JEE Main 2020 Paper

Date of Exam: **7th January 2020 (Shift 1)**
Time: **9:30 A.M. to 12:30 P.M.**
Subject: **Mathematics**

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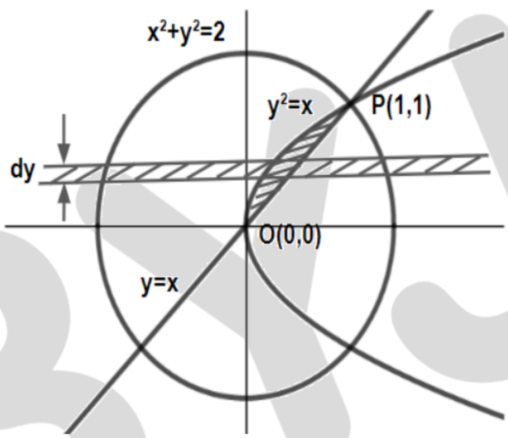
1. The area of the region, enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$, is

   a. $\frac{1}{3}(12\pi - 1)$
   b. $\frac{1}{6}(12\pi - 1)$
   c. $\frac{1}{3}(6\pi - 1)$
   d. $\frac{1}{6}(24\pi - 1)$

   Answer: (b)

   Solution:

   Required area = area of the circle – area bounded by given line and parabola



   Required area = $\pi r^2 - \int_0^1 (y - y^2)\, dy$

   Area $= 2\pi - \left(\frac{y^2}{2} - \frac{y^3}{3}\right)_0^1 = 2\pi - \frac{1}{6} = \frac{1}{6}(12\pi - 1)$ sq. units

2. Total number of six-digit numbers in which only and all the five digits 1,3,5,7 and 9 appear, is

   a. 5^6
   b. $\frac{1}{2}(6!)$
   c. $6!$
   d. $\frac{5}{2}(6!)$

   Answer: (d)

   Solution:

   Selecting all 5 digits $= {}^5C_5 = 1$ way

   Now, we need to select one more digit to make it a 6 digit number $= {}^5C_1 = 5$ ways

Total number of permutations $= \frac{6!}{2!}$

Total numbers $= {}^5C_5 \times {}^5C_1 \times \frac{6!}{2!} = \frac{5}{2}(6!)$

3. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for $k = 3, 4, 5$, otherwise X takes the value -1. The expected value of X, is

a. $\frac{1}{8}$

b. $\frac{3}{16}$

c. $-\frac{1}{8}$

d. $-\frac{3}{16}$

Answer: (a)

Solution:

$k =$ no. of consecutive heads

$P(k = 3) = \frac{5}{32}$ (HHHTH, HHHTT, THHHT, HTHHH, TTHHH)

$P(k = 4) = \frac{2}{32}$ (HHHHT, HHHHT)

$P(k = 5) = \frac{1}{32}$ (HHHHH)

$P(\bar{3} \cap \bar{4} \cap \bar{5}) = 1 - \left(\frac{5}{32} + \frac{2}{32} + \frac{1}{32}\right) = \frac{24}{32}$

$\sum XP(X) = \left(-1 \times \frac{24}{32}\right) + \left(3 \times \frac{5}{32}\right) + \left(4 \times \frac{2}{32}\right) + \left(5 \times \frac{1}{32}\right) = \frac{1}{8}$

4. If $\text{Re}\left(\frac{z-1}{2z+i}\right) = 1$, where $z = x + iy$, then the point (x, y) lies on a

a. circle whose centre is at $\left(-\frac{1}{2}, -\frac{3}{2}\right)$.

b. straight line whose slope is $\frac{3}{2}$.

c. circle whose diameter is $\frac{\sqrt{5}}{2}$.

d. straight line whose slope is $-\frac{2}{3}$.

