JEE Main 2020 Paper

Date : 3rd September 2020

Time : 02 : 00 pm - 05 : 00 pm

Subject : Maths

Q.1 If $x^3dy+xy\,dx=x^2dy+2y\,dx$; $y(2)=e$ and $x>1$, then $y(4)$ is equal to:

(1) $\frac{\sqrt{e}}{2}$ (2) $\frac{3}{2}\sqrt{e}$ (3) $\frac{1}{2}+\sqrt{e}$ (4) $\frac{3}{2}+\sqrt{e}$

Sol. **2**

$(x^3 - x^2)dy = (2 - x)\,ydx$

$$\int\frac{dy}{y}=\int\frac{2-x}{x^2(x-1)}dx$$

$$\int\frac{dy}{y}=-\int\frac{x-2}{x^2(x-1)}dx$$

$$\int\frac{dy}{y}=-\int\left(\frac{p}{x}+\frac{q}{x^2}+\frac{r}{x-1}\right)dx$$

$p=1,\ q=2,\ r=-1$

$$\int\frac{dy}{y}=-\int\frac{1}{x}dx-\int\frac{2}{x^2}dx-\int\frac{(-1)}{x}dx$$

$$\ln|y| = -\ln x+\frac{2}{x}+\ln(x-1)+c.$$

$x = 2,\ y = e$

$1 = 1 - \ln 2 + c \Rightarrow c = \ln 2$

$$\ln|y| = \frac{2}{x} - \ln|x| + \ln|x - 1| + \ln 2$$

put $x = 4$

$$\ln|y| = \frac{1}{2} - 2\ln 2 + \ln 3 + \ln 2$$

$$\ln y = \ln\left(\frac{3}{2}\right)+\frac{1}{2}$$

$$y = \frac{3}{2}\cdot e^{\frac{1}{2}} = \frac{3}{2}\sqrt{e}$$

Q.2 Let A be a 3×3 matrix such that $\text{adj}\,A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$ and B=adj(adj A).

If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to:

(1) $\left(9, \frac{1}{81}\right)$ (2) $\left(9, \frac{1}{9}\right)$ (3) $\left(3, \frac{1}{81}\right)$ (4) (3, 81)

Sol. **3**

adjA = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$ $\Rightarrow$ |adjA| = 9

$\Rightarrow |A|^2 = 9 \Rightarrow |A| = \pm 3 = |\lambda|$

$\det(\text{adj}(\text{adj}A)) = (|A|)^{(n-1)^2}$

$|B| = (|A|)^{(3-1)^2} = (|A|)^4 = 3^4 = 81$

$|(B^T)^{-1}| = \frac{1}{|B^T|} = \frac{1}{|B|} = \frac{1}{81} = \mu$

$|\lambda|, \mu = \left(3, \frac{1}{81}\right)$

Q.3 Let a,b,c $\in R$ be such that $a^2+b^2+c^2=1$, If $a\cos\theta = b\cos\left(\theta + \frac{2\pi}{3}\right) = c\cos\left(\theta + \frac{4\pi}{3}\right)$, where $\theta = \frac{\pi}{9}$, then the angle between the vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{2\pi}{3}$ (3) $\frac{\pi}{9}$ (4) 0

Sol. **1**

$\cos\alpha = \frac{\vec{p}.\vec{q}}{|p||q|} = \frac{ab+bc+ca}{a^2+b^2+c^2} = \frac{ab+bc+ca}{1}$

$\cos\alpha = ab + bc + ca$

$\cos\alpha = abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ .....(1)

$a\cos 20° = b\cos(140°) = c\cos(260°) = \lambda$

$\Rightarrow \frac{1}{a}=\frac{\cos 20°}{\lambda}, \frac{1}{b}=\frac{\cos(140°)}{\lambda}, \frac{1}{c}=\frac{\cos(260°)}{\lambda}$ put in eq. (1)

$\Rightarrow \cos\alpha = \frac{abc}{\lambda}(\cos 20° + \cos 140° + \cos 260°)$

$\Rightarrow \cos\alpha = \frac{abc}{\lambda}(\cos 20° + 2\cos 200°. \cos 60°)$

$\Rightarrow \cos\alpha = \frac{abc}{\lambda}(\cos 20° - \cos 20°)$

$\cos\alpha = 0$

$\Rightarrow \alpha = \frac{\pi}{2}$

Q.4 Suppose f(x) is a polynomial of degree four, having critical points at −1,0,1. If $T = \{x \in R \mid f(x) = f(0)\}$, then the sum of squares of all the elements of T is:

