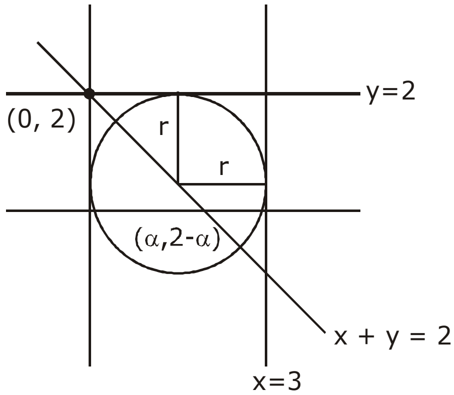
$$\lim_{x\to 0}\frac{\left(1-\cos\dfrac{x^2}{2}\right)}{\left(\dfrac{x^2}{2}\right)^2}\frac{\left(1-\cos\dfrac{x^2}{4}\right)}{\left(\dfrac{x^2}{4}\right)^2}\cdot\frac{\left(\dfrac{x^2}{2}\right)^2\cdot\left(\dfrac{x^2}{4}\right)^2}{x^8}$$

$$\lim_{x\to 0}\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{16} \Rightarrow \frac{1}{256} = 2^{-k}$$

$2^{-8} = 2^{-k} \Rightarrow k = 8$

Q.22 The diameter of the circle, whose centre lies on the line x + y = 2 in the first quadrant and which touches both the lines x=3 and y=2, is ....

Sol. **3**



p = r

for y = 2

$$r = \left|\frac{2-\alpha-2}{1}\right| = |\alpha|$$

for x = 3

$$r = \left|\frac{\alpha-3}{1}\right| = |\alpha - 3|$$

$|\alpha| = |\alpha - 3|$

$\Rightarrow \alpha^2 + \alpha^2 - 6\alpha + 9 \Rightarrow \alpha = \frac{3}{2}$

$2\alpha = 3 = 2r$

Q.23 The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^2}+\text{.........to } \infty\right)}$ is equal to..............

Sol. **4**

$$\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\text{........}\infty = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$$

$$\log_{2.5}\left(\frac{1}{2}\right) \Rightarrow \log_{\frac{5}{2}}\frac{1}{2}$$

$$.16 = \frac{16}{100} = \frac{4}{25} = \left(\frac{2}{5}\right)^2$$

$$\Rightarrow \left(\frac{2}{5}\right)^{2\log_{\frac{5}{2}}\frac{1}{2}} = \left(\frac{5}{2}\right)^{-2\log_{\frac{5}{2}}\frac{1}{2}}$$

$$\Rightarrow \left(\frac{5}{2}\right)^{\log_{\frac{5}{2}}\left(\frac{1}{2}\right)^{-2}}$$

$= 4$

Q.24 Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}, x \in R$ and $A^4 = [a_{ij}]$. If a_{11}=109, then a_{22} is equal to ....

Sol. **10**

$$A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} x^3+x+x & x^2+1 \\ x^2+1 & x \end{bmatrix}$$

$$A^4 = \begin{bmatrix} x^3+2x & x^2+1 \\ x^2+1 & x \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x^4+2x^2+x^2+1 & x^3+2x \\ x^3+x+x & x^2+1 \end{bmatrix}$$

$a_{11} \Rightarrow\ x^4 + 3x^2 + 1 = 109$
$x^4 + 3x^2 - 108 = 0$
$\Rightarrow (x^2 + 12)(x^2 - 9) = 0$
$x = \pm 3$
$a_{22} = x^2 + 1 = 10$

Q.25 If $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}} = \left(\frac{1+i}{i-1}\right)^{\frac{n}{3}} = 1$, (m, n $\in$ N) then the greatest common divisor of the least values of m and n is ....

Sol. **4**

$$\left[\frac{(1+i)(1+i)}{(1+i)(1-i)}\right]^{\frac{m}{2}} = \left[\left(\frac{1+i}{-1+i}\right)\left(\frac{-1-i}{-1-i}\right)\right]^{\frac{n}{3}} = 1, \qquad \left(i=\sqrt{-1}\right)$$

$$= \left(\frac{2i}{2}\right)^{\frac{m}{2}} = 1 \quad \Bigg| \quad \left(\frac{-1-i-i+1}{1+1}\right)^{\frac{n}{3}} = 1$$

m = 8

$(-i)^{n/3} = 1 \qquad \left(i=\sqrt{-1}\right)$

n = 12

greatest common divisor of m & n is 4