Answer: (c)

Solution:

$z = x + iy$

$$\frac{x + iy - 1}{2x + 2iy + i} = \frac{(x-1) + iy}{2x + i(2y+1)}\left(\frac{2x - i(2y+1)}{2x - i(2y+1)}\right)$$

$$\frac{2x(x-1) + y(2y+1)}{4x^2 + (2y+1)^2} = 1$$

$$2x^2 + 2y^2 - 2x + y = 4x^2 + 4y^2 + 4y + 1$$

$x^2 + y^2 + x + \frac{3}{2}y + \frac{1}{2} = 0$

Circle's centre will be $\left(-\frac{1}{2}, -\frac{3}{4}\right)$

Radius $= \sqrt{\frac{1}{4} + \frac{9}{16} - \frac{1}{2}} = \frac{\sqrt{5}}{4}$

Diameter$= \frac{\sqrt{5}}{2}$

5. If $f(a+b+1-x) = f(x)\ \forall\ x$, where a and b are fixed positive real numbers, then $\frac{1}{(a+b)}\int_a^b x(f(x) + f(x+1))\ dx$ is equal to

a. $\int_{a-1}^{b-1} f(x)\ dx$

b. $\int_{a+1}^{b+1} f(x+1)\ dx$

c. $\int_{a-1}^{b-1} f(x+1)\ dx$

d. $\int_{a+1}^{b+1} f(x)\ dx$

Answer: (c)

Solution:

$f(a+b+1-x) = f(x) \qquad (1)$

$x \to x+1$

$f(a+b-x) = f(x+1) \qquad (2)$

$I = \frac{1}{a+b}\int_a^b x(f(x) + f(x+1))dx \qquad (3)$

From (1) and (2)

$I = \frac{1}{a+b}\int_a^b (a+b-x)(f(x+1) + f(x))dx \qquad (4)$

Adding (3) and (4)

$$2I = \int_a^b (f(x) + f(x+1))dx$$

$$2I = \int_a^b f(x+1)dx + \int_a^b f(x)dx$$

$$2I = \int_a^b f(a+b-x+1)dx + \int_a^b f(x)dx$$

$$2I = 2\int_a^b f(x)dx$$

$$I = \int_a^b f(x)dx \qquad ; \quad x = t+1, dx = dt$$

$$I = \int_{a-1}^{b-1} f(t+1)dt$$

$$I = \int_{a-1}^{b-1} f(x+1)dx$$

6. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is

a. $2\sqrt{3}$
b. $\sqrt{3}$
c. $\frac{3}{\sqrt{2}}$
d. $3\sqrt{2}$

Answer: (d)

Solution:

Let the equation of ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b)$

Now $2ae=6$ & $\frac{2a}{e}=12$

$\Rightarrow ae=3$ & $\frac{a}{e}=6$

$\Rightarrow a^2=18$

$\Rightarrow a^2e^2=c^2=a^2-b^2=9$

$\Rightarrow b^2=9$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{\sqrt{18}}=3\sqrt{2}$

7. The logical statement $(p\Rightarrow q)\wedge(q\Rightarrow\sim p)$ is equivalent to

a. $\sim p$
b. p
c. q
d. $\sim q$

Answer: (a)

Solution:

p	q	$p\Rightarrow q$	$\sim p$	$q\Rightarrow\sim p$	$(p\Rightarrow q)\wedge(q\Rightarrow\sim p)$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

Clearly $(p\Rightarrow q)\wedge(q\Rightarrow\sim p)$ is equivalent to $\sim p$

8. The greatest positive integer k, for which 49^k+1 is a factor of the sum $49^{125}+49^{124}+\cdots+49^2+49+1$, is

a. 32
b. 60
c. 65
d. 63

Answer: (d)

Solution:

$1 + 49 + 49^2 + \cdots + 49^{125} = \frac{49^{126} - 1}{49 - 1}$

$= \frac{(49^{63} + 1)(49^{63} - 1)}{48}$

$= \frac{(49^{63} + 1)((1 + 48)^{63} - 1)}{48}$

$= \frac{(49^{63}+1)(1+48I-1)}{48}$; where I is an integer

= $(49^{63} + 1)I$

Greatest positive integer is $k = 63$

9. A vector $\vec{a} = \alpha\hat{\imath} + 2\hat{\jmath} + \beta\hat{k}$ $(\alpha, \beta \in \boldsymbol{R})$ lies in the plane of the vectors, $\vec{b} = \hat{\imath} + \hat{\jmath}$ and $\vec{c} = \hat{\imath} - \hat{\jmath} + 4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then

a. $\vec{a}.\hat{\imath} + 3 = 0$
b. $\vec{a}.\hat{k} + 4 = 0$
c. $\vec{a}.\hat{\imath} + 1 = 0$
d. $\vec{a}.\hat{k} + 2 = 0$