(1) 6 (2) 2 (3) 8 (4) 4

Sol. **4**

f′(x) = k (x + 1)x(x-1)

f′(x) = k [x^3 - x]

Integrating both sides

$f(x) = k\left[\frac{x^4}{4} - \frac{x^2}{2}\right] + c$

f(0) = c

$f(x) = f(0) \Rightarrow k\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + c = c$

$\Rightarrow k\frac{x^2}{4}\left(x^2 - 2\right) = 0$

$\Rightarrow x = 0, \pm\sqrt{2}$

sum of all of squares of elements = $0^2 + (\sqrt{2})^2 + (-\sqrt{2})^2 = 4$

Q.5 If the value of the integral $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}}dx$ is $\frac{k}{6}$, then k is equal to:

(1) $2\sqrt{3}+\pi$ (2) $3\sqrt{2}+\pi$ (3) $3\sqrt{2}-\pi$ (4) $2\sqrt{3}-\pi$

Sol. **4**

$\int_0^{\frac{1}{2}} \frac{x^2}{\left(1-x^2\right)^{\frac{3}{2}}}dx$

x = sinθ

$$\int_0^{\frac{\pi}{6}} \frac{\sin^2\theta}{\cos^3\theta}.\cos\theta d\theta$$

$$\int_0^{\frac{\pi}{6}} \tan^2\theta d\theta = \left[\tan\theta - \theta\right]_0^{\frac{\pi}{6}}$$

$$\Rightarrow \left(\frac{1}{\sqrt{3}} - \frac{\pi}{6}\right) = \frac{k}{6}$$

$$\frac{2\sqrt{3}-\pi}{6} = \frac{k}{6}$$

$k = 2\sqrt{3} - \pi$

Q.6 If the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k, then 18 k is equal to:

(1) 5 (2) 9 (3) 7 (4) 11

Sol. 3

$$T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(\frac{-1}{3x}\right)^r$$

$$= {}^9C_r \frac{3^{9-2r}}{2^{9-r}} (-1)^r .x^{18-3r}$$

$18 - 3r = 0$

$\Rightarrow r=6$

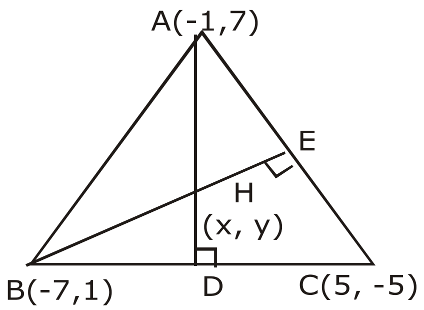
$$= {}^9C_r \left(\frac{3^{-3}}{2^3}\right) = k$$

$$= \frac{7}{18} = k \Rightarrow 18k = 7$$

7. If a ΔABC has vertices A(−1,7), B(−7,1) and C(5,−5), then its orthocentre has coordinates:

(1) (−3,3) (2) $\left(-\frac{3}{5}, \frac{3}{5}\right)$ (3) $\left(\frac{3}{5}, -\frac{3}{5}\right)$ (4) (3,−3)

Sol. 1



$m_{AH}.\ m_{BC}=-1$

$\Rightarrow \left(\frac{y-7}{x+1}\right)\left(\frac{1+5}{-7-5}\right)=-1$

$\Rightarrow 2x-y+9=0$ ....(1)

and $m_{BH}.\ m_{AC}=-1$

$\Rightarrow \left(\frac{y-1}{x+7}\right)\left(\frac{7-(-5)}{-1-5}\right)=-1$

$\Rightarrow x-2y+9=0$ .....(2)

Solving equation (1) and (2) we get

$(x, y) \equiv (-3,3)$

Q.8. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ (b<5) and the hyperbola, $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ respectively satisfying $e_1e_2=1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α,β) is equal to:

(1) (8,12) (2) $\left(\frac{24}{5},10\right)$ (3) $\left(\frac{20}{3},12\right)$ (4) (8,10)