Answer: (BONUS)

Solution:

The angle bisector of vectors $\vec{b}$ and $\vec{c}$ is given by:

$$\vec{a} = \lambda(\hat{b} + \hat{c}) = \lambda\left(\frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}} + \frac{\hat{\imath} - \hat{\jmath} + 4\hat{k}}{3\sqrt{2}}\right) = \lambda\left(\frac{4\hat{\imath} + 2\hat{\jmath} + 4\hat{k}}{3\sqrt{2}}\right)$$

Comparing with $\vec{a} = \alpha\hat{\imath} + 2\hat{\jmath} + \beta\hat{k}$, we get

$$\frac{2\lambda}{3\sqrt{2}} = 2 \Rightarrow \lambda = 3\sqrt{2}$$

$\therefore \vec{a} = 4\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$

None of the options satisfy.

10. If $y(\alpha) = \sqrt{2\left(\frac{\tan\alpha + \cot\alpha}{1 + \tan^2\alpha}\right) + \frac{1}{\sin^2\alpha}}$ where $\alpha \in \left(\frac{3\pi}{4}, \pi\right)$, then $\frac{dy}{d\alpha}$ at $\alpha = \frac{5\pi}{6}$ is

a. $-\frac{1}{4}$
b. $\frac{4}{3}$
c. 4
d. -4

Answer: (c)

Solution:

$$y(\alpha) = \sqrt{2\left(\frac{\tan\alpha + \cot\alpha}{1 + \tan^2\alpha}\right) + \frac{1}{\sin^2\alpha}}$$

$$y(\alpha)=\sqrt{2\frac{1}{\sin\alpha\cos\alpha\times\frac{1}{\cos^2\alpha}}+\frac{1}{\sin^2\alpha}}$$

$$y(\alpha)=\sqrt{2\cot\alpha+\operatorname{cosec}^2\alpha}$$

$$y(\alpha)=\sqrt{(1+\cot\alpha)^2}$$

$$y(\alpha)=-1-\cot\alpha$$

$$\frac{dy}{d\alpha}=0+\operatorname{cosec}^2\alpha\Big|_{\alpha=\frac{5\pi}{6}}$$

$$\frac{dy}{d\alpha}=\operatorname{cosec}^2\frac{5\pi}{6}$$

$$\frac{dy}{d\alpha}=4$$

11. If $y=mx+4$ is a tangent to both the parabolas, $y^2=4x$ and $x^2=2by$, then b is equal to

a. -64
b. 128
c. -128
d. -32

Answer: (c)

Solution:

Any tangent to the parabola $y^2=4x$ is $y=mx+\frac{a}{m}$

Comparing it with $y=mx+4$, we get $\frac{1}{m}=4\Rightarrow m=\frac{1}{4}$

Equation of tangent becomes $y=\frac{x}{4}+4$

$y=\frac{x}{4}+4$ is a tangent to $x^2=2by$

$\Rightarrow x^2=2b\left(\frac{x}{4}+4\right)$

Or $2x^2-bx-16b=0$,

$D=0$

$b^2+128b=0$,

$\Rightarrow b=0$ (not possible),

$\Rightarrow b=-128$

12. Let α be a root of the equation $x^2+x+1=0$ and the matrix $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1&1&1\\1&\alpha&\alpha^2\\1&\alpha^2&\alpha^4\end{bmatrix}$, then the matrix A^{31} is equal to

a. A
b. A^2
c. A^3
d. I_3

Answer: (c)

Solution:

The roots of equation $x^2+x+1=0$ are complex cube roots of unity.