Sol. 4

$\left.\begin{aligned}\alpha &= 10e_1\\ \beta &= 8e_2\end{aligned}\right]$

$\left.\begin{aligned}b^2 &= 25\left(1-e_1^2\right)\\ b^2 &= 16\left(e_2^2-1\right)\end{aligned}\right]$

$(e_1e_2)^2 = 1$

$\left(1-\frac{b^2}{25}\right)\left(1+\frac{b^2}{16}\right)=1$

$\Rightarrow 1+\frac{b^2}{16}-\frac{b^2}{25}-\frac{b^4}{400}=1$

$\Rightarrow \frac{9b^2}{16.25} - \frac{b^4}{400} = 0$

$\Rightarrow 9b^2 - b^4 = 0$

$\Rightarrow b^2(9 - b^2) = 0, \quad b \neq 0$

Therefore, $9 - b^2 = 0$
$b^2 = 9$

$$\left.\begin{matrix} e_1 = \frac{4}{5} \\ e_2 = \frac{5}{4} \end{matrix}\right] = \left.\begin{matrix} \alpha = 2ae_1 = 10 \times \frac{4}{5} = 8 \\ \beta = 2ae_2 = 8 \times \frac{5}{4} = 10 \end{matrix}\right] = (\alpha, \beta) = (8, 10)$$

Q.9 If z_1, z_2 are complex numbers such that $Re(z_1) = |z_1 - 1|$, $Re(z_2) = |z_2 - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $Im(z_1 + z_2)$ is equal to:

(1) $2\sqrt{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{\sqrt{3}}{2}$

Sol. **1**

$z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$
$x_1^2 = (x_1 - 1)^2 + y_1^2$ .....(1)
$\Rightarrow y_1^2 - 2x_1 + 1 = 0$
$x_2^2 = (x_2 - 1)^2 + y_2^2$
$y_2^2 - 2x_2 + 1 = 0$ ....(2)
from equation (1) – (2)
$(y_1^2 - y_2^2) + 2(x_2 - x_1) = 0$
$(y_1 + y_2)(y_1 - y_2) = 2(x_1 - x_2)$

$y_1 + y_2 = 2\left(\frac{x_1 - x_2}{y_1 - y_2}\right)$

$\arg(z_1 - z_2) = \frac{\pi}{6}$

$\tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{6}$

$\Rightarrow \frac{y_1 - y_2}{x_1 - x_2} = \frac{1}{\sqrt{3}}$

$\therefore y_1 + y_2 = 2\sqrt{3}$

Q.10 The set of all real values of λ for which the quadratic equations, $\left(\lambda^2+1\right)x^2-4\lambda x+2=0$ always have exactly one root in the interval (0,1) is:

(1) (–3,–1) (2) (2,4] (3) (1,3] (4) (0,2)

Sol. 3

$f(0)\ f(1) \le 0$

$\Rightarrow (2)\ [\lambda^2 - 4\lambda + 3] \le 0$

$(\lambda - 1)\ (\lambda - 3) \le 0$

$\Rightarrow \lambda \in [1, 3]$

at $\lambda = 1$

$2x^2 - 4x + 2 = 0$

$\Rightarrow (x - 1)^2 = 0$

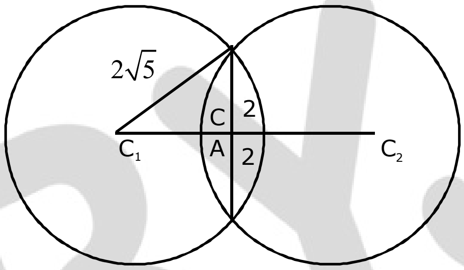
$x = 1, 1$

$\therefore \lambda \in (1, 3]$

Q.11 Let the latus ractum of the parabola $y^2=4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is:

(1) 8 (2) $8\sqrt{5}$ (3) $4\sqrt{5}$ (4) 12

Sol. 1



$C_1C_2 = 2C_1A$

$(C_1A)^2 + 4 = (2\sqrt{5})^2$

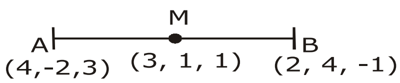
$C_1A = 4$

$C_1C_2 = 8$

Q.12 The plane which bisects the line joining the points (4,–2,3) and (2,4,–1) at right angles also passes through the point:

(1) (0,–1,1) (2) (4,0,1) (3) (4,0,–1) (4) (0,1,–1)

Sol. 3



$a = 2,\ b = -6$

$c = 4$

equation of plane

$2(x - 3) + (-6)\ (y - 1) + 4(z - 1) = 0$

$\Rightarrow 2x - 6y + 4z = 4$

passes through (4, 0, -1)

Q.13 $\lim_{x\to a}\frac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}}(a\neq 0)$ is equal to :

(1) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$ (2) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$ (3) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$ (4) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

Sol. **3**

Apply L-H Rule

$$\lim_{x\to a}\frac{\frac{2}{3}(a+2x)^{\frac{-2}{3}}-3^{\frac{1}{3}}.\frac{1}{3}x^{-\frac{2}{3}}}{\frac{1}{3}(3a+x)^{\frac{-2}{3}}-4^{\frac{1}{3}}.\frac{1}{3}x^{-\frac{2}{3}}}$$

$$\Rightarrow\frac{\frac{2}{3}(3a)^{\frac{-2}{3}}-\frac{1}{3^{\frac{2}{3}}}.\left(a^{-\frac{2}{3}}\right)}{\frac{1}{3}(4a)^{\frac{-2}{3}}-\frac{1}{3}.4^{\frac{1}{3}}\left(a^{-\frac{2}{3}}\right)}$$

$$=\frac{2}{3}.\left(\frac{2}{9}\right)^{\frac{1}{3}}$$

Q.14 Let $x_i(1\le i\le 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10}(x_i-p)=3$ and $\sum_{i=1}^{10}(x_i-p)^2=9$ where $0\neq p\in R$, then the standard deviation of these observations is :

(1) $\frac{7}{10}$ (2) $\frac{9}{10}$ (3) $\sqrt{\frac{3}{5}}$ (4) $\frac{4}{5}$

Sol. **2**

Standard deviation
is free from shifting
of origin

S . D = $\sqrt{\text{variance}}$

$$=\sqrt{\frac{9}{10}-\left(\frac{3}{10}\right)^2}$$

$$=\sqrt{\frac{9}{10}-\frac{9}{100}}=\sqrt{\frac{81}{100}}=\frac{9}{10}$$

Q.15 The probability that a randomly chosen 5–digit number is made from exactly two digits is :

(1) $\frac{134}{10^4}$ (2) $\frac{121}{10^4}$ (3) $\frac{135}{10^4}$ (4) $\frac{150}{10^4}$

Sol. **3**

Total case = $9(10^4)$

First case : Choose two non-zero digits = ${}^9C_2$

Now, number of 5-digit numbers containing both digits = $2^5 - 2$

Second case : Choose one non-zero & one zero as digit = ${}^9C_1$

Number of 5-digit numbers containing one non-zero & one zero both = $2^4 - 1$.

fav. case = ${}^9C_2\,(2^5 - 2) + {}^9C_1\,(2^4 - 1)$

= 1080 + 135 = 1215

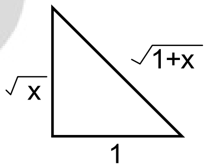
Prob = $\frac{1215}{9 \times 10^4} = \frac{135}{10^4}$

Q.16 If $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right)dx = A(x)\tan^{-1}\left(\sqrt{x}\right) + B(x) + C$, where C is a constant of integration, then the ordered pair (A(x),B(x)) can be:

(1) $\left(x+1, -\sqrt{x}\right)$ (2) $\left(x-1, -\sqrt{x}\right)$ (3) $\left(x+1, \sqrt{x}\right)$ (4) $\left(x-1, \sqrt{x}\right)$

Sol. **1**

$$\int \sin^{-1}\sqrt{\frac{x}{1+x}}\,dx$$



$$\int \underset{I}{\tan^{-1}\sqrt{x}}.\ \underset{II}{1}\,dx$$

$$\left(\tan^{-1}\sqrt{x}\right).x - \int \frac{x}{1+x}.\frac{1}{2\sqrt{x}}\,dx$$

put $x = t^2 \Rightarrow dx = 2t\,dt$

$$= x\tan^{-1}\sqrt{x} - \int \frac{\left(t^2\right)\left(2t\,dt\right)}{\left(1+t^2\right)\left(2t\right)}$$