$\therefore \alpha = \omega$ or ω^2

$$A = \frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\1 & \alpha & \alpha^2\\1 & \alpha^2 & \alpha^4\end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\1 & \omega & \omega^2\\1 & \omega^2 & \omega\end{bmatrix}$$

$$A^2 = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\1 & \omega & \omega^2\\1 & \omega^2 & \omega\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\1 & \omega & \omega^2\\1 & \omega^2 & \omega\end{bmatrix}$$

$$A^2 = \frac{1}{3}\begin{bmatrix}3 & 0 & 0\\0 & 0 & 3\\0 & 3 & 0\end{bmatrix}$$

$$A^2 = \begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}$$

$$A^4 = \begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}$$

$$A^4 = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = I$$

$A^4 = I$

$A^{28} = I$

Therefore, we get

$A^{31} = A^{28}A^3$

$A^{31} = IA^3$

$A^{31} = A^3$

13. If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to

a. $-\frac{3}{2}$

b. $-\frac{1}{2}$

c. $\frac{1}{2}$

d. $\frac{3}{2}$

Answer: (b)

Solution:

$g(x) = x^2 + x - 1$

$gof(x) = 4x^2 - 10x + 5$

$g\big(f(x)\big) = 4x^2 - 10x + 5$

$f^2(x) + f(x) - 1 = 4x^2 - 10x + 5$

Putting $x = \frac{5}{4}$ & $f\left(\frac{5}{4}\right) = t$

$$t^2 + t + \frac{1}{4} = 0$$

$t = -\frac{1}{2}$ or $f\left(\frac{5}{4}\right) = -\frac{1}{2}$

14. Let α and β are two real roots of the equation $(k+1)\tan^2 x - \sqrt{2}\,\lambda\tan x = 1-k$, where $(k \neq -1)$ and λ are real numbers. If $\tan^2(\alpha+\beta) = 50$, then value of λ is

a. $5\sqrt{2}$
b. $10\sqrt{2}$
c. 10
d. 5

Answer: (c)

Solution:

$(k+1)\tan^2 x - \sqrt{2}\lambda\tan x = 1-k$
$\tan^2(\alpha+\beta) = 50$
$\because \tan\alpha$ and $\tan\beta$ are the roots of the given equation.
Now,

$$\tan\alpha + \tan\beta = \frac{\sqrt{2}\lambda}{k+1}, \qquad \tan\alpha\tan\beta = \frac{k-1}{k+1}$$

$$\Rightarrow \left(\frac{\frac{\sqrt{2}\lambda}{k+1}}{1-\frac{k-1}{k+1}}\right)^2 = 50$$

$$\Rightarrow \frac{2\lambda^2}{4} = 50$$

$\Rightarrow \lambda^2 = 100$
$\Rightarrow \lambda = \pm 10$

15. Let P be a plane passing through the points $(2,1,0)$, $(4,1,1)$ and $(5,0,1)$ and R be any point $(2,1,6)$. Then the image of R in the plane P is:

a. $(6,5,2)$
b. $(6,5,-2)$
c. $(4,3,2)$
d. $(3,4,-2)$

Answer: (b)

Solution:

Points $A(2,1,0)$, $B(4,1,1)$ $C(5,0,1)$
$\overrightarrow{AB} = (2,0,1)$
$\overrightarrow{AC} = (3,-1,1)$
$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = (1,1,-2)$
Equation of the plane is $x + y - 2z = 3 \ldots (1)$
Let the image of point $(2,1,6)$ is (l,m,n)

$$\frac{l-2}{1} = \frac{m-1}{1} = \frac{n-6}{-2} = \frac{-2(-12)}{6} = 4$$

$\Rightarrow l = 6, m = 5, n = -2$
Hence the image of R in the plane P is $(6,5,-2)$

16. Let $x^k + y^k = a^k$, $(a, k > 0)$ and $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then k is

a. $\frac{1}{3}$

b. $\frac{3}{2}$

c. $\frac{2}{3}$

d. $\frac{4}{3}$

Answer: (c)

Solution:

$x^k + y^k = a^k$

$kx^{k-1} + ky^{k-1}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{k-1}$

$\Rightarrow \frac{dy}{dx} + \left(\frac{y}{x}\right)^{1-k} = 0$

$\Rightarrow 1 - k = \frac{1}{3}$

$\Rightarrow k = \frac{2}{3}$

17. Let the function, $f: [-7, 0] \to \mathbf{R}$ be continuous on $[-7, 0]$ and differentiable on $(-7, 0)$. If $f(-7) = -3$ and $f'(x) \leq 2$, for all $x \in (-7, 0)$, then for all such functions f, $f(-1) + f(0)$ lies in the interval:

a. $[-6, 20]$

b. $(-\infty, 20]$

c. $(-\infty, 11]$

d. $[-3, 11]$

Answer: (b)

Solution:

$f(-7) = -3$ and $f'(x) \leq 2$

Applying LMVT in $[-7,0]$, we get

$\frac{f(-7) - f(0)}{-7} = f'(c) \leq 2$

$\frac{-3 - f(0)}{-7} \leq 2$

$f(0) + 3 \leq 14$

$f(0) \leq 11$

Applying LMVT in $[-7, -1]$, we get

$\frac{f(-7) - f(-1)}{-7 + 1} = f'(c) \leq 2$

$\frac{-3 - f(-1)}{-6} \leq 2$

$f(-1) + 3 \leq 12$

$f(-1) \leq 9$

Therefore, $f(-1) + f(0) \leq 20$

18. If $y = y(x)$ is the solution of the differential equation, $e^y\left(\frac{dy}{dx} - 1\right) = e^x$ such that $y(0) = 0$, then $y(1)$ is equal to

a. $\log_e 2$
b. $2e$
c. $2 + \log_e 2$
d. $1 + \log_e 2$

Answer: (d)

Solution:

$e^y(y' - 1) = e^x$

$\Rightarrow \frac{dy}{dx} = e^{x-y} + 1$

Let $x - y = t$

$1 - \frac{dy}{dx} = \frac{dt}{dx}$

So, we can write

$\Rightarrow 1 - \frac{dt}{dx} = e^t + 1$

$\Rightarrow -e^{-t}\, dt = dx$

$\Rightarrow e^{-t} = x + c$

$\Rightarrow e^{y-x} = x + c$

$1 = 0 + c$

$\Rightarrow e^{y-x} = x + 1$

at $x = 1$

$\Rightarrow e^{y-1} = 2$

$\Rightarrow y = 1 + \log_2 2$

19. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is

a. 16
b. 27
c. 7
d. $\frac{21}{2}$

Answer: (a)

Solution:

Let 5 numbers be $a - 2d, a - d, a, a + d, a + 2d$

$5a = 25$

$a = 5$

$(a - 2d)(a - d)a(a + d)(a + 2d) = 2520$

$(25 - 4d^2)(25 - d^2) = 504$

$4d^4 - 125d^2 + 121 = 0$

$4d^4 - 4d^2 - 121d^2 + 121 = 0$

$d^2 = 1 \; or \; d^2 = \frac{121}{4}$

$d = \pm\frac{11}{2}$

For $d = \frac{11}{2}, a + 2d$ is the greatest term, $a + 2d = 5 + 11 = 16$

20. If the system of linear equations

$2x + 2ay + az = 0$

$2x + 3by + bz = 0$

$2x + 4cy + cz = 0,$

where $a, b, c \in \mathbf{R}$ are non-zero and distinct; has non-zero solution, then

a. $a + b + c = 0$

b. a, b, c are in A.P.

c. $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

d. a, b, c are in G.P.

Answer: (c)

Solution:

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 2 & 2a & a \\ 0 & 3b - 2a & b - a \\ 0 & 4c - 2a & c - a \end{vmatrix} = 0$$

$\Rightarrow (3b - 2a)(c - a) - (4c - 2a)(b - a) = 0$

$\Rightarrow 3bc - 2ac - 3ab + 2a^2 - [4bc - 4ac - 2ab + 2a^2] = 0$

$\Rightarrow -bc + 2ac - ab = 0$

$\Rightarrow ab + bc = 2ac$

$\Rightarrow \frac{1}{c} + \frac{1}{a} = \frac{2}{b}$

21. $\lim\limits_{x \to 2} \frac{3^x + 3^{3-x} - 12}{3^{\frac{-x}{2}} - 3^{1-x}}$ is equal to ___________

Answer: (36)

Solution:

$$\lim_{x \to 2} \frac{3^x + \frac{27}{3^x} - 12}{\frac{1}{3^{\frac{x}{2}}} - \frac{3}{3^x}}$$

Put $3^{\frac{x}{2}} = t$

$$\lim_{t\to 3}\frac{t^2+\frac{27}{t^2}-12}{-\frac{3}{t^2}+\frac{1}{t}} = \lim_{t\to 3}\frac{(t^2-9)(t^2-3)}{(t-3)} = \lim_{t\to 3}(t^2-3)(t+3) = 36$$

22. If variance of first n natural numbers is 10 and variance of first m even natural numbers is 16, $m + n$ is equal to________.