$= x\tan^{-1}\sqrt{x} - t + \tan^{-1}t + c$

$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + c$

$A(x) = x + 1,\ B(x) = -\sqrt{x}$

Q.17 If the sum of the series $20+19\frac{3}{5}+19\frac{1}{5}+18\frac{4}{5}+\ldots$ upto n^{th} term is 488 and the n^{th} term is negative, then:

(1) n=60 (2) n=41 (3) n^{th} term is -4 (4) n^{th} term is $-4\frac{2}{5}$

Sol. **3**

$$20+\frac{98}{5}+\frac{96}{5}+\ldots\ldots$$

$S_n = 488$

$$\Rightarrow \frac{n}{2}\left[2\times 20+(n-1)\left(\frac{-2}{5}\right)\right] = 488$$

$$\Rightarrow 20n - \frac{n^2}{5}+\frac{n}{5} = 488$$

$\Rightarrow 100n - n^2 + n = 2440$
$= n^2 - 101n + 2440 = 0$
$\Rightarrow n = 61$ or 40

for $n = 40$, $T_n = 20 + 39\left(\frac{-2}{5}\right) = +ve$

$n = 61$, $T_n = 20 + 60\left(\frac{-2}{5}\right) = 20 - 24 = -4$

Q.18 Let p, q, r be three statements such that the truth value of $(p\wedge q)\rightarrow(\sim q\vee r)$ is F. Then the truth values of p, q, r are respectively :

(1) F, T, F (2) T, F, T (3) T, T, F (4) T, T, T

Sol. **3**

$(p \wedge q) \rightarrow (\sim q \vee r)$
Possible when
$p \wedge q \rightarrow T$
$\sim q \vee r \rightarrow F$

$$\left.\begin{array}{l} p\rightarrow T \\ q\rightarrow T \\ r\rightarrow F \end{array}\right] \qquad \begin{array}{l} p\wedge q \Rightarrow T \\ \sim q\vee r \rightarrow F\vee F\Rightarrow F \\ T\rightarrow F\Rightarrow F \end{array}$$

Q.19 If the surface area of a cube is increasing at a rate of 3.6 cm^2/sec, retaining its shape; then the rate of change of its volume (in cm^3/sec), when the length of a side of the cube is 10cm, is :

(1) 9 (2) 10 (3) 18 (4) 20

Sol. **1**

$A = 6a^2$

$a \to$ side of cube

$$\frac{dA}{dt} = 6\left(2a\frac{da}{dt}\right) \Rightarrow 3.6 = 12 \times 10\ \frac{da}{dt} \Rightarrow \frac{da}{dt} = \frac{3}{100}$$

$v = a^3$

$$\frac{dV}{dt} = 3a^2\frac{da}{dt}$$

$$= 3 \times 100 \times \frac{3}{100}$$

$= 9cm^3$ / sec

Q.20 Let R_1 and R_2 be two relations defined as follows:

$R_1 = \{(a,b) \in R^2 : a^2 + b^2 \in Q\}$ and

$R_2 = \{(a,b) \in R^2 : a^2 + b^2 \notin Q\}$, where Q is the set of all rational numbers. Then :

(1) R_1 is transitive but R_2 is not transitive
(2) R_1 and R_2 are both transitive
(3) R_2 is transitive but R_1 is not transitive
(4) Neither R_1 nor R_2 is transitive

Sol. **4**

for R_1

Let $a = 1 + \sqrt{2}$, $b = 1 - \sqrt{2}$, $c = 8^{\frac{1}{4}}$

$aR_1b \quad a^2 + b^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 6 \in Q$

$bR_1c \quad b^2 + c^2 = (1 - \sqrt{2})^2 + \left(8^{\frac{1}{4}}\right)^2 = 3 \in Q$

$aR_1c \Rightarrow a^2+c^2 = \left(1+\sqrt{2}\right) + \left(8^{1/4}\right)^2 = 3 + 4\sqrt{2} \notin Q$

R_1 is not transitive

R_2

let $a = 1 + \sqrt{2}$, $b = \sqrt{2}$, $c = 1- \sqrt{2}$

$aR_2b \quad a^2 + b^2 = 5 + 2\sqrt{2} \notin Q$

$bR_2c \quad b^2 + c^2 = 5 - 2\sqrt{2} \notin Q$

$aR_2c \quad a^2 + c^2 = 6 \in Q$

R_2 is not transitive

Q.21 If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th A.M. is equal to 2nd G.M., then m is equal to___