Answer: (18)

Solution:

For n natural number variance is given by

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

$$\frac{\sum x_i^2}{n} = \frac{1^2+2^2+3^2+\ldots . n\ term}{n} = \frac{n(n+1)(2n+1)}{6n}$$

$$\frac{\sum x_i}{n} = \frac{1+2+3+\ldots . n\ terms}{n} = \frac{n(n+1)}{2n}$$

$$\sigma^2 = \frac{n^2-1}{12} = 10 \text{ (given)}$$

$\Rightarrow n = 11$

Variance of $(2, 4, 6 \ldots)$ = $4 \times$ variance of $(1, 2, 3, 4 \ldots) = 4\times\frac{m^2-1}{12} = \frac{m^2-1}{3} = 16$ (given)

$\Rightarrow m = 7$

Therefore, $n + m = 11 + 7 = 18$

23. If the sum of the coefficients of all even powers of x in the product $(1 + x + x^2 + x^3 \ldots . +x^{2n})(1 - x + x^2 - x^3 \ldots . + x^{2n})$ is 61, then n is equal to ________

Answer: (30)

Solution:

Let $(1 + x + x^2 + \cdots + x^{2n})(1 - x + x^2 - \cdots + x^{2n}) = a_o + a_1x + a_2x^2 + \cdots$

Put $x = 1$

$2n + 1 = a_o + a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots$ (1)

Put $x = -1$

$2n + 1 = a_o - a_1 + a_2 - a_3 + \ldots\ldots\ldots\ldots$ (2)

Add (1) and (2)

$2(2n + 1) = 2(a_o + a_2 + a_4 + \ldots\ldots\ldots\ldots$

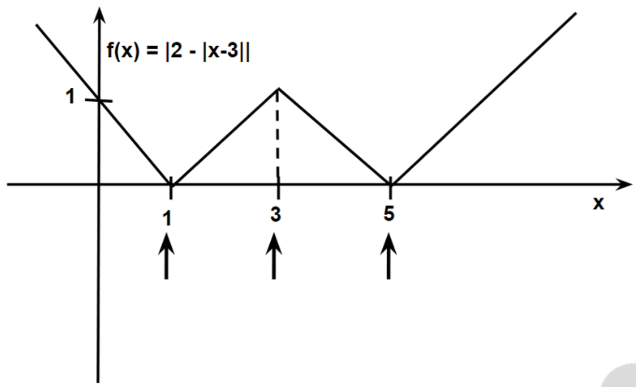
$2n + 1 = 61$

$n = 30$

24. Let S be the set of points where the function, $f(x) = \left|2 - |x-3|\right|, x \in \mathbf{R}$, is not differentiable. Then, the value of $\sum_{x \in S} f(f(x))$ is equal to ________.

Answer: (3)

Solution:



There will be three points $x = 1, 3, 5$ at which $f(x)$ is non-differentiable.

So $f(f(1)) + f(f(3)) + f(f(5))$

$= f(0) + f(2) + f(0)$

$= 1 + 1 + 1$

$= 3$

25. Let $A(1,0)$, $B(6,2)$, $C\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line the segment PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$, is ___________

Answer: (5)

Solution:

P is the centroid which is $\equiv \left(\frac{1+6+\frac{3}{2}}{3}, \frac{1+5+2}{3}\right)$

$P = \left(\frac{17}{6}, \frac{8}{3}\right)$

$Q = \left(-\frac{7}{6}, -\frac{1}{3}\right)$

$PQ = \sqrt{(4)^2 + (3)^2} = 5$