Sol. **39**

3, ..., 243 (m A.M.)

$$d = \frac{b-a}{n+1} = \frac{243-3}{m+1} = \frac{240}{m+1}$$

4tn A.M = 3 + 4d = $3 + 4\left(\frac{240}{m+1}\right)$

$$3 + \frac{960}{m+1} = 27$$

$$= \frac{960}{m+1} = 24$$

$\Rightarrow$ m = 39

3,............,243 (3 G.M)

243 = $3(r)^4$

r = 3

2nd G.M. = ar^2 = 27

Q.22 Let a plane P contain two lines $\vec{r} = \hat{i} + \lambda(\hat{i}+\hat{j}), \lambda \in R$ and $\vec{r} = -\hat{j} + \mu(\hat{j}-\hat{k}), \mu \in R$.

If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn from the point M(1,0,1) to P, then $3(\alpha+\beta+\gamma)$ equals _____

Sol. **5**

$$\left.\begin{array}{l}\vec{r} = \hat{i} + \lambda(\hat{i}+\hat{j}) \\ \vec{r} = -\hat{j} + \mu(\hat{j}-\hat{k})\end{array}\right]$$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$$

= (-1, 1 ,1)

equation of plane

-1(x - 1) + 1(y - 0) + 1(z - 0) = 0

$\Rightarrow$ x - y - z - 1 = 0

foot of $\perp^r$ from m(1, 0, 1)

$$\frac{x-1}{1} = \frac{y-0}{-1} = \frac{z-1}{-1} = -\frac{(1-0-1-1)}{3}$$

$$x - 1 = \frac{1}{3} \quad \left|\frac{y}{-1} = \frac{1}{3}\right| \quad = \frac{z-1}{-1} = \frac{1}{3}$$

$$x = \frac{4}{3}, y = \frac{-1}{3}, z = \frac{2}{3}$$

$$\Rightarrow \left.\begin{array}{l}\alpha = \frac{4}{3} \\ \beta = \frac{-1}{3} \\ \gamma = \frac{2}{3}\end{array}\right]$$

$\alpha + \beta + \gamma = \frac{4}{3} - \frac{1}{3} + \frac{2}{3} = \frac{5}{3}$

$3(\alpha + \beta + \gamma) = 5$

Q.23 Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x - 2y + 5z = 0$

$-2x + 4y + z = 0$

$-7x + 14y + 9z = 0$

such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to ___

Sol. **8**

$x - 2y + 5z = 0$ .....(1)

$-2x + 4y + z = 0$ ....(2)

$-7x + 14y + 9z = 0$ .....(3)

2.(1) + (2) we get z = 0, x = 2y

$15 \le 4y^2 + y^2 \le 150$

$\Rightarrow 3 \le y^2 \le 30$

$y \in \left[-\sqrt{30}, -\sqrt{3}\right] \cup \left[\sqrt{3}, \sqrt{30}\right]$

y = ±2, ±3, ±4, ±5

number of elements in S is 8.

Q.24 The total number of 3–digit numbers, whose sum of digits is 10, is ____

Sol. **54**

Let xyz be 3 digit number

$x + y + z = 10$ where $x \ge 1$, $y \ge 0$, $z \ge 0$

$\Rightarrow t + y + z = 9$ $\left.\begin{matrix} x - 1 \ge 0 \\ t \ge 0 \end{matrix}\right] x - 1 = t$

${}^{9+3-1}C_{3-1} = {}^{11}C_2 = 55$

but for t = 9, x = 10 not possible

total numbers = 55 - 1 = 54

Q.25 If the tangent to the curve, $y=e^x$ at a point (c,e^C) and the normal to the parabola, $y^2=4x$ at the point (1,2) intersect at the same point on the x-axis, then the value of c is ____

Sol. **4**

Tangent at (c, e^c) $y - e^c = e^c(x - c)$ ....(1)

normal to parabola $y - 2 = -1(x - 1)$

$x + y = 3$ ...(2)

at x-axis y = 0 | at x-axis y = 0

in (1), x = c - 1 | in (2), x = 3

$c - 1 = 3 \Rightarrow c = 4$